Acceleration due to Gravity variation with depth within the Earth

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Abstract;

Acceleration due to gravity of the earth depends on the mass and distance of object from the centre of mass of the Earth. In fact acceleration due to gravity varies with density and distance from centre. Since our earth is non-uniform and the vertical mass distribution within the earth is heterogeneous, with increasing depth, density of the earth increases, although the distance of a particular point from centre decreases. But somewhere increasing density is more effective than the distance, so it increases somewhere, so that acceleration due to gravity firstly increases up to certain distance and then decreases, again increases and finally from core mantle boundary it decreases and become tending to zero at the centre of the earth.

In this paper gravity variation within the earth varies with depth is presented in mathematically and graphically. It is inter relation of acceleration due to gravity of Earth depth and density.

Here two types of mathematical relation has been developed in relation with 1.) Depth and acceleration due to gravity and second is 2.) Depth, density and acceleration due to gravity in linear form.

Within the earth acceleration due to gravity variation is shown by mathematical equations, by which acceleration due to gravity can be found out easily at any point with in the Earth.

Introduction;

Acceleration due to gravity is an intrinsic property of any body and its only depends on mass and radius of the body more preciously it depends on mass of a body and distance of object from centre of mass. Similarly acceleration due to gravity of the Earth depends on its mass and radius or distance of the object from the centre of the Earth. As the earth is a spheroid body and it is also called by Geoid in geophysical world, Average radius of the Earth is 6371 Km and its mass is approximated 5.96×10^{24} Kg. The acceleration due to gravity on the earth surface is nearly 9.8 $m/sec^2$and roughly it is considered. Our earth is consisting heterogeneously throughout the centre of the earth and mass distribution is non uniform laterally on surface but with depth it becomes homogeneous. Due to heterogeneity with depth of the earth acceleration due to gravity, increases somewhere and it decreases somewhere. With increasing depth density increases irregularly so that acceleration due to gravity changes.

Acceleration due to Gravity;

Acceleration due to gravity is the amount of attraction force which is acted on a body of unit mass by the earth.

Acceleration due to gravity is represented by g.

Attraction force generated by earth on a body be F and the mass of that body let M than

Attraction force $F = M.g$

$$g = F/M.$$

Acceleration due to gravity of a body depends on the mass and radius of that body which is attracted to another body.

So acceleration due to gravity

$$g \propto \text{mass of body}(M)\text{........................}(1)$$

$$g \propto 1/\text{distance between two objects}(R^2)\text{...................}(2)$$

From the eqn. (1) and (2),

We get,

$$g \propto M/R^2$$

$$g = GM/R^2\text{..............................} (3)$$

Where G is constant and it is called as universal gravitational constant and its value is

$$G = 6.67\times10^{-11}\ kg^{-1}\text{-}m^3\text{-}sec^{-2}$$

In general acceleration due to gravity depends on distance between two bodies and mass of that body by which is attracted towards own self.

Let the mass of the earth is M_e and radius is R_e then,

Acceleration due to gravity of the earth on its surface is

$$g = G.M_e/R_e^2$$

If the earth is considered as a homogeneous body, the acceleration due to gravity decreases continuously but our earth is heterogeneous with height so it doesn't happen.

Formula for acceleration due to gravity with depth for assuming earth as a homogeneous body;

Let the gravity at the earth surface is g,

$$g = 4/3G \times\pi\times R_e \times d \text{..........................} (4)$$

and at depth H is g' and the average density below the depth H is d then,

$g' = 4/3G \times \pi \times (R_e - H) \times d$..................(5)

