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A STUDY ON EDGE EVEN GRACEFUL LABELING OF SOME GRAPHS

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ABSTRACT

A function f is called *edge even graceful labeling* of a graph G if $f:E(G) \to \{2, 4, 6, .., 2q\}$ is bijective and the induced function $f^*: V(G) \to \{0, 2, 4, .., 2q - 2\}$ defined as $f^*(u) = \sum_{e=u(v)\in E(G)} f(e) \bmod 2r$, where $r = \max\{p, q\}$ is injective. This type of graph labeling is first introduced by Elasonbaty and Dauod in 2017. In this paper we proved that even edge graceful labeling for the graphs $P_n \odot K_1 + P_1$, $C_n + nS_2$, $P_n \odot K_1 + (n - 2)P_2$ & $2C_n + P_{2n}$.

Keywords:

Graceful labeling, Even graceful labeling, Edge even graceful labeling.

1. INTRODUTION:

The graph G considered here will be finite, undirected, and simple where V(G) and E(G) will denote the vertex set and edge set of a graph G. If p = V(G), q = E(G).

The field of graph theory plays an important role in various areas of pure and applied sciences. One of the importance areas in graph theory is graph labeling of a graph G which is an assignment of integers either to the vertices or edges or both subject to certain conditions. Graph labeling is a very powerful tool that eventually makes things in different fields very ease to be handled in mathematical way.

Nowadays graph labeling has much attention from different brilliant researches in graph theory which has rigorous applications in many disciplines, e.g, communication networks, coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, data base management and graph decomposition problems.

2. DEFINITIONS:

2.1 Graceful Labeling:

A function is called a graceful labeling of a graph G if $f: V(G) \to \{0, 1, 2, 3, ..,, q\}$ is injective and the induced function $f^*:E(G) \to \{1, 2, 3, \ldots, q\}$ defined as $f^*(e = uv) = |f(u) - f(v)|$ is bijective. This type of graph labeling first introduced by Rosa in 1967[1] as a β-valuation, later on Solomon W.Golomb called as graceful labeling[1].

2.2 Even Graceful Labeling:

The even graceful labeling of a graph G with p vertices and q edges is defined that there is an injection f: E(G) → {2, 4, 6, ..,, 2q} so that induced map f*:V(G) → {0, 2, 4, ….(2k-2)} defined by f*(x) = $\sum f(x,y) mod\ 2k$, where k = max{p, q} makes all the edges distinct and even.

2.3 Edge Even Graceful Labeling:

A function f is called edge even graceful labeling of a graph G if f: E(G) → {2, 4, , ..,,, 2q} is bijective and the induced function f*: V(G) → {0, 2, 4, ..,, 2q - 2} defined as

f*(u) = $\sum_{e=u(v)\in E(G)} f(e) \bmod 2r$,where r = max{p, q} is injective. This type of graph labeling was first introduced by Elasonbaty and Dauod in 2017[3].

3. MAIN RESULTS:

3.1 Theorem

The connected graph $P_n \odot K_1 + P_1$ is edge even graceful graph.

Proof:

The graph $P_n \odot K_1 + P_1$ have 2n+1 vertices and 2n edges.

To find edge even graceful labeling, we define f: E($P_n \odot K_1 + P_1$) → {2, 4, 6, ..,, 2q}

as f(ei) = 2i; i = 1, 2, 3, …, 2n ---- (1)

