International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 04 Issue: 08 | August -2020 ISSN: 2582-3930

DIRECT AREA METHOD
SLOPE & DEFLECTION OF SIMPLY SUPPORTED BEAM CARRYING "U.V.L"

N.BABU, R.PRASAD, AK.MD MUBARAK

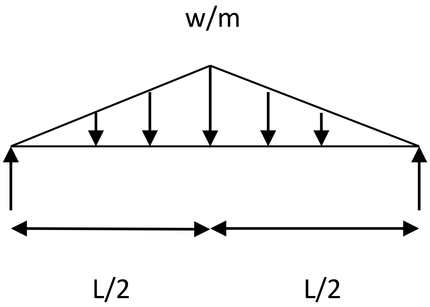
N.Babu M.E.,Department of Civil Engineering,Pollachi Institute of Engineering and Technlogy,Pollachi,Coimbatore.
R.Prasad B.E., Department of Civil Engineering,Sri Ramakrishna Mission Vidyalaya Polytechnic College,Coimbatore
AK.Md Mubarak B.E., Civil Engineer,,Coimbatore.

---***---

Abstract - Direct area method is combination of Mohr's theorem and double integration method. This method is used to find out area constant and centre of gravity of bending moment diagram for simply supported beam carrying uniformly varying load.

1. INTRODUCTION

We using double integration method to find out slope and deflection for the simply supported beam carry point loads, UDL, symmetrical and unsymmetrical uniformly varying load acting along the span.



Symmetrical U.V.L

Fig1.

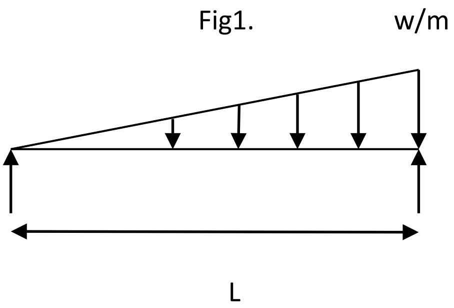


Fig2. Unsymmetrical U.V.L

Mohr's theorem is one of the methods to find out slope and deflection by using graphical representation of bending moment diagram.

In Mohr's theorem for UVL except moment diagram by parts the direct formula which is not mentioned in symmetrical UVL bending moment diagram of 3^0 curves.

2. Body of Paper

Mohr's –I Theorem:

$$i_{AB} = \frac{(\textbf{AREA OF B.M.D})_{AB}}{\textbf{EI}}$$

The change in angle of slope between the tangents at any two points (A and B) on the elastic curve is equal to the area of bending moment diagram in between those two points divided by flexural rigidity (EI).

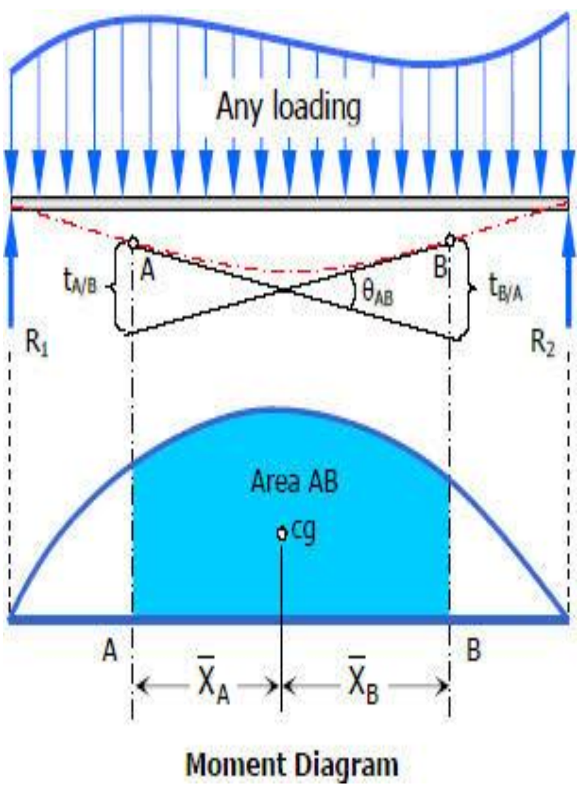


Fig 3.

