

A Study of Mathematical Game Theory

Minal Suhas Chandwadkar

SY BSc (Computer Science),

Ashoka Center for Business and Computer Studies,

Nashik, India

Abstract :

Game theory, although we play games we never thought of this is also related to mathematics. Game theory is a branch of mathematics which deal with mathematical models which are used for strategic interaction among rational decision-makers. Game Theory has applications in all fields like social science, logic, economics, psychology, and also in computer science. Games like chess, blackjack, and poker are some examples. Also, there are some situations where one has to decide what to do in that situation within a framework of rules. Examples like auctions, negotiations between countries, and military decisions. This paper will define what a game theory is and also with some examples of games. Next, introduce a concept called Nash equilibrium, which is a situation of a game in which no player has the motivation to change his approach after examining the strategies of other players.

Keywords - Decision, Game, Game Theory, Nash Equilibrium, Rational

I. INTRODUCTION

Game :

Game is object of game theory. Complete set of rules that describes a game. A player is instance of game. Mostly games played for entertainment, enjoyment and fun. Mostly games like chess, judo and squash which has two players or teams and only one winner is announced. Games are not usually taken as serious but due to games is it easy to understand how conflicts and corporation in teams or team players.

Game Theory :

Game Theory is approach of predicting the decisions of players. It is logical analysis of many situations which occurs in games like conflict and cooperation. So, game theory actually describes how players play any game with which strategy or with which approach for getting their outcome.

Game Theory Introduced in last century by mathematicians and economists as tool to analyze economical and political conflicts. Game Theory are situations where the decisions taken by player does not affect the outcome but he also has to analyze the behavior of other player. It is the theory where outcome come depending upon independent and interdependent decision making. It actually structure and analyze the different scenarios and strategies. Game theory is a scientific discipline that analyze the conflicts situations, interaction between players and their decisions which th Key take in conflict situation.

II .CLASSIFICATION OF GAMES

1.Co-operative & Non-Co-operative Games :

Cooperative games are usually explained through the structure of the cooperative game theory, which focuses on predicting which combinations will form, the joint actions that groups take, and the resulting group payoffs. In the non-cooperative game theory, it focuses on foretelling individual players' actions and analyzing Nash equilibrium.

2.Symmetric and Non-symmetric Games :

In a symmetric game outcome is of playing a game in a particular approach depending upon the approach used not on who is playing them. Asymmetric games are games where there are not similar approaches defined for both players.

3.Zero-sum and Non-Zero Games :

In zero-sum games, outcome adds to zero always for every strategy they apply.

Non-zero-sum games because the result has greater or less than zero. It's a no-loss game where the gain of one player does not create the other player's loss.

4.Perfect Information & Imperfect Information Games :

Perfect information and Imperfect information :

In this, all players have perfect information about the moves made by previous players.

Examples: tic-tac-toe, checkers, infinite chess, and Go

Similarly, In card games, there is no such perfect information about the previous moves so it is known as Imperfect Information. Example: poker and bridge

5.Combinational Games :

Games like chess and go have multiple ways to win, to find out the best solution from many ways this type of game is known as combinational games.

6.Discrete and Continuous Games :

The game theory which played by rules like a finite number of players, finite moves, outcomes, etc is a Discrete game.

Continuous games let players take an approach from a continuous approach ways.

Example : tic-tac-toe(crosses) or checkers.

III .REPRESENTATION OF GAMES

A. Extensive form

It is used to formalize games with time sequence move it also capture simultaneous move games with imperfect information. It is multiplayer generalization of a decision tree. For solving this backward induction is used.

A complete extensive-form representation specifies:

1. players of a game
2. for every player every opportunity they have to move
3. what each player can do at each of their moves
4. what each player knows for every move
5. the payoffs received by every player for every possible combination of moves

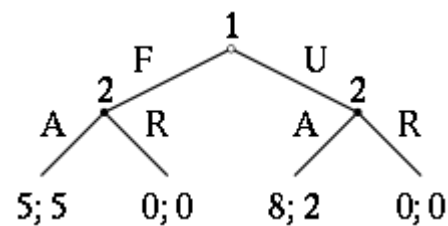


Fig.1 Representation of extensive form

B. Normal form

The normal game is represented by a matrix which has players, strategies and payoffs. It has combination of actions. It can represented by any Function associate with player. When game is in normal form ,each player acts simultaneously without knowing actions of other.

