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A Study on Critical Path Analysis and Project Evaluation Review Technique

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Abstract –This Research paper provides the extension form of Network diagram and Project Completion time as well as Critical path computation process. It also provides the basic definitions of Activity, Successor activity, Concurrent activity and dummy activity in network work. It also provides the required norms and applicable rules for drawing network diagram. It estimates the critical path in network analysis. We also computer critical path using the network analysis.It also provides the probability situation in project evaluation, Review Technique, Time Cost Analysis, Time crashing methodology.

***Key Words*:**Critical path, Dummy activity, Network diagram,Successor, Time Cost Analysis, Time crashing

1.INTRODUCTION

A project defines a combination of interrelated activities which must be executing certain sequence before the entire task can be computed. Global development, complexities of modern projects have demanded more systematic and effective planning techniques with the objective of optimizing the efficiency of leading the project. Project management has evolved as new field with the development of two analytic techniques for planning, scheduling and controlling of projects. Theseare the Critical Path Method and Project Evaluation and Review Technique.

- ❖ In general individual operation, which utilizes resources and has an end and a beginning, is called method of activity.
- ❖ Activities that must be completed immediately prior to the start of other activity are predecessor activities.
- ❖ Activities which can be accomplished concurrency are known as concurrent activities. It may be noted that an activity can be a predecessor or a successor to an event or it may be concurrent with one or more of the other activities.
- ❖ In determination of critical path diagramsactivities that cannot be started until one or more of the other activities are completed, which are known as successor activities.
- ❖ An activity which does not consume any kind of resource but merely depends on the technological approach is called a dummy activity. Null(dummy) activity is added in the network to clarify the activity pattern.
- ❖ To identify and maintain the proper precedence relationship between activities that a are not connected by events one of the situation of Dummy activity application.
- ❖ To make activities with common starting and finishing points distinguishable is the other method in dummy activity application.

2. RULES FOR NETWORK DIAGRAM

- ❖ Every movement or activity is represented by arrow symbol in the network. No single activity can be represented in the network. This is to be distinguished from the case where one activity is broken into different segments. In such case, each segment may be represented by a separate arrow.
- ❖ No two activities can be identified by the same start and end events. Zero(dummy) activities are also useful in establishing logical relationship in the network diagram.
- ❖ In order to ensure the correct precedence relationship in the arrow diagram we need to verify the basic conditions as below mentioned :

  a) Need to find out the activity that must be completed before the activity to start.

  b) Activities that must follow.

  c) Activities must occur simultaneously with the present activity.

- ❖ If two or more individuals draw the same network for agiven project it seldom happens that some of them will look alike as matter of fact, no two of them may even look similar.
- ❖ The reason is that there are many ways to draw the same network. However, there will be same network representation drawn from above set of rules which are much easier to follow than the other.
- ❖ In large network, it is mandatory that certain ‘ good habits’ be practiced to draw an easy to follow critical situation network paths.
  - ▪ Cross over arrows need to be avoided.
  - ▪ Only apply straight arrows.
  - ▪ Do not attempt to represent period of activity by its length of arrow.
  - ▪ Vertical and standing arrows may be used if necessary, and from left to right direction.

- Final network should not have any redundant dummies as used in draft one.
- The network has only one entry point called the start event and one point of emergency called the end event.
- Events numbers must be given from left to right and top to bottom.

3. TIME ESTIMATES IN NETWORK

The main objective of the time analysis is to prepare a planning schedule or the project. The major factors in planning are :

- Total completion time for the project.
- Need to identify the critical activities,
- Float for each activity,
- Latest time when each activity can be started without dealing the total project.
- Finalize earliest time to start the activity.

Case-I:

a) Earliest Event Time – Before starting computations, the occurrence time of initial network event is fixed. Then the forward pass computation yields the earliest start and earliest finish time for each activity, and indirectly the earliest expected occurrence time for each event.
b) Step-1: The computations begin from the START node and move towards the END node.
c) Step-2: Earliest starting time of activity is the earliest event time of the tail end event.
d) Step-3: Earliest finish time of activity is the earliest starting time plus the activity time
e) Step-4: Earliest event time for event j is the maximum of the earliest finish times of all activities ending into that event.