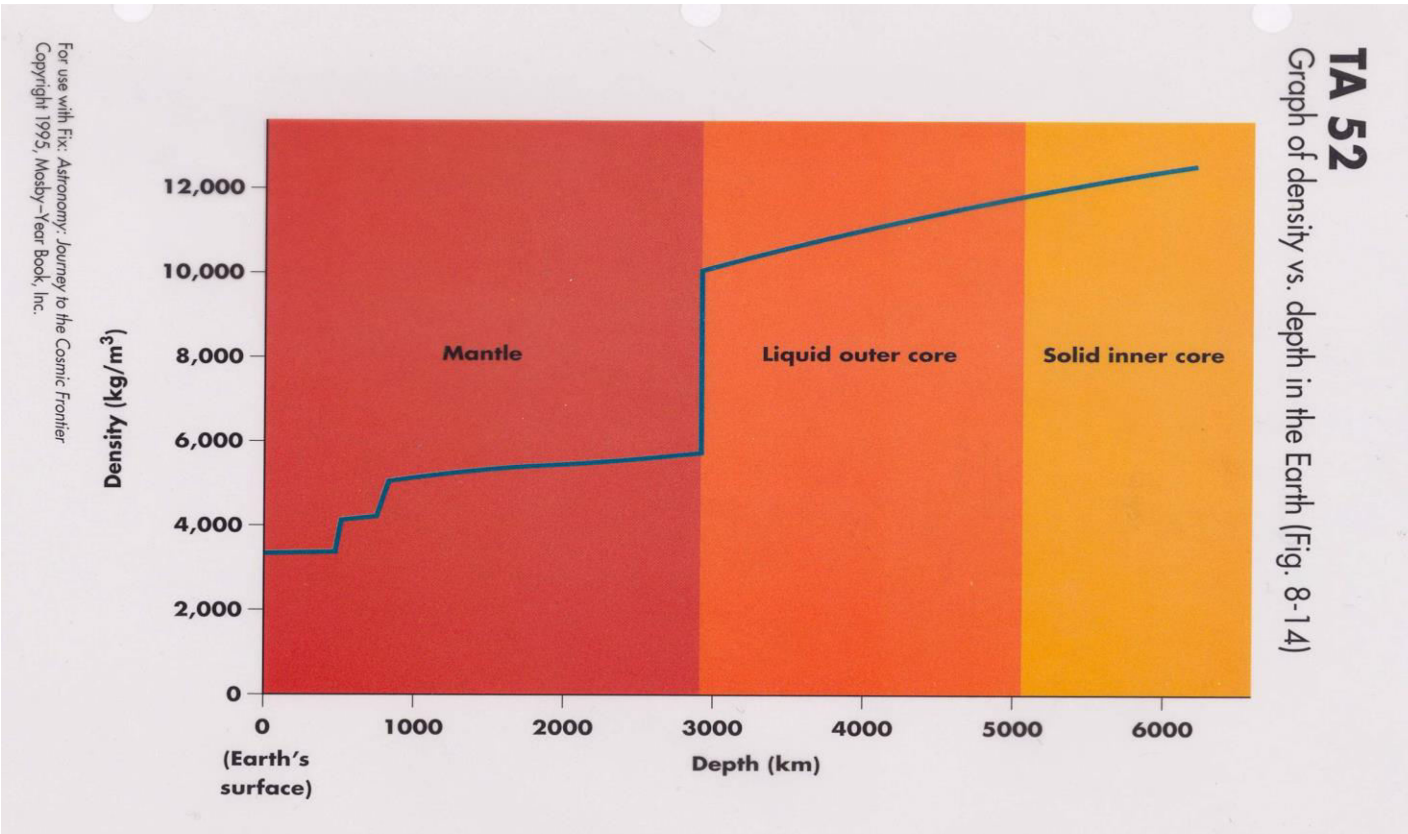
on solving eqn.(4) and (5)

$g' = g \times (1 - H/R_e)$ ....(6)

Equation (6) is valid only for homogeneous spherical body, but because of heterogeneity eqn. cannot be use for longer depth. So the acceleration due to gravity of the earth decreases with increases depth does not strictly valid and the rate of change of acceleration due gravity is not uniform throughout the centre of the earth.

In actual practise,

$dg/dH \neq$ constant.