International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 04 Issue: 08 | August -2020 ISSN: 2582-3930

Mohr's -II theorem:

$$Y_{AB} = \frac{(\text{AREA OF B.M.D})_{AB}\, X_B}{EI}$$

The tangential deviation of any point B on the elastic curve from a tangent at any other point A on the elastic curve, perpendicular to the original axis of the beam is equal to the moment of inertia of BMD in between those two points about B divided by flexural rigidity.

i or θ = slope

Y = Deflection

Double integration method:

The Double Integration Method, also known as Macaulay's Method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.

In calculus, the radius of curvature of a curve y = f(x) is given by

$$\rho = \frac{(1 + ((dy/dx)^2)^{3/2}}{(d^2y/dx^2)}$$

The radius of curvature of a beam is given as

$$\rho = \frac{EI}{M}$$

E = modulus of elasticity

M = moment

Deflection of beams is so small, such that the slope of the elastic curve dy/dx is very small, and squaring this expression the value becomes practically negligible, hence

$$\rho = \frac{1}{(d^2y/dx^2)} = \frac{1}{y''}$$

Thus, (EI/M) = (1/y")

$$y'' = \frac{M}{EI}$$

If EI is constant, the equation may be written as:

EIy"=M

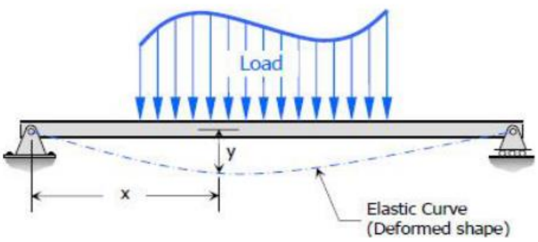


Fig 4.

where x and y are the coordinates shown in the above Figure of the elastic curve of the beam under load, y is the deflection of the beam at any distance x. E is the modulus of elasticity of the beam, I represent the moment of inertia about the neutral axis, and M represents the bending moment at a distance x from the end of the beam. The product EI is called the flexural rigidity of the beam.

The first integration y' yields the slope of the elastic curve and the second integration y" gives the deflection of the beam at any distance x. The resulting solution must contain two constants of integration since EI y" = M is of second order. These two constants must be evaluated from known conditions concerning the slope deflection at certain points of the beam. For instance, in the case of a simply supported beam with rigid supports, at x = 0 and x = L, the deflection y = 0, and in locating the point of maximum deflection, we simply set the slope of the elastic curve y' to zero.

Boundary Conditions: Generally, the deflection is known as y-values and a slope is known as dy/dx. The values are called boundary conditions, which normally are:

1.For simply supported beams:

(i)At the x-values of the two supports, deflection is zero, i.e. y = 0.

(ii)If the point (i.e. the x-value) of maximum deflection is known, then at the x-value of the point, the slope is zero, i.e. dy/dx= 0.

2. For cantilever beams, at the x-value of the built-in end:

(i)The deflection is zero, i.e. y = 0.

(ii)The slope is zero, i.e. dy/dx= 0

From the double integration method:

Finding slope and deflection of beam shown in fig1.

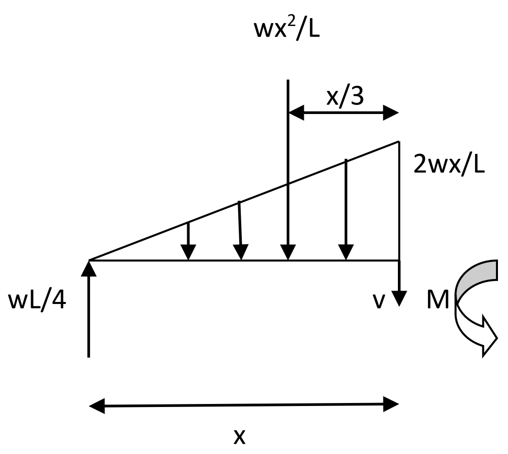


Fig5.

Reaction at A and B = wL/4

M $=\frac{WLx}{4}-\frac{wx^2}{L}(\frac{X}{3})=\frac{w}{12L}(12L^2x-4x^3)$

EIy'' $=\frac{w}{12L}(12L^2x-4x^3)$

EIy' $=\frac{w}{12L}\left(\frac{3L^2x^2}{2}-x^4\right)+C_1$

EIy' $=\frac{w}{12L}\left(\frac{L^2x^3}{2}-\frac{x^5}{5}\right)+C_1x+C_2$

Applying condition:

x = 0 (deflection @ A = 0)

EIy' $=\frac{w}{12L}\left(\frac{L^2x^3}{2}-\frac{x^5}{5}\right)+C_1x+C_2$ $\quad C_2=0$

x = L/2

EIy' $=\frac{w}{12L}\left(\frac{3L^2x^2}{2}-x^4\right)+C_1$ $\quad C_1 = 5wL^3/192$

The equation of elastic curve segment AB

EIy' $=\frac{w}{12L}\left(\frac{L^2x^3}{2}-\frac{x^5}{5}\right)+\frac{5wL^3}{192}$

EIy $=\frac{-wx}{960L}(25L-40L^2x^2+16x^4)$

Symmetrically loaded beam, so the maximum bending moment occurs at centre.