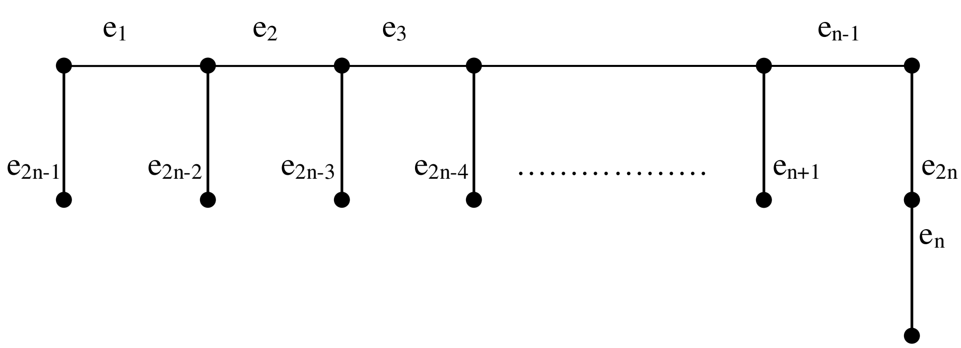

Fig 1: Edge even graceful graph of $P_n \odot K_1 + P_1$

Hence, for the graph G if f: E(G) → {2, 4, 6, ..,, 2q} is bijective and the induced function f*:V(G) → {0, 2, 4, …., 2q - 2} defined as f*(u) = $\sum_{e=u(v)\in E(G)} f(e) \bmod 2r$, where r = max{p, q} is injective. i.e. the connected graph $P_n \odot K_1 + P_1$ admits edge even graceful labeling. Hence the graph $P_n \odot K_1 + P_1$ is edge even graceful graph.

3.1 Example: The connected graph $P_7 \odot K_1 + P_1$ is edge even graceful graph.

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Solution: The following connected graph has 15 vertices and 14 edges.

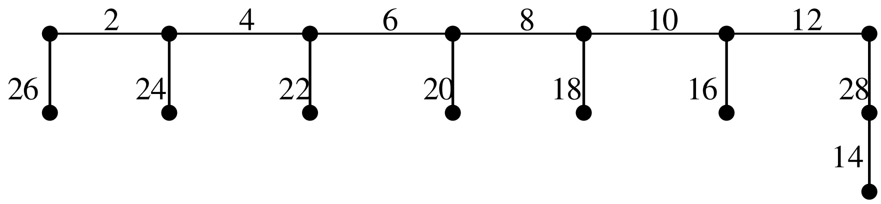

Fig 2: Edge even graceful graph of $P_n \odot K_1 + P_1$

As per the rule (1) the connected graph $P_n \odot K_1 + P_1$ admits the even edge graceful labeling. Hence the graph is even edge graceful.

3.2 Theorem: The connected graph $C_n + nS_2$ is edge even graceful graph if n is odd.

Proof: The connected graph $C_n + nS_2$ has 3n vertices and 3n edges.

To find edge even graceful labeling:

We define edge labeling f: E($C_n + nS_2$) $\rightarrow$ {2, 4, 6, …, 2q}

as f(e_m) = 2m; m = 1, 2, …, 3n.

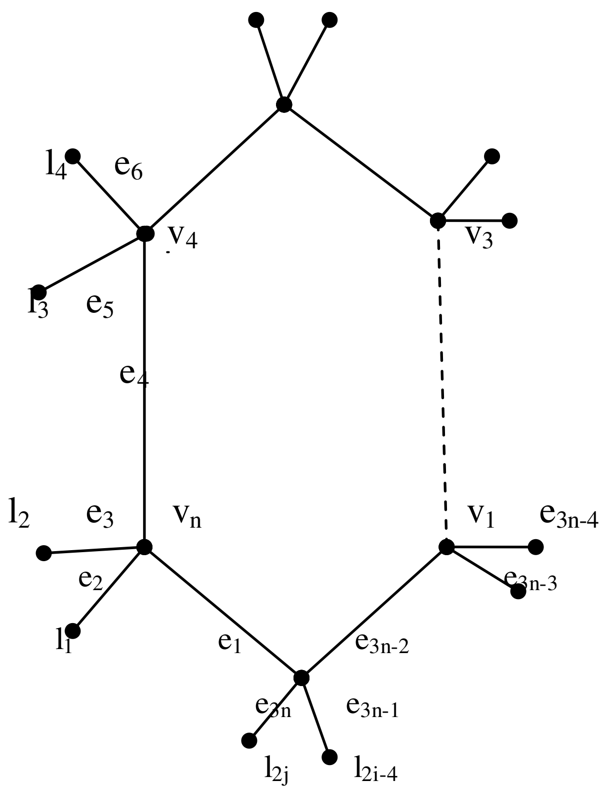

Fig 3: Edge even graceful graph of $C_n + nS_2$

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Hence, the graph G if f: E(G) $\rightarrow$ {2, 4, 6, …, 2q} is bijective and the induced function f*: V(G) $\rightarrow$ {0, 2, 4, …, 2q - 2} defined as f*(u) = $\sum_{e=u(v)\in E(G)} f(e) \bmod 2r$, where r = max{p, q} is injective and the labeling is distinct.

This is defined as follows:

$l_{2i-1} = 6i - 2$; i =1, 2, 3, ..., 2n-1.

$l_{2j} = 6j$; i = 1, 2, 3, …, 2n.

