Model Based Controller Design for Inverted Pendulum

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Abstract*: The Inverted-pendulum mechanism is commonly found in many applications in real time, such as robotics weapon, bicycle, missile launch, etc. Without proper control strategy, this system remains unstable. The dynamic structure shows highly nonlinear behaviour. The main purpose of the proposed controller is to stabilize the Inverted-pendulum by applying the output signal of the controller to the cart attaching the pendulum. The Inverted-pendulum concept directly relates the booster rocket attitude control during take-off. Thus it applies the traditional and model-based control methodology and further addresses the effects. The parameter tuning of the controller was done correctly by implementing the controller output-PID, Fractional PID and further with Model Predictive Controller.

***Keywords* : Inverted pendulum control, Modeling, MPC, PID, Fractional PID, Transfer function.**

I . INTRODUCTION

The Inverted-pendulum system which could be viewed as a classic example of non linear control problem is unstable, high order, multivariable and highly coupled and under actuated system . To analyze specific control theories or a standard solution Inverted-pendulum system is the well established benchmark problem and thus new theories are promoted. Various fields such as artificial intelligence, semiconductors, rockets and missiles, shipyard heavy cranes lifting containers, self-balancing robots, earthquake-resistant building design, potential transport systems such as segways and jetpacks, etc., make use of the inverted pendulum. Since its center of gravity lies behind the center of drag that causes aerodynamic instability, the design of the Inverted-pendulum parallels that of the missile or rocket launcher. The Inverted-pendulum is called the single-input multi-output operation (SIMO) with control voltage as input and cart position and pendulum angle as output. Although the device is easy from a design point of view, due to the following characteristics, there is a lot of control difficulty.

Vertical up and vertical down: Two equilibrium states are present for the Inverted-pendulum system where the vertical up is an unstable balance point, and the vertical down is a stable balance point. The inverted position is the point of instability, as shown by the non linear dynamic equations.

Highly non-linear: Inverted-pendulum is considered as a non-linear system as non-linear terms are composed of the dynamic equations. Usually in real control the system model is linearized.

Non-minimal phase systems: Inverted-pendulum system transfer mechanism includes right-hand plane zeros affecting stability margins like robustness

Under actuated: The system has two degrees of freedom, but a single actuator only, i.e. the D.C. Drive. This machine is under-actuated, making the system cost-effective, but raising the control problem.

Uncertainty: Most uncertainties are due to uncertainty of the model, error of transmission and other resistance. By controlling errors, these uncertainties are reduced in real control.

II. SYSTEM MODELLING

This system consists of a motorized cart fixed sideways to the pendulum. The inverted pendulum system is extremely popular in science literature and control systems textbooks. This popularity gives way to the fact that it is unstable without power, that is, If the cart is not balanced, the pendulum will actually fall over. In addition, the inverted pendulum system's dynamics are non-linear. The control system's purpose is to stabilize the inverted pendulum by applying a force on the cart to which the pendulum is attached. An example of real-time evidence of the inverted pendulum process is the guiding attitude of a booster rocket at takeoff. The control input for this system is the force F that pushes the cart horizontally, and Outputs are pendulum angular orientation and the horizontal direction of the carts x. The pendulum in this case is restricted to moving through the vertical plane as shown below.

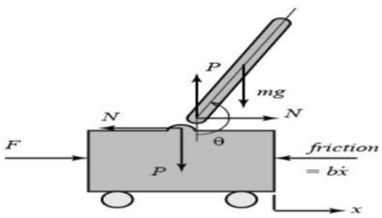

Fig.1 Free Body Diagram of the Inverted Pendulum System.

We will be only interested in controlling the pendulum location in the PID, response of the frequency and root locus response of the frequency parts of this problem. This is because the methods used in these parts are best suited for SISO systems.Hence, no other design criteria relates to the location of the cart.Once the controller has been designed, investigation on the effect of the controller on the position of the cart will take place. We must build a controller in these sections to restore the pendulum upwards vertically following an impulsive bump in the cart.

A. System Equation

Summing the forces in the cart's free-body diagram in horizontal direction, following equation of motion is as follows.

$$M\ddot{x} + b\dot{x} + N = F \quad (1)$$

Note that the cart forces can also be summed up vertically but no useful information would be obtained.

The following expression for the reaction force N will be the summation of the forces in the pendulum in horizontal direction

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \quad (2)$$

By replacing this equation with the first equation one of the two governing equations will be obtained for that system.