From the above we conclude that :

$$(E_s)_{ij} = E_j \quad \ldots (1)$$

$$(E_f)_{ij} = (Es)_{ij} + D_{ij} \quad \ldots (2)$$

$$E_j = \max[E_i + D_{ij}] \quad \ldots (3)$$

Case-II.

Latest Allowable Time- The latest event times(L) indicates the time by which all activities entering into that event must be completed without delaying the completion of the project.

Step-1: Assume for ending event: i.e., E = L.

Step-2: Latestfinish time for activity is equal to the latest event time of event $(Lf)_{ij} = L_j$

Step-3: Latest starting time of activity - the latest completion time of – the activity time.

$$(Ls)_{ij} = (Lf)_{ij} - D_{ij}$$

Step-4: Let'I' is the minimum of the latest start time of all path activitiesthat is

$$Lj = \min[(Ls)ij \text{ immediate successors of } (I,j)]$$

$$= \min[L(Lf)_{ij} - D_{ij}]$$

4. COMPUTING FLOATS AND SLACK TIMES

When the network diagram is completely drawn, properly labeled, and earliest(E) and latest(L) event times are computed as discussed so far, the next object is to determine the flats and slack times are considered.

The time which could be delayed beyond the earliest expected completing time without affecting the overall project duration time is known as Total float.

Mathematically, the total float of an activity (i-j) is the difference between the latest start time and earliest start time of the activity. Hence the total float for an activity (i-j) denoted by $(T_f)_{ij}$ can be calculated by the formula:

$$(Tf)_{ij} = (\text{Latest start} - \text{Earliest start}) \text{ for } (i\text{-}j)$$

$$= (Ls)_{ij} - (Es)_{ij}$$

The time which can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity. Which is known as Free float.

In mathematically the free float for activity(i,j) denoted by $(F_f)_{ij}$ can be computed by wehre$K_{ij} = (E_j - E_i)$

$$(F_f)_{ij} = K_i - D_{ij}$$

The amount of time by which the start of an activity can be delayed without effecting the earliest start time of any immediately following activities such type of situation is known as Independent float.In mathematically the Independent float is denoted by :

$$(I_f)_{ij} = K_{ij} - D_{ij}$$

We consider negative independent float as zero.
For any given event the event slack is the difference between the latest event and earliest event times such type of situation is known as Event Slacks.

Mathematically it is denoted as :

$$\text{Event slack} = L_j - E_j$$

Tail Event slack = $L_i - E_i$

Total flaot $= L_j - E_i - D_{ij}$

Free float = $M_{ij} - D_{ij}$; $(M_{ij} = E_j - E_i)$

Independent float = $E_j - L_j - D_{ij}$

5. DETERMINATIN OF CRITICAL PATH

- Critical Event – Since the slack of an event is the difference between the latest and earliest event times, that is slack(i) = $L_i - E_i$, here zero slack times are called critical events.

  If E_i = Li then it is said to be critical.

- The difference between the latest start time and earliest start time of an activity is usually called as total float and zero float is known as critical activity.

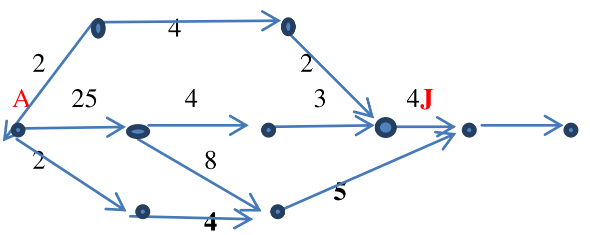
- if a delay in its start will cause a further delay in the completion date of the entire project is also known as critical path.

- A non-critical activity is such that the time between its earliest start and its latest completion dates is longer than its actual duration.

- Critical Path – The sequence of critical activities in a network is called the critical a path.