		PLAYER 2	
		Strategy A	Strategy B
PLAYER 1	Strategy A	p_{1A}, p_{2A}	p_{1A}, p_{2B}
	Strategy B	p_{1B}, p_{2A}	p_{1B}, p_{2B}

Fig. 2 Example of Normal form

Definition : A game in a normal form is a structure

$$G = \{P, S, F\}$$

Where:

$$P = \{1, 2, \dots, m\} \quad \dots \text{set of players}$$

$$S = \{S_1, S_2, \dots, S_m\} \quad \dots \text{set of pure strategy, one for each player}$$

$$F = \{F_1, F_2, \dots, F_m\} \quad \dots \text{payoff functions}$$

C. Characteristic function form

In this game form there is no removable utility. Only united one will get rewarded rather than separate player. These characteristic functions expand to describe games which have no removable utility.

It is primarily used to divide difficulties and eliminating distractions.

A characteristic function can be seen as (N, v) , where N represents the group of people and is a normal utility.

There are also some alternative game forms like Congestion game, Sequential form, Timed games, gala, Local effect games, GDL, Game Petri-nets, Continues games, PNSI, Action graph games and Graphical games.

IV. NASH EQUILIBRIUM

Nash Equilibrium was discovered by American mathematician, John Nash. He was awarded the Nobel Prize

in Economics in 1994 for his contributions to the development of other players.

Nash Equilibrium is a decision - making theorem in game theory which states that player can achieve desired outcome by not changing their initial strategy. It determines optimal solution in a non-cooperative game.

In the Nash Equilibrium, each players strategy is optimal than decisions of other players. Every player wins because everyone get desired outcome.

		Player B	
		Option 1	Option 2
Player A	Option 1	6, 6	4, 7
	Option 2	7, 4	5, 5

Fig. 4 Nash Equilibrium Example

The prisoner's dilemma is game theory example which showcases the effect of Nash Equilibrium.

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V. SOME APPLICATION OF GAME THEORY

Game Theory is applied in many areas like economics, business, biology, computer science, political science,

psychology and philosophy. It also describes phenomena like interpersonal relations, competition, war and political affairs. Game Theory is proven powerful to detect conflicts and cooperation interactions in various fields.

A. Economics and Business:

Economics use The Game Theory to analyze various situations like competition ,economical phenomena like bargaining, mechanism design, voting theory, political economy, behavioral economics, auctions ,etc

Game Theory offers tools for solving conflicts and problems. Business can achieve success if it follows right strategy in business world and can suffer also if taken wrong strategy. In business world strategic behavior occurs in managers, executives and investors .They have to manage the situation and take decision regarding any difficulty. Game Theory is potential which force a person to take considerable action to pick their strategy to get outcome.

B. Game Theory in Politics :

Game Theory is mostly used in political affairs which beneficial in war strategy, war bargaining, social choice theory , Strategic voting, political economy etc. Game theory is effectively used to solve conflicts in political individuals ,companies, states, political parties. Game Theory used to be useful to research on various problems like terrorism cause it track the strategies ,analyze attacking situations their approach.

By using Prisoner's Dilemma, government used to choose between terrorism policies. The result of prisoner's dilemma game is active terrorism policies of countries. It can define that which country has active and reactive terrorism policies.

C. Game Theory in Philosophy :

Game Theory is connected with Philosophy in many ways. It has been used as a tool in philosophy to find out the outcome. In philosophical discussions, philosophers always use game theory to interact for finding out the conception. As many philosophers have their own opinions to find out optimal solution and find out the result for the conflicts. There are many different areas of Philosophy that are connected with Game Theory.

V. CONCLUSION

Game Theory is a study of conflict and interaction between decision –makers. It is tool from which we can find out the approach used by people to take decision and their behavior. Game models are becoming more powerful and useful to track the strategy. It is already applied in many different areas. Game Theory provide to predict things in various fields. Game Theory helped in to sharp our paths that which path , values and norms will make sustain for any business in business world. From this I can conclude that Game Theory is powerful in many ways.

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