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On X- axis represented by depth of the earth interior (In metres)

On Y-axis represented by density of the earth interior (in Kg/M^3)

Equation of each lines as fallows;

Equation of first line is formed by point $(0, 3.2\times10^3)$ and $(4.5\times10^5, 3.4\times10^3)$

$Y = 1/100\{(2/45).x+3.2\times10^3\}$................ (1)

Equation of second line is formed by point $(4.5\times10^5, 3.4\times10^3)$ and $(5.1\times10^5, 4\times10^3)$

$Y = 1/100\times(2x-5.6\times10^5)$....................(2)

Equation of third line is formed by point $(5.1\times10^5, 4\times10^3)$ and $(7.1\times10^5, 4.2\times10^3)$

$Y = 1/1000(x+3.49\times10^6)$................. (3)

Equation of fourth line is formed by point $(7.1\times10^5, 4.2\times10^3)$) and $(8.4\times10^5, 4.5\times10^3)$

$Y = 3/1300(x+11.1\times10^6)$ .... (4)

Equation of fifth line is formed by point $(8.4\times10^5, 4.5\times10^3)$ and $(29\times10^5, 5.7\times10^3)$

$Y = 1/1716(x+68.75\times10^5)$ .... (5)

Equation of sixth line is formed by point $(29\times10^5, 5.7\times10^3)$ and $(31\times10^5, 9.9\times10^3)$

$Y = 1/100(2.1x-5.52\times10^6)$ ....(6)

Equation of seventh line is formed by point $(31\times10^5, 9.9\times10^3)$ and $(63.71\times10^5, 12.5\times10^3)$

$Y = 1/1258(x+292.82\times10^5)$ .... (7)

The determination of the acceleration due to gravity at each discontinuity can be measured by the calculation of the mass of the different layered body, which is more or less homogeneously distributed laterally on that particular depth.

According to the above figure, the mass distribution with depth is as fallows;

Here earth surface is considered as a 0 Km and further is divided as fallows;

1. First density discontinuity is located at a depth of 450 km,

2. Second density discontinuity is located at a depth of 510 Km,
3. Third density discontinuity is located at a depth of 710 Km,
4. fourth density discontinuity is located at a depth of 840 Km,
5. fifth density discontinuity is located at a depth of 2900 Km,
6. sixth density discontinuity is located at a depth of 3100 Km,

Since the earth is a spherical body, the mass particular region can be determined by the product of volume and average density,

The volume of core (inner core as well as outer core) of the earth is,

Volume $V_1 = (4/3).\pi.R_1^3$.

Volume $V_1 = (4/3).\pi.(3.271\times10^6)^3$

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Volume $V_1 = 146.6\times10^{18}$

Volume $V_1 = 1.466\times10^{20}$ m^3.

Mass of the sphere surrounded by first discontinuity from the centre of the earth,

Mass M_1 = volume (V_1) × Average density (d_1)

Mass $M_1 = 1.466\times10^{20} \times 11.2\times10^3$

$M_1 = 1.466\times10^{20} \times 11.2\times10^3$

$M_1 = 1.6419\times10^{24}$ Kg.

Again the volume of the earth interior from the 3271 Km to 3471,Km which is at a depth of 3100 Km to 2900 Km, and the average density of that region is 7.8×10^3 Kg/m^3.

The volume of transition zone (outer core -mantle boundary) of the earth is,

Volume $V_2 = (4/3).\pi.(R_2^3 – R_1^3)$.

Volume $V_2 = (4/3).\pi.\{(3.471\times10^6)^3 - (3.27\times10^6)^3\}$

Volume $V_2 = (4/3).\pi.\{(41.818\times10^{18} – 34.998\times10^{18}\}$

Volume $V_2 = (4/3).\pi.(6.82\times10^{18})$

Volume $V_2 = 28.5758\times10^{18}$ m^3.

Mass of body covered by second discontinuity from the 3271 Km to 3471Km is

Mass $M_2 = 28.5758\times10^{18}\times7.8\times10^3$

Mass $M_2 = 0.2229\times10^{24}$ Kg.

Again the volume of the earth interior from the 3471 Km to 5531Km, which is at a depth of 840 Km to 2900 Km, and the average density of that region is 5.1×10^3 Kg/m^3.