Example-Problem-1:

Compute the time taken to complete the project.



$E_1 = 1$; $E_2 = (0+2) =$ [from : $E_1 + D_{12}$] = 2

$E_3 = E_1 + D_{13} = 0+2 = 2$; $E_4 = E_1 + D_{14} = 0 + 2 = 2$

$E_5 = E_2 + D_{25} = 2+4 = 6$; $E_6 = E_3 + D_{36} = 2 + 5 = 7$

$E_7 = E_3 + D_{37} = 2+8 = 10$; $E_8 = L_6 + D_{68} = 7 + 4 = 11$

$E_9 = E_7 + D_{79} = 10+5 = 15$; $E_{10} = E_9 + D_{9,10} = 15+4=19$

Total Project time is 19 days to finish as this is the longest path of the network.

Longest path(Critical path) :1-3,3-7,7-9, and 9-10.

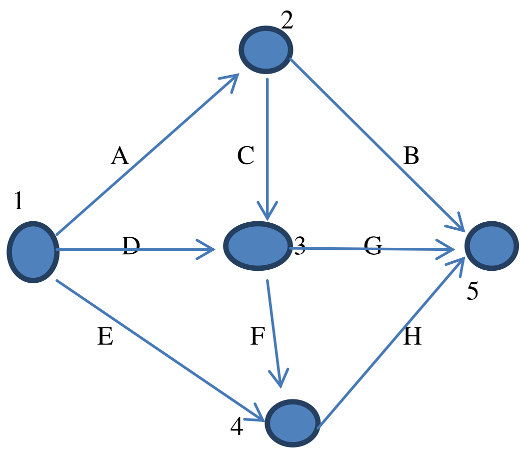
Example-Problem-2:

Draw a network for the project whose activities and their relationships are as below :

Activities :Tot start: A, D, E
Activities Relation : B, C >A ; H > E, FG, F > D, C ;

Sol:- Start Activities: using (1-2, 1-3, 1-4)

End Activities :(2-5, 3-5, 4-5)

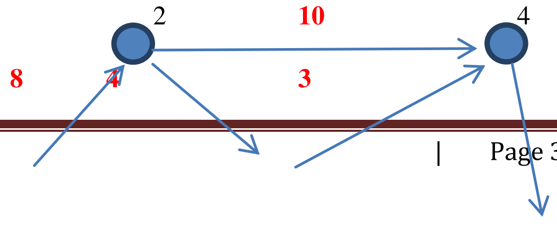


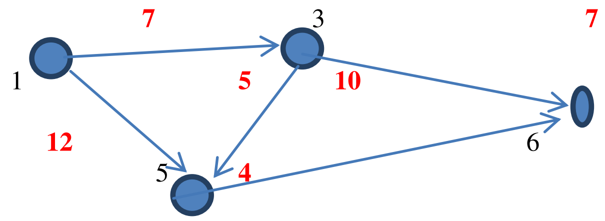
Example-Problem-3:

Determine the Earliest Start, Earliest Finish, Latest Start and Latest Finish of each activity of the given project and also the Critical Path of the Project.

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration (Weeks)	8	7	12	4	10	3	5	10	7	4

Sol:-





Let Earliest Start; Earliest Finish

Let Latest start; Latest Finish

(1,2) = 8; (2,4) = 10 ; (4,6) = 7 ;

(1,5) = 12 ; (1,3) = 7; (2,3) = 4 ;

(3,5) = 5 ; (3,4) = 3 ; (3,6) = 10 ;

(5,6) = 4

Activity	Duration	Earliest		Latest	
		S	F	S	F
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Critical Path: (1,2)(2,4),&(4,6)

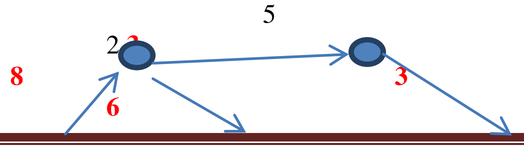
Max Value : 25

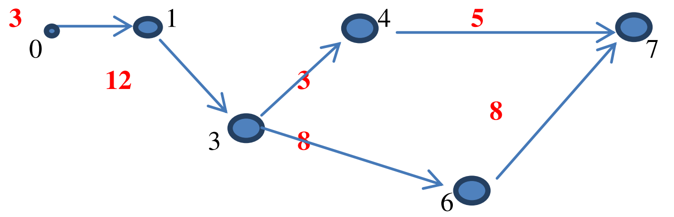
Example-Problem-4:

Construct the network for the project whose activities are given in table and compute the total, free and independent float of each activity and hence determine the critical path and the project duration weeks.