$v_i = 6i - 4$; i = 1, 3, 5, …, n & i = 2, 4, 6, …, n.

Hence the connected graph $C_n + nS_2$ admits edge even graceful labeling. Hence the graph is $C_n + nS_2$ is edge even graceful graph.

3.2 Example:

The connected graph C_9+9S_2 is edge even graceful graph.

Solution:

The connected graph have 27 vertices and 27 edges.

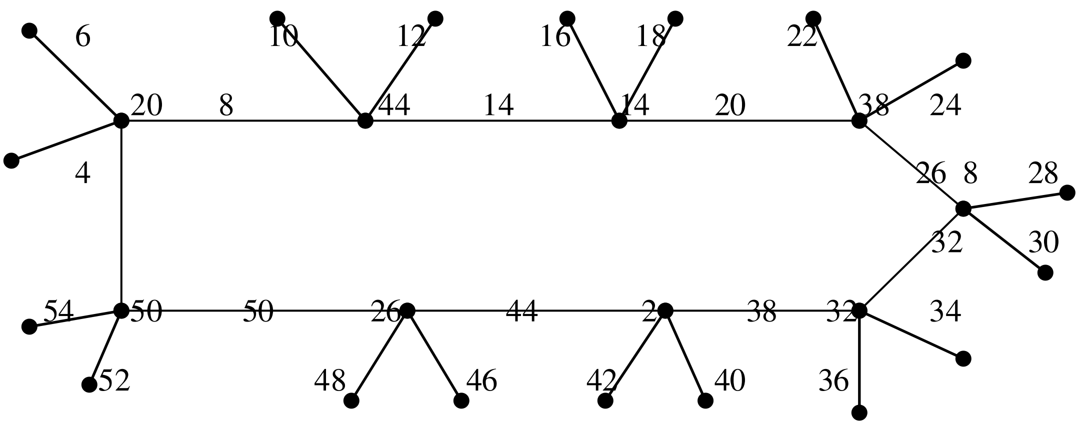

Fig 4: Even edge graceful graph of C_9+9S_2

Hence the connected graph C_9+9S_2 is edge even graceful graph.

3.3 Theorem:

The connected graph $P_n \odot K_1$+(n -2)P_2 edge even graceful graph.

Proof: The connected graph $P_n \odot K_1$+(n-2)P_2 has 2n vertices and 3n - 3 edges.

To find edge even graceful graph.

We define f: $[P_n \odot K_1 + (n - 2)P_2] \rightarrow \{2, 4, 6, \ldots, 2q\}$.

Define the edge labeling $f(e_i) = 2i$; $i = 1, 2, 3, \ldots, 3n - 3$.

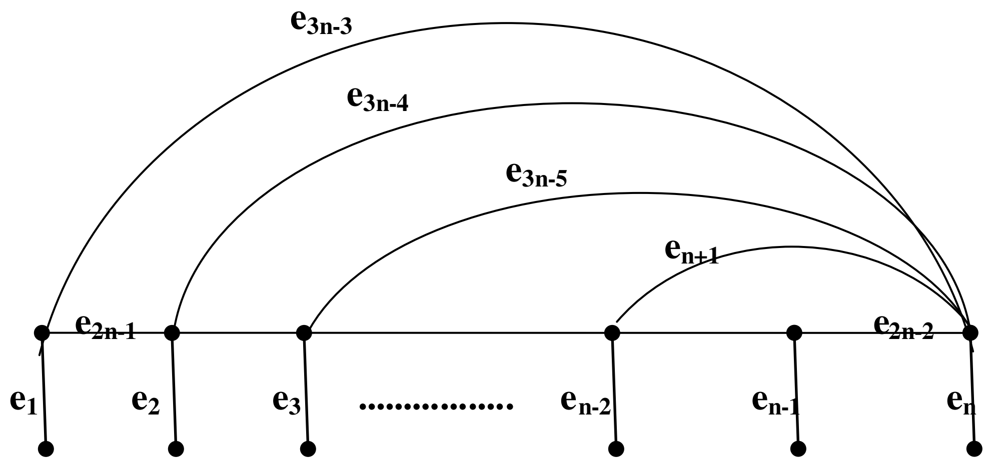

Fig 5: Edge even graceful graph of $P_n \Theta K_1 + (n-2)P_2$

Hence, the graph G if $f : E(G) \rightarrow \{2, 4, 6, \ldots, 2q\}$ is bijective and the induced function $f^*: V(G) \rightarrow \{0, 2, 4, \ldots, 2q - 2\}$ defined as $f^*(u) = \sum_{e=u(v)\in E(G)} f(e) \bmod 2r$, where $r = \max\{p, q\}$ is injective and the edge labeling are distinct.

