International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 03 Issue: 12 | Dec -2019 ISSN: 2582-3930

Least weight Design of a Brittle Cantilever Loaded Uniformly

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ABSTRACT: The minimum weight design of a cantilever beam for a specified probability of failure is considered. The higher the specified probability of failure the lesser is the depth.

Introduction: Brittle materials are characterized by their pronounced strength dependence on the specimen size as well as the decrease in mean strength and the associated scatter with increase in specimen size[1-4]. In designing structural elements with such materials, safety is expressed more rationally in terms of probability of failure. Among the many statistical theories of strength advanced[2], the most widely used one is due to Weibull[1], according to which the probability of failure P_F is given by

$$P_F=1-\exp\left[-\int_V (\sigma/\sigma_0)^m dv\right]$$

whereσ is the strength in atypical element,σ_0 is a characteristic strength and m is a constant that reflects the scatter in observed strength values. The integral in

Eq. (1) extends over the volume of the material that is stressed in tension. Consequently in designing a structural element like a beam, For example, the proportioning of the elements affects the stress distribution and the probability of failure.

In this paper minimum weight design of a cantilever beam for a specified probability of failure is considered and the method is illustrated with an example. In the limiting case the results reduce to those of deterministic design. The design method developed permits incorporating scatter in strength quantitatively in the optimization process.

The design method is the same as the advanced earlier[4-5].

Optimality criterion -Consider a cantilever beam uniformly loaded, of length *L*, of uniform width *b* and of variable depth h (*x*) as shown in Fig. 1. It is required probability of failure of the beam has a preassigned value P^*_F

From Eq. (1) the condition that the beam should have a failure probability P^*_F implies that

$$\int_V (\sigma/\sigma_0)^m dv = \log\left(\frac{1}{1-P_F^*}\right)$$

or

$$\int_V \sigma^m dv = K^* \sigma_0^{*m} \qquad (2)$$

Where

$$K^* = \log\left(\frac{1}{1-P_F^*}\right)$$

The total weight of the beam φ is shown by

$\varphi \;\; = \rho \int_0^L b \;\; h(x)dx$ (3)

whereρ is the unit weight of the material. The bending moment M(x) at a typical section is $wx^2/2$ and the moment of inertia I(x)=b $h^3(x)/12$, so that the tensile stress in an element located at a depth Y below the neutral axis is given by σ =M(x).Y/ I(x). For simplicity only bending stress distribution is considered. Substituting for σ in Eq. (2) and setting the limits of integration over the volume stressed in tension we obtain, after simplification,

$\beta \;\; \int_0^L x^{2m}\, h^{1-2m}\, dx = \; K^*$ (4)

Where

$\beta \;\; = \; 3^m w^m (m+1) b^{m-1} \sigma_0^m$

The optimization problem now reduces to finding a$h(x)$ that satisfies Eq. (4) and minimizes φ given by Eq. (3)

The optimality condition is given by

$\rho\text{b}+\beta\;\lambda x^{2m}\;(1-2m)h^{2m} = 0$

h=$[\lambda\;\beta\;(2m-1)/\rho b]^{1/2m}\;x$ (5)

Where λ is the Lagrangian multiplier which can be is is evaluated from Eq^{n}s (5) and (4); the optimal variation of depth $h(x)$ can be found to be

h (x) = $[2K^* /\beta L^2]^{1/(1\text{-}2m)} x$ (6)

Now introducing the variables H, B, Q and X as given by

h= HL; b= BL; Q= $(wL^2/\sigma_0)^m$; $x = XL$

we can rewrite Eq. (6) as

H=$[2K^*\,(m+1)/3^m]^{1/(1\text{-}2m)}$

$[B^{m\text{-}1}/Q^m]^{1/(1\text{-}2m)}$ X (7)

The variation of depth along the span is shown schematically in fig. 1,with

H= $D[B^{m\text{-}1}\,/\,Q^m]^{1/(1\text{-}2m)}X$ (8)

Where

D=$[2K^*\,(m+1)/3^m]^{1/(1\text{-}2m)}$

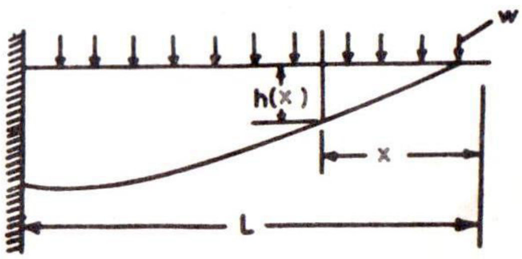

Fig. 1-Least Weight design of cantilever with uniform loan (schematic)

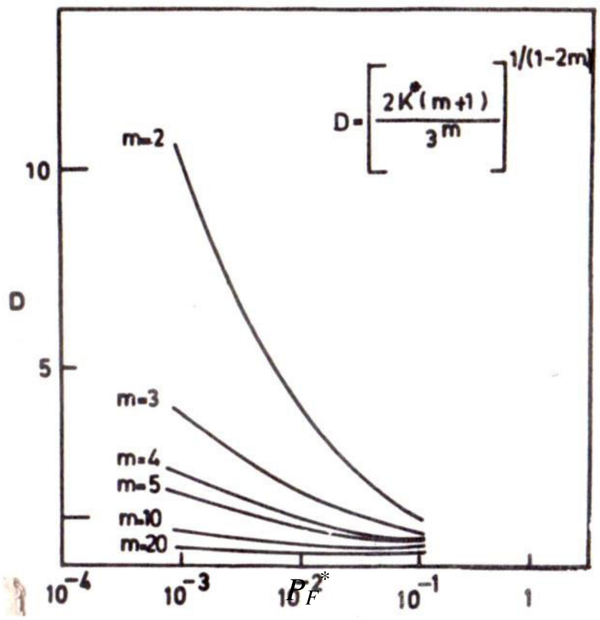

Fig. 2- variation of constant D with probability of failure P_F*

The variation of the factor D in Eq. (8) with probability of failure is shown in fig. 2 correspondingto some chosen values of m of practical interest. A material having lower value of m has more scatter in strength. From Fig. 2 and Eq^{n}s (7) and (8) it can be seen that the more is the specified level of P_F less is the depth h and hence less is the weight of the beam. This conclusion was arrived at with references to other problems solved as well[4,5].

This work was supported financially by the state Council of science and Technology, Uttar Pradesh.

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