Modification of the General Expression to IndicateEquivalence of Energy

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Abstract

The law of conservation of energy holds true in classical mechanics as well as in special relativity as long as the frame of reference remains unchanged. But as the Newtonian equivalents of classical energy are not equivalent to the relativistic expressions at high velocities there arises the need for a modified expression which works both for classical mechanics as well as relativity. In the following paper such a relation has been derived from the work done by Albert Einstein and Paul Dirac. This derived expression also is applicable to Classical Mechanics as well as Special Relativity. The new expression is equated to an expression of relativistic potential energy and has been later verified by calculations.All calculations have been done for a case where a body moves from rest to motion in an inertial single frame of reference.Also these experiments and calculations are carried out for an isolated system.

Derivation

Assuming that a body is in motion, the total energy (E_t) is equal to the sum of Rest Energy and Kinetic Energy

∴ $E_t = mc^2 + K.E$

~ $E_t = mc^2 + (\gamma mc^2 - mc^2)$ [$K.E = \gamma mc^2 - mc^2$ (from Einstein's Equation for Relativistic K.E)]

~ $E_t = mc^2 - mc^2 + \gamma mc^2$

~ **$E_t = \gamma mc^2$ ---------------- (i)**

Assuming that the body changes from rest to motion

∴ E_t = Rest Energy + Kinetic Energy (K.E) + Potential Energy (P.E)

~ **$E_t = \gamma mc^2 + P.E$** [Rest Energy + K.E = γmc^2 (from i)] ----------**(ii)**

From Dirac's Original form of Energy-Momentum Relation

∴ $E_t = \sqrt{c^2p^2 + (mc^2)^2} + v$ [where v= P.E and p= momentum]

~ $E_t = \sqrt{c^2p^2 + (mc^2)^2} + P.E$ ------------ **(iii)**

As both the equation describe the Total Energy (E_t) of a system containing Potential Energy (P.E)

From eq(ii) and eq(iii)

∴ $E_t = \gamma mc^2 + P.E = \sqrt{c^2p^2 + (mc^2)^2} + P.E$(here P.E is cancelled on both sides as P.E shall be equal for the same body during the same course of motion at some instant of motion.)

$\Longrightarrow \sqrt{c^2p^2 + (mc^2)^2} = \gamma mc^2$

Squaring both sides

$= c^2p^2 + (mc^2)^2 = (\gamma mc^2)^2$

$= (cp)^2 + (mc^2)^2 = (\gamma mc^2)^2$

$= (cp)^2 = (\gamma mc^2)^2 - (mc^2)^2$

$= (cp)^2 = (\gamma mc^2 - mc^2)(\gamma mc^2 + mc^2)$ [$a^2 - b^2 = (a+b)(a-b)$, here $a = \gamma mc^2$ and $b = mc^2$]

~ $(cp)^2 = K.E(\gamma mc^2 + mc^2)$]

[K.E = $\gamma mc^2 - mc^2$ (from
Einstein’s Equation for
Relativistic K.E)

$$\sim K.E = \frac{(pc)^2}{(\gamma mc^2 + mc^2)}$$

$$\sim K.E = \frac{(\gamma mvc)^2}{(\gamma mc^2 + mc^2)} \qquad [p = \gamma mv]$$

$$\sim K.E = \frac{m(\gamma v)^2}{(\gamma + 1)}$$

This can be further expanded with taylor series to take the most extreme cases the new expression shall look like this

K.E = $m(\gamma v)^2 (1 - \gamma + \gamma^2 - \gamma^3 + \gamma^4 - \gamma^5 + higher\ order\ expansion\ terms)$(this is taylor series expansion of function 1/(1+x) here x is taken to be γ)

Hence arriving at a more precise form of the equation.

Considering the same one particle system taken above

In the above derivation we do not know anything about the particle's rest state. In a fixed frame of reference let's consider that the particle is at position (x,y,z) in a 3-D surface plot containing some amount of Potential Energy 'V'.

Total Energy of the particle at position (x,y,z) = Rest Energy + Potential Energy

= mc^2+ V (for a single particle where the potential energy of the particle can be calculated by subtracting rest mass energy of the particle from the total system.)

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As Potential Energy (V) is an added form of energy barring Rest Energy for this single particle it leads to an increase of mass of the particle. Hence the increase in mass can be termed as Δm.

$\because \Delta m = \frac{E_{potential}}{c^2}$

$\Rightarrow E_{potential}(V) = \Delta mc^2$ --------- (ii)

So the expression of Potential energy can be given by Δmc^2

Hence total energy at rest for a single particle can be given by:

$mc^2 + V$ (for a single particle)

$\Rightarrow E_{total}(\text{Rest}) = \Delta mc^2 + mc^2$

$\Rightarrow E_{total}(\text{Rest}) = (\Delta m + m)c^2$ (this form of the equation is only valid for a single particle in an isolated system)

For a system the equation shall assume more complex forms where $(\Delta m + m)c^2$ could be calculated individually for every particle in the system and added up in general cases for a system at rest in an inertial frame of reference .