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \quad (3)$$

For this method, sum up the forces perpendicular to the pendulum to obtain the second equation of motion. Solving the problem along this axis simplifies mathematics considerably. The following equation should be obtained.

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \quad (4)$$

To get rid of the terms P and N in the above equation, sum up the moments about the pendulum centroid to get the equation that follows.

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \quad (5)$$

By combining the last two terms, the second governing equation is achieved.

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \quad (6)$$

Since the analytical methods and control methods we will use in this example only apply to linear systems, we need to linearize this set of equations. In particular, we will linearize the equations of the equilibrium location vertically up, θ = , thus infer, the system remains within this tiny equilibrium. This statement should be fairly true, because we do not want the pendulum to deviate more than 20 degrees under control from the vertically upward position. Let $\boldsymbol{\phi}$ reflect a deviation from equilibrium in the pedulum position, that is,θ =$\boldsymbol{\pi}$+ $\boldsymbol{\phi}$. Again, the following small angle can be used in our system equations ($\boldsymbol{\phi}$) approximations of nonlinear functions if we presume a small deviation from the equilibrium:

$$\cos\theta = \cos(\pi + \phi) \approx -1 \quad (7)$$

$$\sin\theta = \sin(\pi + \phi) \approx -\phi \quad (8)$$

$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0 \quad (9)$$

After replacing the above approximations in our nonlinear governing equations we arrive at the two linearized equations of motion. Note u was replaced with Input F.

$$(I + ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x} \quad (10)$$

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u \quad (11)$$

B. System Parameters

Let's assume that the parameters of the system are:

mass of the cart (M) = 0.6 kg
mass of the pendulum (m) = 0.25 kg
coefficient of friction of cart (b) = 0.15 N/m/sec
length to the pendulum center of mass (l) = 0.4 m
mass moment of inertia of the pendulum (I) = 0.007 kg.m^2
force applied to the cart (F)
cart position coordinate (x)
pendulum angle from vertical (down) (theta)

C. Transfer function model

To achieve the transfer functions of a linear system equation, we have to take the transform system equations of Laplace first assuming zero initial conditions. The corresponding Laplace transformations are to be seen below.

$$(I+ml^2)\phi(s)s^2 - mgl\phi(s) = mlX(s)s^2 \quad (12)$$

$$(M+m)X(s)s^2 + bX(s)s - ml\phi(s)s^2 = U(s) \quad (13)$$

Note that a transfer function is the relation between a single input and a single output at one time. We need to subtract X(s) from the above equations To find our first performance transfer function and an entry for U(s). First Equation Solution for X(s).

$$X(s) = \left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s) \quad (14)$$

The above equation is then substituted into the second equation.

$$(M+m)\left[\frac{I+ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s - ml\phi(s)s^2 = U(s)$$

(15)

By rearranging, the below transfer function is obtained.

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad (16)$$

Where,

$$q = [(M+m)(I+ml^2) - (ml)^2] \quad (17)$$

From the above transfer function it can be seen that both a pole and a zero are at the origin. These can be cancelled and the role of transfer becomes the next.

$$P_{pend}(s) = \frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \quad \left[\frac{rad}{N}\right] \quad (18)$$

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad \left[\frac{m}{N}\right] \quad (19)$$

III. SYSTEM ANALYSIS AND CONTROLLER DESIGN

In case of obtaining the time domain data of cruise control system, the step input has been provided to first order transfer function and state space model[4][5]. The output response has been plotted and the time domain parameter has been tabulated.

(i) Mass of the vehicle (m) = 2000 kg
(ii) Damping coefficient (b) = 60 N.s/m
(iii) Nominal control force (u) = 600 N

A. Design Criteria

Given the pendulum response to a 1-N sec wheel impulse, the pendulum design requirements are,

1. Rise time < 0.5s
2. Settling Time <5s
3. Steady state error < 2% for Position & angle
4. Deviation angle disturbance <0.05 rad/sec

B. Conventional PID Controller Design

The mathematical expression of PID controller is,

$$u(t) = K_p e(t) + K_p K_i \int_0^t e(t)dt + K_p K_d \frac{d}{dt} e(t)$$

(20)

Proportional controller Kp provides the speed of the response but introduce offset error. In order to eliminate this offset error, the integral controller Ki is added. Further in order to eliminate the overshoot being provided the derivative mode Kd is introduced. The combination of PID provides better controller action. For better design the selection of controller parameter values i.e., the values of Kp , Ki , Kd is very much important.