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (Weeks)	3	8	12	6	3	3	8	5	3	8

Sol:-





Activity	Duration	Earliest		Latest		Floats		
		S	F	S	F	T	F	I
0-1	3	0	3	0	3	0	0	0
1-2	8	3	11	12	20	9	0	0
1-3	12	3	15	3	15	0	0	0
2-4	6	11	17	20	26	9	1	-8
2-5	3	11	14	25	28	14	0	-9
3-4	3	15	18	23	26	8	0	0
3-6	8	15	23	15	23	0	0	0
4-7	5	18	23	26	31	8	8	0
5-7	3	14	17	28	31	14	14	0
6-7	8	23	31	23	31	0	0	0

Critical Path : (0-1),(1-3)(3-6) & (6-7)

Project Duration : 31 Weeks

6. PROJECT EVALUATION REVIEW TECHNIQUE

Analysis through PERT is to find out the completion time for a particular event within specified date. A reliable time estimate is difficult to get due to rapid change of technology.

- The possible timeinwhich the activity can be finished in short time is optimistic and it is denoted by t_0.
- The most likely time is the estimate of the normal time the activity would take most likely time will represent the highest frequency fo occurrence. This is denoted by t_m.
- The pessimistic time represents the longest timethe activity could take and is denoted by t_p.
- Expected time is the average time an activity will take if it were to be repeated on large number of times and is based on the assumption that the activity time follows Beta distribution. Computed by :

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$t_e = (K + 4M + L) / 6.$

- Variance :: $\sigma^2 = [(L - K) / 6]^2$

Considering : Optimistic time : t_o=K

Pessimistic time : t_p= L

Most likely time : t_m= M

Expected time : t_e= A

Variance : σ^2

7. TIME COST ANALYSIS

In real life projects it may be required to complete the project even before its optimal completion time. To meet this requirement the evaluator has to cut down some time of the project which is known as TIME CHASHING. In general this type of activity adds some cost to the project. Project time crashing involves the basic steps to proceed and compute the time cost optimization procedure.

Step-1: Compute the Critical path using the given activities.

Step-2: Calculate the **Cost slope** for the different activities by using notation: Normal Cost = N_c ;Normal time=N_t

= [(Crash cost–N_c)] / N_t-Crash time

Step-3: Based on the activities need to arrange the Rank in ascending order

Step-4: Activity having lower cost slope would be crashed first to the maximum extent possible. We useCumulativelyadding the cost of crashing to the normal cost to determine direct cost.

Step-5: As the critical path duration is reduced by the crashing in Step3. Other paths also become critical. We get parallel critical paths. Project duration can be reduced in the parallel critical paths.

Step-6: Optimal duration. Crashing as per Step-3, and Step-4 an optimal project duration is determined. IT would be the time duration corresponding to the total cost.

Example-Problem-5:

Draw a network for the project whose activities and the three time estimates of these activities in weeks are given

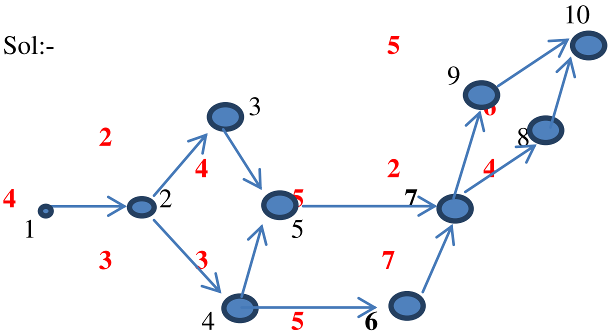
i) Compute Expected duration of each activity

ii) Compute Expected variance of each activity

iii) Compute Expected variance of the project length.