$EIy_{x=L/2}=\frac{-w}{960L}\cdot\frac{L}{2}\left(25L-40L^2\frac{L^2}{4}+16\frac{L}{16}^4\right)$

$Y_C=\frac{wL^4}{120EI}$

Maximum deflection at centre

$\mathbf{Y_{MAX}=\frac{wL^4}{120EI}}$

We know the formula for slope and deflection of beam. The formula substitute in the Mohr's theorem to finding area constant and centre of gravity for 3^0curves.

From Mohr's theorems:

Finding area constant and centre of gravity of beam shown in fig1.by using Mohr's theorem.

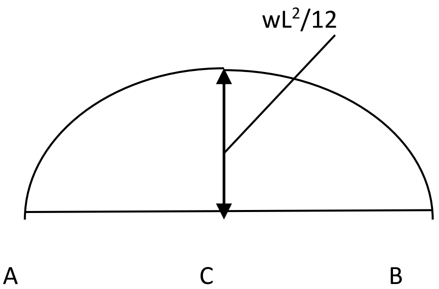


Fig 6. BMD

Symmetrically loaded so taking section "CB"

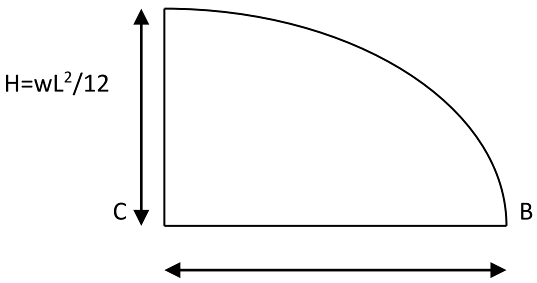


Fig7. B=L/2

By using Mohr's1 theorem find out area constant (X)

$$\mathbf{i_B=\frac{(AREA\ OF\ B.M.D)_{CB}}{EI}}$$

$$\frac{5wL^3}{192EI}=\frac{X\ x\ B\ x\ H}{EI}$$

$$\frac{5wL^3}{192EI}=\frac{X\ x\frac{L}{2}\ x\ \frac{wL^2}{12}}{EI}$$

$$\frac{5wL^3}{192EI}=\frac{\frac{XwL^3}{24}}{EI}$$

$$X=\frac{5\ x\ 24}{192}=\mathbf{\frac{5}{8}}$$

By using Mohr's -2 theorem find out centroid (X_B) :

$$\mathbf{Y_C=\frac{(AREA\ OF\ B.M.D)_{CB}\,X_B}{EI}}$$

$$\frac{wL^4}{120EI}=\frac{5wL^3\,X_B}{192EI}$$

$$X_B=\frac{192}{5x120}$$

$$\mathbf{X_B=\frac{8L}{25}}$$

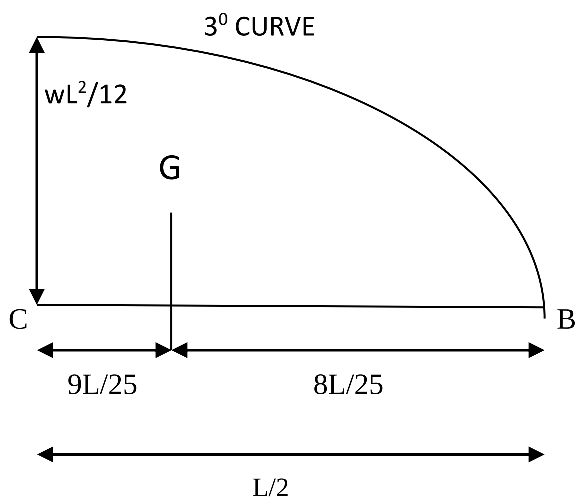


Fig 8

Area and Centre of gravity for 3^0 curve to length 'L':