The volume of transition zone (outer core -mantle boundary) of the earth is,

Volume $V_3 = (4/3).\pi.(R_4^3 - R_3^3)$

Volume $V_3 = (4/3).\pi.\{(5.531\times10^6)^3 - (3.471\times10^6)^3\}$

Volume $V_3 = (4/3).\pi.\{(169.204\times10^{18} - 41.818\times10^{18}\}$

Volume $V_3 = (4/3).\pi.(127.386\times10^{18})$

Volume $V_3 = 533.75\times10^{18}$ m^3.

Mass of the body which is surrounded by 3471 Km to 5531 Km from the centre of the earth, with average density is 5.1×10^3 Kg/m^3.

Mass $M_3 = 533.75\times10^{18}\times5.1\times10^3$ Kg

Mass $M_3 = 2.7221\times10^{24}$ Kg

further the volume of the earth interior from the 5531Km 5661 Km to, which is at a depth of 710 Km to 840 Km, and the average density of that region is 4.35×10^3 Kg/m^3.

The volume of transition zone (outer core -mantle boundary) of the earth is,

Volume $V_4 = (4/3).\pi.(R_5^3 - R_4^3)$

Volume $V_4 = (4/3).\pi.\{(5.661\times10^6)^3 - (5.531\times10^6)^3\}$

Volume $V_4 = 51.172\times10^{18}$

Mass of the body which is surrounded by 5531 Km to 3471 km with corresponding to the average density is 4.35×10^3 Kg/m^3.

Mass $M_4 = 0.2226\times10^{24}$ Kg

Further the volume of the earth interior from the 5661Km 5861 Km to, which is at a depth of 510 Km to 710Km, and the average density of that region is 4.1×10^3 Kg/m^3.

Mass of the body surrounded by 5661Km to 5861 Km with corresponding density is 3.8×10^3 Kg/m^3.

Mass $M_5 = 83.448\times10^{18}\times4.1\times10^3$

Mass $M_5 = 0.3421\times10^{24}$ Kg.

further the volume of the earth interior from the 5861Km to 5921 Km, which is at a depth of 450 Km to 510 Km, and the average density of that region is 4.1×10^3 Kg/m^3.

The volume of upper part of upper mantle of the earth is,

$$\text{Volume } V_6 = (4/3).\pi.(R_7^3 - R_6^3)$$
$$\text{Volume } V_6 = (4/3).\pi.\{(5.921\times10^6)^3 - (5.861\times10^6)^3\}$$
$$\text{Volume } V_6 = 26.170\times10^{18}\ m^3$$

Mass of the body which is surrounded by 5861 Km to 5921Km and the density is 3.8×10^3 Kg/m^3.

$$\text{Mass } M_6 = 26.170\times10^{18}\times4.1\times10^3 \text{ kg}$$

$$\text{Mass } M_6 = 107.29\times10^{21} \text{ kg}$$

further the volume of the earth interior from the 5921Km to 6371 Km, which is at a depth of 0 Km to 450 Km, and the average density of that region is 3.3×10^3 Kg/m^3.

The volume of upper part of upper mantle of the earth is,

$$\text{Volume } V_7 = (4/3).\pi.(R_6^3 - R_5^3)$$
$$\text{Volume } V_7 = (4/3).\pi.\{(6.371\times10^6)^3 - (5.921\times10^6)^3\}$$
$$\text{Volume } V_7 = 213.761\times10^{18}\ m^3$$

Mass of the surrounded by surface of the earth and till the depth of 450 Km and corresponding density 3.3×10^3 Kg/m^3.

$$\text{Mass } M_7 = 213.761\times10^{18}\times 3.3\times10^3$$
$$\text{Mass } M_7 = 0.705\times10^{24} \text{ Kg/m}^3.$$

Here mass and volume is listed with respect to its thickness.

