ADAPTIVE NOISE CANCELLING ALGORITHMS: A COMPARISON

by

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Abstract:

Adaptive filters have found its applications in radar, sonar, spectral estimation, seismic processing, speech analysis, communication, control theory, noise cancellation, system identification, etc. An algorithm with faster convergence rate should be utilized for using the adaptive filter in real time applications. This paper discusses and compares the performance of three adaptive digital filter algorithms based on the method of steepest descent. The speed of adaptation was measured in terms of signal to noise ratio at the input and output for various iterations and it is observed that normalized least mean square is the fastest of all the algorithms discussed.

1. Introduction:

The use of Adaptive filters for the reduction of noise in the real time systems have increased much since the development of first adaptive noise cancelling system at Stanford University in 1965 by two students.Adaptive filters have been successfully applied to a wide range of applications including radar, sonar, spectral estimation, seismic processing, speech analysis, communications, control theory, noise cancellation, system identification, image processing, neural networks, etc. In its usual form, an adaptive filter is a device that adjust its parameters and optimizes its performance according to the statistical characteristic of its input signals. The parameters are adjusted using adaptive algorithms such as least mean square, signed error least mean square, normalized least mean square, etc.

The purpose of this paper is to compare the properties of certain algorithms which are available for use with adaptive filters. The method of steepest descent is being considered. The performance of these algorithms is compared in terms of signal to noise ratio.

2. Adaptive Digital FIR Filter:

In this paper, we consider the transversal finite impulse response (FIR) filter shown in Figure 1. The filter is having an input x(k) and a set of N adjustable gain coefficients W_o through W_{N-1}. The filter output is given by the expression:

$$y(k) = \sum_{i=0}^{N-1} w_i \, . \, x(k-i) \qquad \text{------ (1)}$$

If the input data vector X(k) and the impulse response vector W are represented

by $X(k) = [x(k), x(k-1), \text{-----}\ x(k-N+1)\ ]^t$

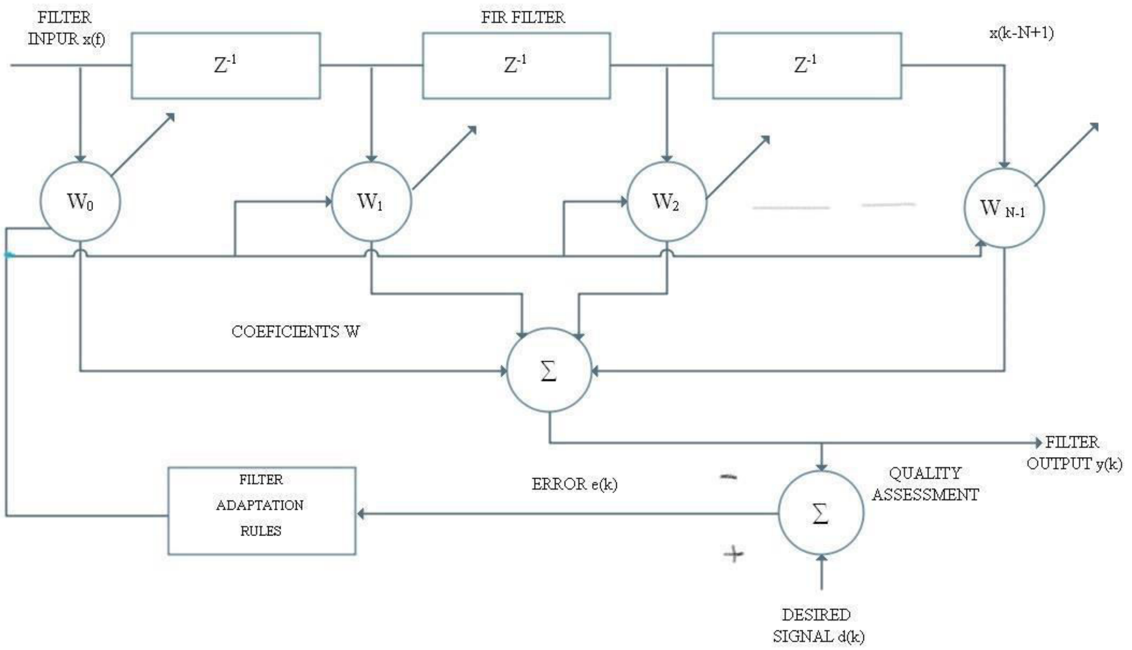
And $W^t = (w_0, w_1, \text{----}, W_{N-1})$,