C. Fractional PID Controller Design

The mathematical expression of Fractional PID controller is

$$G_c(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \qquad (21)$$

For the generalization of the traditional integer pid controller the fractional order PID controller denoted by was introduced. Where **λ** and **μ** are two additional integral parameters and the conventinal PID controller derivative components, the complexity of tuning these parameters will increase. For better design the selection of controller parameter values i.e., the values of Kp , Ki , Kd is very much important.

D. Model based Predictive Controller Design

In Model Predictive Control (MPC) prediction horizon and control horizon selection plays a vital role in order to control the entire process at desired pre set value. MPC is known to be one of the developed and popular model based control technique as it can manage different forms of control problems such as interactions, constrains, multi-inputs and multi-outputs etc.

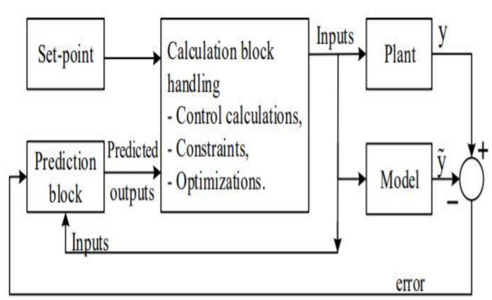

Fig. 2 General Structure of MPC

The two well known companies Shell Oil and ADERSA holds the credit of proposing the elementary idea regarding MPC around 1980 [12]. Model of the plant is the major component of such control strategy that affects the performance of control loop. Figure 2 shows the general architecture of MPC control scheme [8]. Hence it is an advantage to have accurate model of the plant compared to inaccurate model.

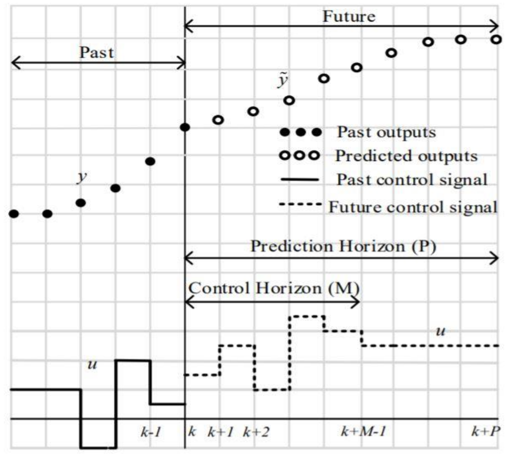

Fig.3 Horizon based approach on MPC

Figure 3.shows the control and prediction Horizon based approach of MPC to control speed at 10m/sec. MPC takes action on future moves based on its prediction and control horizon moves [9].

IV.RESULT AND DISCUSSION

Open loop step response for inverted pendulum system

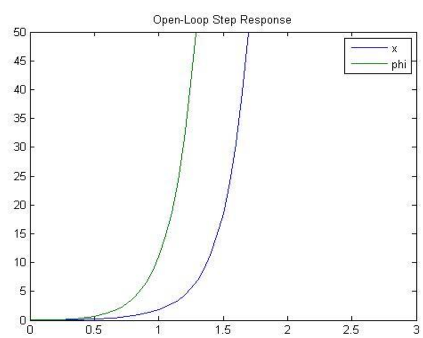

Fig.4 Step response of inverted pendulum system for the control force of 1N

Open loop impulse response for inverted pendulum System.