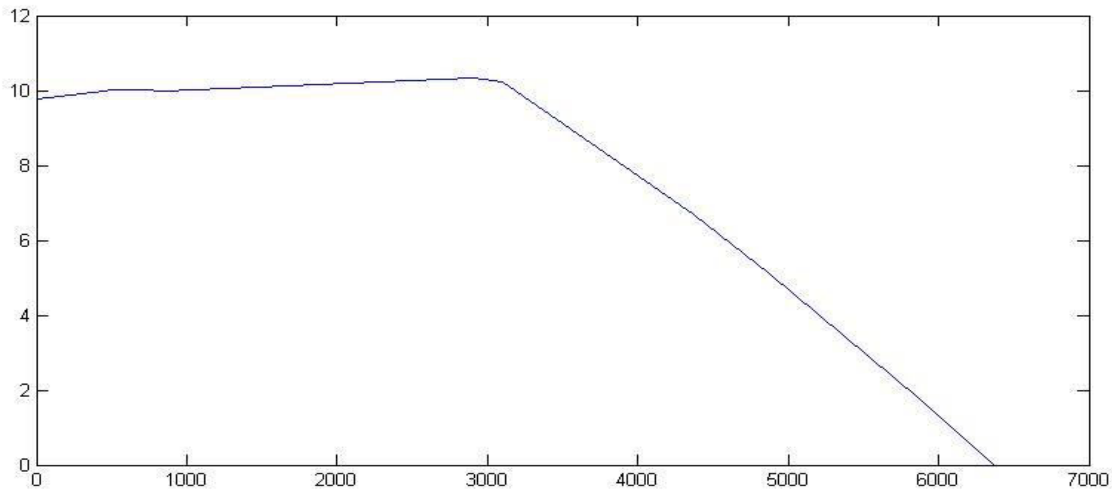
S. No.	Layer region in Km	Volume ($\times10^{18}$ m^3)	Density ($\times10^3$ kg/m^3)	Mass ($\times10^{24}$ Kg)	Cumulative mass ($\times10^{24}$Kg)
1.	0-450	213.761	3.3	0.705	0.705
2.	450-510	26.170	3.8	0.0994	0.8044

3.	510-710	83.448	4.1	0.3421	1.1465
4.	710-840	51.172	4.35	0.2226	1.3691
5.	840-2900	533.75	5.1	2.7221	4.0912
6.	2900-3100	28.5758	7.8	0.2229	4.3141
7.	3100-6371	146.6	11.2	1.6419	5.9560

Measured gravity at depth of density discontinuity is listed below;

S. No.	Distance from the centre of earth	Depth from the earth surface	Mass till the centre of earth ($\times 10^{24}$ Km.)	Gravity (M/s^2)
1.	6371	0	5.9560	9.787
2.	5921	450	5.2510	9.990
3.	5861	510	5.1516	10.003
4.	5661	710	4.8095	10.010
5.	5531	840	4.5866	10.000
6.	3471	2900	1.8645	10.322
7.	3271	3100	1.6416	10.233
8.	2500	3871	0.76	8.10
9.	2000	4371	0.402	6.70
10.	1500	4871	0.172	5.11
11.	1000	5371	0.052	3.46
12.	500	5871	0.0066	1.76
13.	0	6371		0

Graph between the acceleration due to gravity versus depth of the earth interior, throughout the centre;



On X -axis depth from the earth surface (in km.)

And on Y- axis amount of acceleration due to gravity (in m/s^2).

Since it is clear that the acceleration due to gravity with the earth does not change uniformly throughout the centre of the earth, so there can be generate an equation throughout the centre of earth, which will follow the gravity variation with respect to depth.

Acceleration due to gravity of the earth depends on average density at that depth from the centre of earth and distance of that surface from the centre of the earth.

According to the figure (1); there are 7 major density contrasts within the earth, so the equation of gravity with respect to depth can be formed as fallows;

Serial No.	Depth (x) in 10^6 metre	Gravity(y) (in m/sec^2)
1.	0	9.787
2.	450	9.990
3.	510	10.003
4.	710	10.010
5.	840	10.000
6.	2900	10.322
7.	3100	10.233
8.	6371	0

There can be generated general equations which will follow the changing in acceleration due to gravity with depth. Since the gravity varies from surface to the centre of earth, in which till the core mantle boundary this attained a maximum value and from outer core mantle boundary this decrease continuously till the centre of the earth.