Then, Eq.(1) can be written as,

$$y(k) - W^t \, . \, X(k) \qquad \text{----- (2)}$$

where the superscript t represent transpose.

FIGURE 1
SCHEMATIC DIAGRAM OF FINITE IMPULSE RESPONSE FILTER (FIR)



2.1. Steepest Descent (Gradient Search) Technique:

The practical objective of the adaptive process is to find an optimum value of the coefficient vector W, say W^0, such that the value of some appropriately defined performance function (J) is minimum shown in Figure 2. Suppose we start from an initial value of W called W(O) where the performance function is J(0). The next point W(1) is selected in such a way that ∂J(1) is less than ∂J(0) where

$\partial J = J - J_{min}$,

$\partial J(0) = J(0) - J_{min}$ and

$\partial J(1) = J(1) - J_{min}$.

Similarly, the values of W(2), W(3), W(4)……. are selected until the value of ∂J reaches zero approximately so that W approaches W^o.

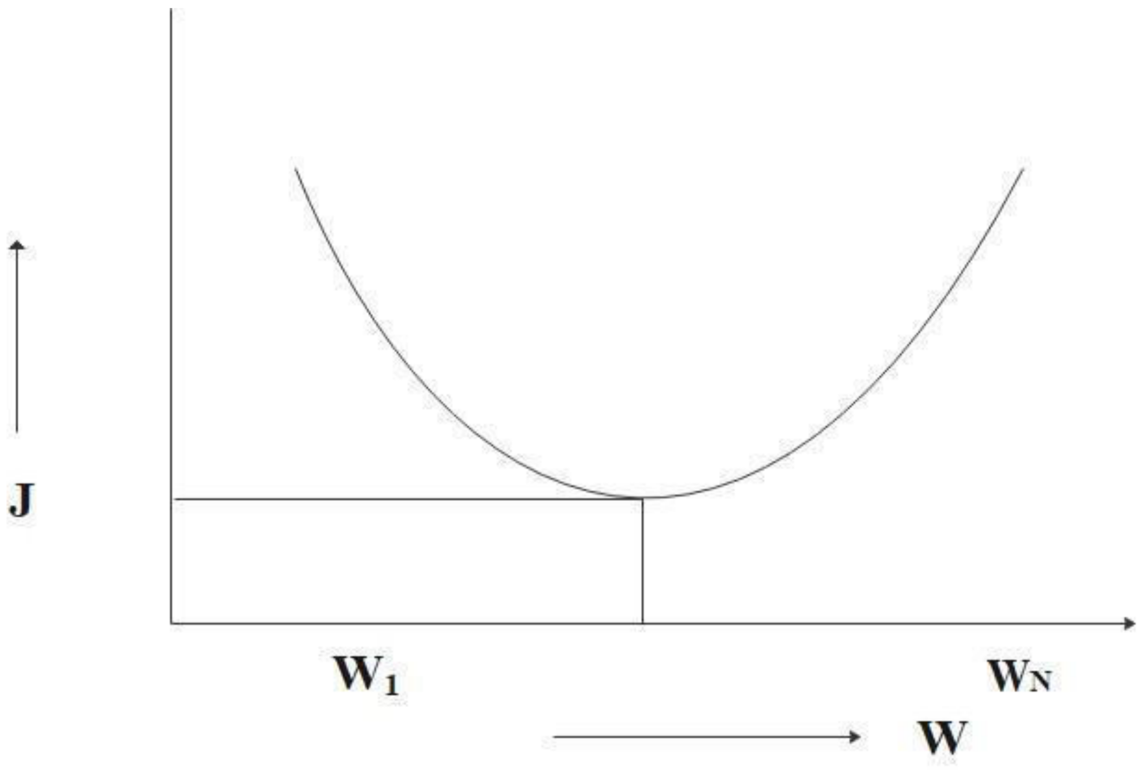


Figure 2
Steepest Descent

3. Adaptive Filter Algorithms:

a. Least Mean Square (LMS):

This is the simplest and the most extensively used adaptive algorithm. It utilizes the steepest descent technique to determine the filter coefficients which minimizes the mean square of the estimation error.

The filter output is

$$y(k) = W^t(k).\ X(k)$$

and the estimation error is

$$e(k) = d(k) - y(k).$$

where d(k) is desired response

The filter coefficient vector is updated by the expression

$$W(k+1) = W(k)+\mu.e(k).\ X(k)$$

Where μ is a bounded step size (constant).

b. Signed Error Least Mean Square (SELMS):

After having looked, at the real arithmetic version of least mean square algorithm, it was felt that the amount of computation should be lowered although this computation is lower than many complicated adaptive algorithms. One such simplified algorithm is the signed error least mean square algorithm which employs only the sign of the error for updating its coefficients. That is

$$e'(K) = \mathrm{sgn}\{d(k)-y(k)\}$$

$$= \begin{cases} 1 \text{ if } d(k)-y(k) > 0 \\ 0 \text{ if } d(k)-y(k) = 0\ - \\ 1 \text{ if } d(k)-y(k) < 0 \end{cases}$$

Filter coefficient vector is updated as

$$W(k+1) = W(k) + \mu.e'(k).X(k).$$

c. Normalized Least Mean Square (NLMS):