For example in an hydrogen atom where the rest mass energy of the system shall be inclusive of the potential energy of the proton electron system

Hence in this case total rest energy = sum of potential energies of the proton electron system and energy arising due to its invariant mass.

Hence the equation assumes different forms as complexity of the system increases.

Calculations

Let's carry out a thought experiment to verify the equation derived above. We will divide this experiment in two parts

- Case I (velocity of object is close to speed of light ,i.e, very high)
- Case I I (velocity of object is far from speed of light ,i.e, very low)

Case I

We shall compare our result with Einstein's equation for Relativistic Kinetic Energy as well as Newton's Equation for Kinetic Energy (K.E). If our derivation is right then our value shall be equal to the value of Einstein's equation for Relativistic Kinetic Energy.

Mass of body (m) = 2 kg

Velocity of body (v) = 2×10^8 m/s

Speed of light (c) = 3×10^8 m/s

Lorentz Factor (γ) = $1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - (2\times10^8/3\times10^8)^2}$ = 1.3416407865

Our Equation

~ K.E = $\frac{m(\gamma v)^2}{(\gamma + 1)}$

~ K.E = $\frac{2\times(1.34164\times2\times10^8)^2}{(1.34164+1)}$

~ K.E = $2\times\frac{7.199991}{2.34164}\times10^{16}$

~ K.E = $\frac{14.39998}{2.34164}\times10^{16}$

~ **K.E = 6.14953 x 10^{16}**

Einstein's Equation for K.E

~ K.E = $\gamma mc^2 - mc^2$

~ K.E = $(\gamma - 1)\, mc^2$

~ K.E = $(1.34164 - 1)\,(2\times(3\times10^8)^2)$

~ K.E = $(0.34164)\,(2\times9\times10^{16})$

~ K.E = $(0.34164)\,(18\times10^{16})$

~ K.E = $0.34164\times18\times10^{16}$

~ **K.E = 6.14953 x 10^{16}**

Newton's Equation for K.E

~ K.E = $\frac{1}{2}mv^2$

~ K.E = $\frac{1}{2}\times2\times(2\times10^8)^2$

~ K.E = $(2\times10^8)^2$

~ **K.E = 4 x 10^{16} J**

Results of Our Equation = Results of Einstein's Equation
Hence our results are verified for high speed

Case II

We shall compare our result with Einstein's equation for Relativistic Kinetic Energy as well as Newton's Equation for Kinetic Energy (K.E). If our derivation is right then our value shall be equal to the value of Newton's Equation for Kinetic Energy.

Mass of body (m) = 2 kg

Velocity of body (v) = 3 m/s

Speed of light (c) = 3×10^8 m/s

Lorentz Factor (γ) = $1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - (3/3\times10^8)^2} = 1$

Our Equation

$$\sim \text{K.E} = \frac{m(\gamma v)^2}{(\gamma + 1)}$$

$$\sim \text{K.E} = \frac{m(1v)^2}{(1+1)}$$

$$\sim \text{K.E} = \frac{mv^2}{2}$$

$$\sim \text{K.E} = \frac{1}{2}mv^2$$

$$\sim \text{K.E} = \frac{1}{2}\times 2\times (3)^2$$

~ **K.E = 9 J**

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Einstein's Equation for K.E

~ K.E = $\gamma mc^2 - mc^2$

~ K.E = $(\gamma - 1)\, mc^2$

By use of Binomial Theorem and Taylor Series Einstein's equation breakdowns to Newton's Expression for Kinetic Energy, i.e, ½ mv^2

Newton's Equation for K.E

~ K.E = $\frac{1}{2} mv^2$

~ K.E = $\frac{1}{2} \times 2 \times (3)^2$

~ **K.E = 9 J**

Results of Our Equation = Results of Newton's Equation
Hence our results are verified for low speed as well

Total energy

Hence now we have 2 equations one describing the total rest energy of a single particle during rest and other describing total energy of a particle during motion as the law of conservation of energy states energy can neither be destroyed nor be created only convert from one form to another in an isolated system we get.

E_{total} (Rest) $=(\Delta m + m)c^2 = E_{total}$ (Motion)

$\Rightarrow (\Delta m + m)c^2 = \sqrt{c^2 p^2 + (mc^2)^2}$ + P.E (P.E in R.H.S can be also taken as V_2)

Hence arriving at a final expression for the total energy of a single particle in an isolated system in a non inertial frame of reference.

For a system of particles we get

Total Rest Energy (inclusive of energy due to invariant mass ,and inherent K.E and P.E of the system) = Sum of final relativistic kinetic and potential energies during course of its motion.

This expression for a system in its most basic form is analogous to the classical equation

$K_i + U_i = K_f + U_f$ (for an isolated system)