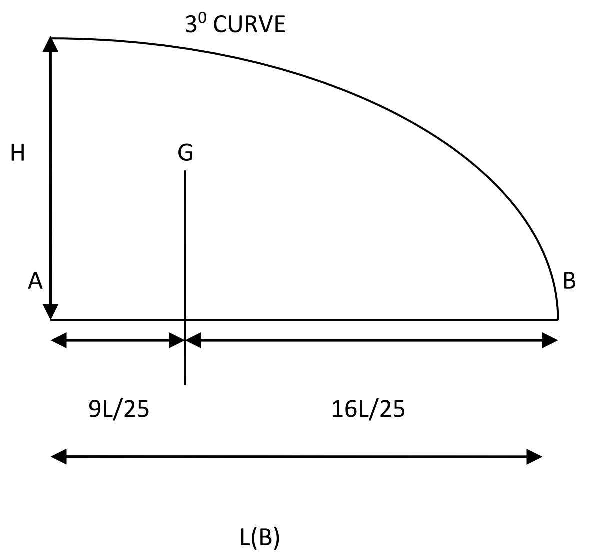


Fig 9

Area of 3^0 curve for simply supported beam

$$= \frac{5}{8} \text{ X B X H}$$

Centre of gravity:

$X_A = \frac{9L}{25}$ and

$X_B = \frac{16L}{25}$

Area and centre of gravity distance for Uneven 3^0 curve bending moment diagram:

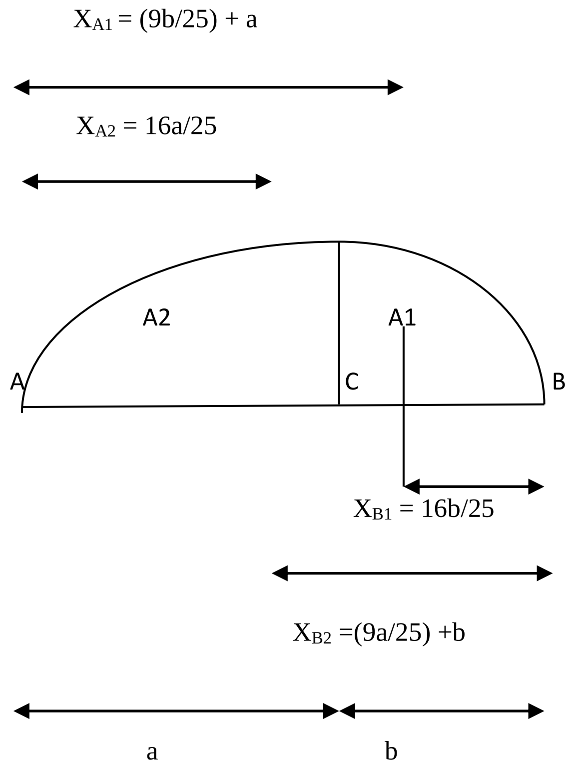


Fig10 BMD

Area formula:

A_1 = **(5/8) x b x H**

A_2 = **(5/8) x a x H**

Centre of gravity:

X_{A1} = **(9b/25) +a**

X_{A2} = **16a/25**

X_{B1} = **16b/25**

X_{B2} =**(9a/25)+b**

3.ADVANTAGES:

1. This method is very simple to find out slope and deflection in UVL acting on the simply supported beam.

2. Area and centre of gravity of BMD is useful for analyzing indeterminate beams by using conjugate beam method.

3. in theorem of three moments used in continuous beam. find out the factors of three moments. Easily when the span carries UVL..

4. Fixed beam carries UVL easily find out the moments, slope and deflection by using moment area method.

4.CONCLUSIONS

From direct area method, the area and centre of gravity formula for bending moment diagram to UVL loading in simply supported beam are directly applying in Mohr's theorem to find out slope and deflection easily and analyzing statically indeterminate beams also.

ACKNOWLEDGEMENT

We would like to express our deep gratitude to our professors and we wish to thank our family for the support and encouragement throughout our study

REFERENCE:

1. .STRENGTH OF MATERIALS- R.NARAYAN and S.RAMAMRUTHAM

1. STRENGTH OF MATERIALS- Dr.RK.BANSAL

2. STRENGTH OF MATERIALS- Dr.RK.RAJPUT

3. STRENGTH OF MATERIAL –R.S.KHURMI