Activity	t_0	t_m	t_p
1-2	3	4	5
2-3	1	2	3
2-4	2	3	4
3-5	3	4	5
4-5	1	3	5
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	1	2	3
8-10	4	6	8
9-10	3	5	7

Sol:-



Activity	t_0	t_m	t_p	Te	Exp. Variance
1-2	3	4	5	4	1/9
2-3	1	2	3	2	1/9
2-4	2	3	4	3	1/9
3-5	3	4	5	4	1/9
4-5	1	3	5	3	4/9
4-6	3	5	7	5	4/9
Activity	t_0	t_m	t_p	Te	Exp. Variance

5-7	4	5	6	5	1/9
6-7	6	7	8	7	1/9
7-8	2	4	6	4	4/9
7-9	1	2	3	2	1/9
8-10	4	6	8	6	4/9
9-10	3	5	7	5	4/9

Where $t_e = (t_o + 4t_m + t_p) / 6$

Expected Variance (σ^2) = $[(t_p – t_0) / 6]^2$

Critical path : 1 - 2 – 4 – 6 -7 – 8 -10.

Expected Project duration = 29 weeks.

Expected Variance of the Project length = Sum of the expected variances of all the critical activities.

[(1/9) +(1/9)+ (4/9) +(1/9)+ (4/9) +(4/9)]

= 1.67 (Approximately)

Example-Problem-6:

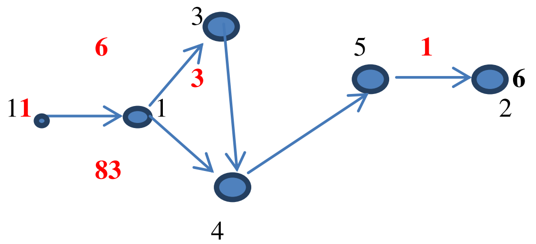
Time Estimates in months of all activities of project given in table find i) Expected duration and standard duration of each activity ii) Construct the Project Network. Iii) Determine the critical path, expected project length and expected variance of the project length.

Activity	a	m	b
1-2	0.8	1.0	1.2
2-3	3.7	5.6	9.9
2-4	6.2	6.6	15.4
3-4	2.1	2.7	6.1
4-5	0.8	3.4	3.6
5-6	0.9	1.0	1.1

Sol:- i)

Activity	a	m	b	Te	σ
1-2	0.8	1.0	1.2	1	0.067
2-3	3.7	5.6	9.9	6	1.03
2-4	6.2	6.6	15.4	8	1.53
3-4	2.1	2.7	6.1	3	0.5
4-5	0.8	3.4	3.6	3	0.47
5-6	0.9	1.0	1.1	1	0.033

ii)



iii) Critical Path: 1-2-3-4-5-6

Expected Project length = 14 months.

Expected σ^2 : $(0.067)^2+(1.03)^2+(0.5)^2+(0.47)^2+(0.033)^2$

= 1.5374

$\sigma_c = \sqrt{1.5374} = 1.2399$

3. CONCLUSION

Project management has evolved as new field with the development of two analytic techniques for planning, scheduling and controlling of projects. The main objective of the time analysis is to prepare a planning schedule or the project. The evaluation beginwith START node and move towards the END node. Earliest starting time of activity is the earliest event time of the tail end event. In real life projects it may be required to complete the project even before its optimal completion time. To meet this requirement the evaluator has to cut down some time of the project which is known as TIME CHASHING. In general this type of activity adds some cost to the project. Project time crashing involves the basic steps to proceed and compute the time cost optimization procedure.

In this Research manuscript the deterministic method of project management namely critical path method and when activity times are not known how to evaluate a project completion times are also explained in the manuscript paper as Project Evaluation Review Technique methods with suitable examples.

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