Scale;

On X- axis, 10^5 Km = 1 unit

On Y- axis, 1 m/s^2 = 1 unit

All these given equations are strictly valid for that particular depth range.

So equation of line pass through (0, 9.787) and (4.5, 9.990) is

$$0.023.x - y + 9.787 = 0 \ldots\ldots\ldots\ldots\ldots\ldots \text{(8)}$$

Equation of line pass through (4.5, 9.990) and (5.1, 10.003) is

$$0.022x - y + 9.893 = 0 \ldots\ldots\ldots\ldots \text{(9)}$$

Equation of line passes through (5.1, 10.003) and (7.1, 10.01) is

$$0.0035x - y + 9.985 = 0 \ldots\ldots\ldots\ldots \text{(10)}$$

Equation of line passes through (7.1, 10.01) and (8.4, 10) is

$$0.0077x + y - 10.064 = 0 \ldots\ldots\ldots\ldots \text{(11)}$$

Equation of line passes through (8.4, 10) and (29,10.322) is

$$0.016x - y + 9.87 = 0 \ldots\ldots\ldots \text{(12)}$$

Equation of line passes through (29, 10.322) and (31, 10.233) is

$$0.045x + y - 11.627 = 0 \ldots\ldots\ldots \text{(13)}$$

Equation of line passes through (31, 10.233) and (63.7, 0) is

$$0.313x + y - 19.94 = 0 \ldots\ldots\ldots \text{(14)}$$

The equations can be listed as fallows in the form of depth and gravity;

Scale;

On X-axis 1 unit 10^5 Km

On Y- axis 1 unit 1 m/s^2

S. No.	Depth (x)	Gravity (y)	Depth between (in Km)	Equations between gravity and depth	Boundary value
1.	0	9.787	0	Y= 9.787	X= 0
2.	4.50	9.990	0-450	Y = 0.023x+9.787	$0\leq x\geq 4.5$
3.	5.10	10.003	450-510	Y = 0.216x+9.992	$4.5\leq x\geq 5.1$
4.	7.10	10.01	510-710	Y = 0.035x-9.7615	$5.1\leq x\geq 7.1$
5.	8.40	10.000	710-840	Y = -0.008x+10.064	$7.1\leq x\geq 8.4$
6.	29.00	10.322	840-2900	Y = 0.016x+9.87	$8.4\leq x\geq 29$
7.	31.00	10.233	2900-3100	Y= - 0.045x + 11.627	$29\leq x\geq 31$
8.	64.00	0	3100-6400	Y = - 0.313x - 19.94	$31\leq x\geq 64$

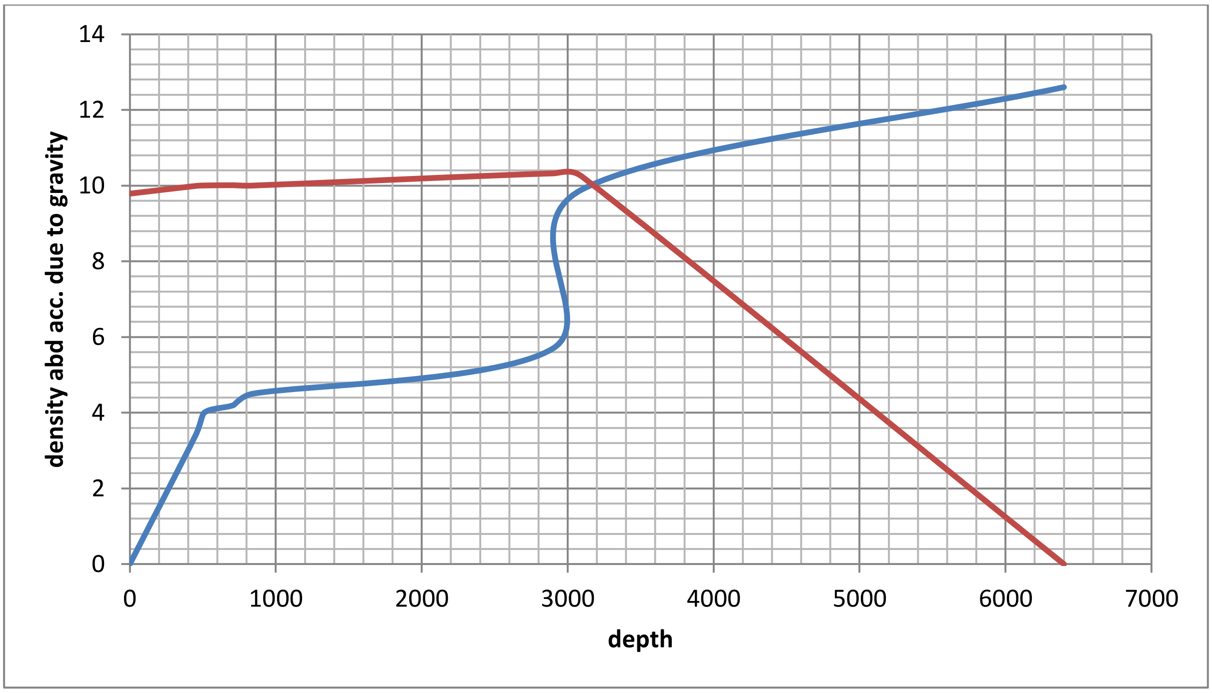
Equation can also be generate in the form of one dependent and two independent variables depth (x), density(y) and gravity (z) which will fallow till the certain depth as fallows;