Here the connected graph $[P_n \odot K_1 + (n - 2)P_2]$ admits edge even graceful labeling. Hence the graph $[P_n \odot K_1 + (n - 2)P_2]$ is edge even graceful graph.

3.3 Example: The connected graph $P_5 \Theta K_1 + 3P_2$ is edge even graceful graph.

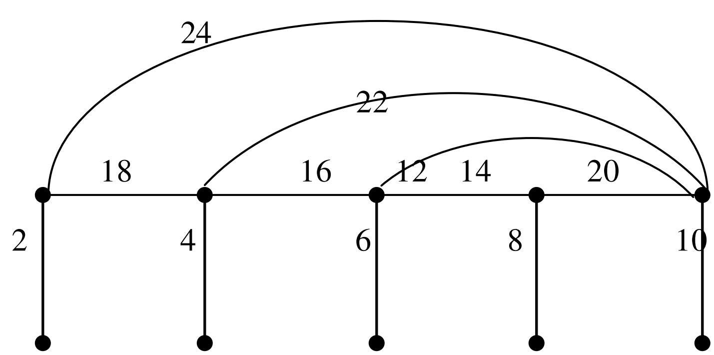

Fig 6: Edge even graceful graph of $P_5 \Theta K_1 + 3P_2$

Hence the connected graph $P_5 \Theta K_1 + 3P_2$ is edge even graceful graph.

3.4 Theorem:

The connected graph $2C_n+P_{2n}$ is edge even graceful graph.

Proof:

The connected graph $2C_n+ P_{2n}$ have 2n+3 vertices and 4n+4 edges.

To find edge even graceful graph.

We define the edge labeling $f : E[2C_n+P_{2n}] \rightarrow \{2, 4, .., 2q\}$

as $f(e_i) = 2i$; $i = 1, 2, 3, \ldots, 4n + 4$.

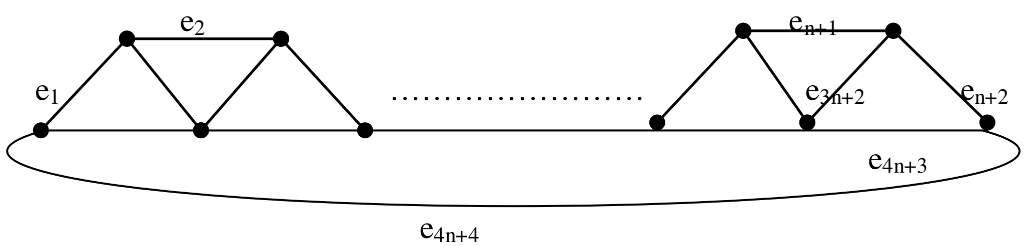

Fig 7: Edge even graceful graph of $2C_n + P_{2n}$

Hence, the graph G if $f : E(G) \rightarrow \{2, 4, 6, \ldots, 2q\}$ is bijective and the induced function $f^*:V(G) \rightarrow \{0, 2, 4, \ldots, 2q - 2\}$ defined as $f^*(u) = \sum_{e=u(v)\in E(G)} f(e) \bmod 2r$; $r = \max(p, q)$ is injective and vertex labeling are distinct.

That is, the connected graph $2C_n+ P_{2n}$ admits edge even graceful graph. Hence the graph $2C_n+ P_{2n}$ is edge even graceful graph.

3.4 Example: The connected graph $2C_2 + P_4$ edge even graceful graph.

Solution: The connected graph 7 vertices and 12 edges.

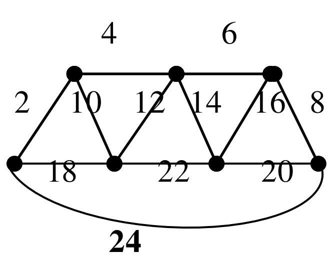

Fig 8: Edge even graceful graph of $2C_2$+P4

4. CONCLUSION:

In this paper we have proved that the connected graphs $P_n \odot K_1+ P_1$, $C_n + nS_2$, $P_n \odot K_1+ (n-1)P_2$, $2C_n+ P_{2n}$ are edge even graceful graphs.

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