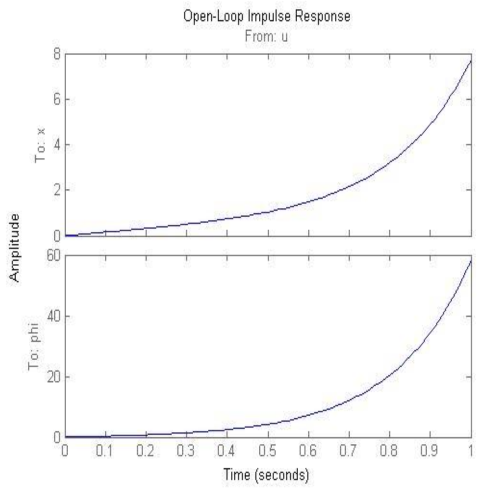

Fig.5 Impulse response of inverted pendulum system for the control force of 1N

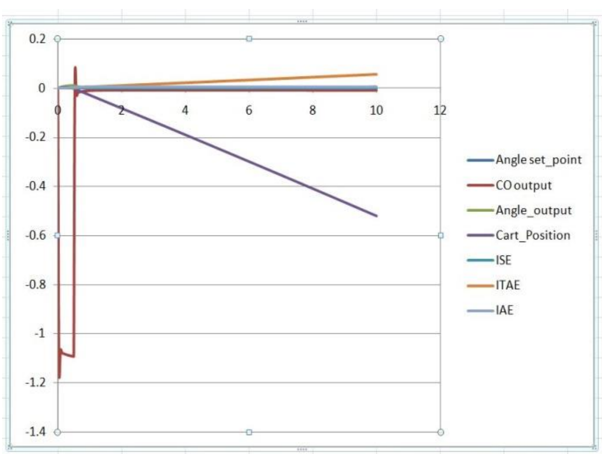

Fig.6 Closed loop response for PID: Kp=100, Ki=1, Kd=20 based inverted pendulum system

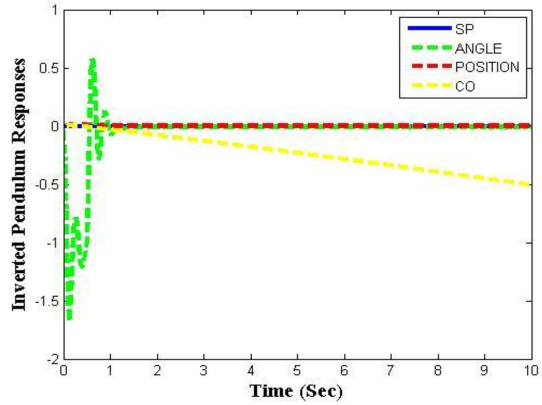

Fig.7 Closed loop response for Fractional PID: Kp=100, Ki=1, Kd=20 based inverted pendulum system

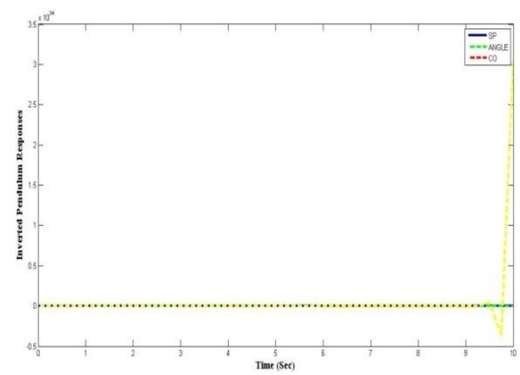

Fig. 8 Closed loop response for MPC based inverted pendulum angle control.

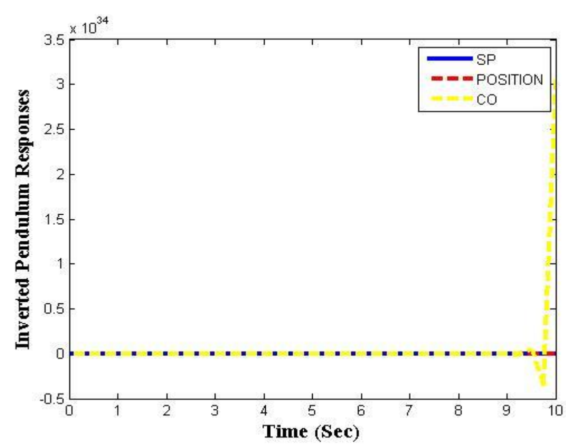

Fig. 9 Closed loop response for MPC based inverted pendulum cart position control

CONTROLLER	ISE	ITAE	IAE	SETTLING TIME (sec)
PID	$3.67E^{-05}$	0.057206	0.005720	5
FRACTIONAL PID	$3.95E^{-05}$	0.046957	0.004336	4
MPC	$2.45E^{-05}$	0.024367	0.002235	2

Table.1 Comparison of PID, Fractional PID, MPC based Inverted Pendulum System

V.CONCLUSION

From the simulation results, MPC provides better set point tracking capabilities under various step and load change conditions. Fractional PID based parameters tuning provides optimum values, which yields efficient control action on Cart Position and Pendulum Angle Inverted pendulum system. Thus when compared to PID and Fractinal PID, MPC gives lesser settling time and overshoot. The minimization of steady state error has been done efficiently in MPC compared to PID, Fractional PID controller tuning parameters.

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BIOGRAPHY

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