Here all the variables x, y and z are treated as in 1 unit $=10^6$ meter on X-axis

On Y- axis 1 unit = 10^3 kg/m^3

And on Z- axis 1 unit = $1m/s^2$

S No.	Depth (x)	Density (y)	Gravity (z)	Equation of line $\{(x-x_1)/(x_2-x_1)\}= \{(y-y_1)/(y_2-y_1)\}= \{(z-z_1)/(z_2-z_1)\}= K$	Boundary value
1.	0	0	9.787	Z= 9.787	
2.	0.450	3.4	9.990	{(x-0.45)/(4.5)}= {(y-3.4)/(3.4)} = {(z-9.787)/(0.103)}=K	0≤ x≥4.5, 3.2≤ y≥3.4,
3.	0.510	4.0	10.003	{(x-0.51)/(60)}= {(y-4.0)/(0.6)} = {(z-10.003)/(0.013)}=K	4.5≤ x≥5.1 3.4≤ y≥4.0,
4.	0.710	4.2	10.010	{(x-0.71)/(0.2)}= {(y-4.2)/(0.2)} = {(z-10.01)/(0.007)}=K	5.1≤ x≥7.1 4.0≤ y≥4.2,
5.	0.840	4.5	10.000	{(x-0.84)/(0.13)}= {(y-4.5)/(0.3)} = {(z-10)/(-0.010)}=K	7.1≤ x≥8.4 4.2≤ y≥4.5,
6.	2.900	5.7	10.322	{(x-29)/(20.6)}= {(y-5.7)/(1.2)} = {(z-10.322)/(0.322)}=K	8.4≤ x≥29 4.5≤ y≥5.7,
7.	3.100	9.9	10.233	{(x-31)/(0.2)}= {(y-9.9)/(4.2)} = {(z-10.233)/(-0.89)}=K	29≤ x≥31 5.7≤ x≥9.9,
8.	6.400	12.6	0	{(x-64)/(3.3)}= {(y-12.6)/(2.7)} = {(z)/(-10.233)}=K	31≤ x≥64 9.9≤ x≥12.6,



Blue curve shows density variation with depth

Brown curve shows acceleration due to gravity variation.

Result;

Acceleration due to gravity varies with depth and this value firstly increases from earth surface till the 710 kilometres and further decreases up to 710 to 840 kilometres, again it increases till outer core mantle boundary and finally it continuously decreases till the centre of the earth.

Using above formulas, one can measure the acceleration due to gravity at any depth beneath the earth surface.

Using these information, we can infer many physical properties of the earth surface just like types of material, pressure, and velocity of the seismic waves& its behaviour.

References;

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