The LMS and SELMS algorithms discussed above use a constant adaptation step-size μ, a small constant which determines among other things the speed of convergence of the algorithm. In NLMS, we choose the adaptation step-size as

$$\mu(k) = \frac{\alpha}{\tau + X^t(k).X(k)}$$

where 'α ' is the new adaptation constant, τ is a small positive term included to ensure that the update term does not become excessively large when $X^t(k).X(k)$ becomes temporarily small.

So, the coefficient vector is updated as

$$W(k+1) = W(k) + \frac{\alpha.\ e(k)\ .\ X(k)}{\tau\ + X^t(k)\ .\ X(k)}$$

4. Signal to Noise Ration (SNR):

The signal to Noise ratio is defined as

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}}$$

So, $(SNR)_{dB} = 10.\ Log_{10}\ (SNR)$.

5. Simulation Results:

The data x(k) was generated as

$$x(k) = \sin(2\ \ (f/fs)k) + (\sqrt{120}).\ v(k)$$

where f is the signal frequency, f_s is the sampling frequency and v(k) is a zero-mean white noise.

The filter output y(k) is obtained for each input sample and was compared to the desired value d(k) which gives the error e(k) which was used in various filter adaptation algorithms.

The comparison in terms of SNR at the output for various algorithms was done and it was found that when a signal of SNR -12.64 dB was considered at the input and the corresponding value of SNR at the output was calculated for various iterations and it was found that NLMS takes only 100 iterations to have an improvement of 7.39 dB whereas the improvement given by LMS is only 4.01 dB after the same number of iterations.

The SNR improvement against the number of iterations for the various algorithms is considered.

6. Conclusions:

The adaptation algorithms, viz. Least Mean Square, Signed Error Least Mean Square and Normalized Least Mean Square which are based on the steepest descent technique were studied and compared using the Signal to Noise ratio at the output as the measure of performance. It was found that Normalized Least Mean Square algorithm is the fastest of all the algorithms discussed as it took less number of iterations to have larger improvement of SNR.

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