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On completely (1,2)*- $\psi\hat{g}$-irresolute functions

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Abstract

In this paper,we introduce the new class of functions called completely(1,2)*- $\psi\hat{g}$-irresolute functions.Some comparative properties of thesefunctions are studied.

Keywords: (1,2)*- $\psi\hat{g}$-irresolute functions ,completely (1,2)*- $\psi\hat{g}$-irresolute functions.

1.Introduction

Crossley and Hildebrand[1] defind irresolute functions by utilizing semi-opensets due to Levine[10].As weak forms of irresoluteness,weak irresoluteness,θ-irresoluteness,almost irresoluteness and quasi irresoluteness have been definedand investigated are equivalent[4].On the other hand,Dube [2,3]et al have introducedthe notion of almost irresolute functions which is independent of thatof almost irresolute functions in the sense of Thakur and Palk[11].Recently authors[5,6,7,8,9,12] studied various functions in bitopologicalspaces.The aimof this paper is introduce and investigate the new class of functions calledcompletely (1,2)*- $\psi\hat{g}$-irresolute functions.

2.Preliminaries

Throughout the present paper(X, τ_1, τ_2) ,(Y, σ_1, σ_2), (Z, η_1, η_2)briefly X,Y,Z bebitopological spaces.

Definition 2.1 AsubsetSofabitopologicalspace(X,τ_1,τ_2)issaidtobe$\tau_{1,2}$-openif S=$A\cup B$whereA$\in\tau_1$andB$\in\tau_2$.AsubsetSofXis$\tau_{1,2}$-closedifthecomplementofSis$\tau_{1,2}$-open.

Definition2.2

(i) The$\tau_{1,2}$-interiorofasubsetAofX,denotedby$\tau_{1,2}$-int(A) isdefinedtobe theunion ofall$\tau_{1,2}$-opensetscontaining A.

(ii) The$\tau_{1,2}$-closureofasubsetAofX,denotedby$\tau_{1,2}$- cl (A)is definedtobe theintersection ofall$\tau_{1,2}$-closedsetscontaining A.

Remark2.3 (i) $\tau_{1,2}$-int(S) is$\tau_{1,2}$-openforeachS⊂Xand$\tau_{1,2}$−cl(S)is $\tau_{1,2}$-closedforeachS⊂X.

(ii) AsetS⊂X is$\tau_{1,2}$-open iffS=$\tau_{1,2}$−int(S)andis$\tau_{1,2}$-closediffS= $\tau_{1,2}$−cl(S).

(iii) $\tau_{1,2}-int(S)=int_{\tau_1}(S)\cup int_{\tau_2}(S)$

(iv)For anyfamilyS_i/i∈IofsubsetsofXwehave

(a) $\bigcup_i \tau_{1,2}\text{-int}(S_i)\subset\tau_{1,2}\text{-int}(\bigcup_i S_i)$

(b) $\bigcup_i \tau_{1,2}\text{-cl}(S_i)\subset\tau_{1,2}\text{-cl}(\bigcup_i S_i)$

(c) $\tau_{1,2}\text{-int}(\bigcap_i S_i)\subset\bigcap_i \tau_{1,2}\text{-int}(S_i)$

(d) $\tau_{1,2}\text{-cl}(\bigcap_i S_i)\subset\bigcap_i \tau_{1,2}\text{-cl}(S_i)$

(v)$\tau_{1,2}$-open setsneednotformatopology

Definition2.4 AsubsetAofabitopologicalspaces(X,τ_1,τ_2) iscalled

1. (1, 2)*-closed if f (V) is $\sigma_{1,2}$-closed in Y, for every $\tau_{1,2}$-closed set V of X.
2. $(1,2)^*$-generalized closed ($(1,2)^*$-g-closed)if$\tau_{1,2}$-cl(A) ⊆UwheneverA⊆Uand Uis $\tau_{1,2}$-openinX.
3. $(1,2)^*$-ψ-closedset if $(1,2)^*$-scl(A) ⊆ U whenever A ⊆ U and U is $(1,2)^*$-sg-open in X.
4. $(1,2)^*$-ψ generalized closed set (briefly $(1,2)^*$-ψ g-closed)if$(1,2)^*$-ψ cl(A) ⊆U whenever A⊆U and U is $\tau_{1,2}$-open in X.
5. $(1,2)^*$-$\hat{g}$-closed set if $(1,2)^*$-cl(A) ⊆ G whenever A ⊆ G and G is $(1,2)^*$- semi-open in X.
6. $(1,2)^*-\psi\hat{g}$ -closedif $(1,2)^*-\psi$ cl(A)⊆UwheneverA⊆UandUis$(1,2)^*-\hat{g}$-openinX.

Definition 2.5 A function f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

1. $(1,2)^*$- continuous if $f^{-1}(V)$ is $(1,2)^*$-closed in X for every $\sigma_{1,2}$-closed set V of Y.
2. $(1,2)^*$– completely-continuous if $f^{-1}(V)$ is $(1,2)^*$–regular-closed in X for every $\sigma_{1,2}$-closed set V of Y.

3. $(1,2)^*-\psi$-continuous if $f^{-1}(V)$ is $(1,2)^*-\psi$-closed in Xfor every $\sigma_{1,2}$-closed set V of Y.
4. $(1,2)^*-\psi g$-continuous if $f^{-1}(V)$ is $(1,2)^*-\psi g$-closed in Xfor every $\sigma_{1,2}$-closed set V of Y.
5. (1,2)*- $\psi\hat{g}$-continuous if $f^{-1}(V)$ is (1,2)*- $\psi\hat{g}$-closed in Xfor every $\sigma_{1,2}$-closed set V of Y.
6. (1, 2)*-irresolute if $f^{-1}(V)$ is (1, 2)*-semi-closed in X for every (1, 2)*-semi-closed set V of Y.

3.(1,2)*- $\psi\hat{g}$ **-Irresolute Functions**

In this section, we introduced (1,2)*- $\psi\hat{g}$-irresolute functions in bitopological spaces and study some of their characterizations and properties.

Definition3.1: Afunctionf : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ fromabitopologicalspaceXintoabitopologicalspaceYiscalled(1,2)*- $\psi\hat{g}$-irresolute iftheinverseimageofevery (1,2)*- $\psi\hat{g}$-closedsetinYis(1,2)*- $\psi\hat{g}$-closedsetinX.

Example3.2: Let X =Y={ a,b,c}withτ ={ X,ϕ,{ c}}and σ={Y,ϕ,{a},{ b},{ a,b}}.Letf : $(X,\tau) \rightarrow (Y,\sigma)$ bean identity function. Then fis (1,2)*- $\psi\hat{g}$-irresolute.

Theorem3.3: Afunction f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is(1,2)*- $\psi\hat{g}$-irresolute ifand onlyiftheinverseimageofevery(1,2)*- $\psi\hat{g}$-open set inYis(1,2)*- $\psi\hat{g}$-openinX.

Proof:Assume that fis(1,2)*- $\psi\hat{g}$-irresolute. LetAbeany (1,2)*- $\psi\hat{g}$-open set in Y. ThenA^c is(1,2)*- $\psi\hat{g}$-closedinY. Since fis(1,2)*- $\psi\hat{g}$-irresolute, $f^{-1}(A^c)$is (1,2)*- $\psi\hat{g}$-closedinX.But$f^{-1}(A^c)=X-f^{-1}(A)$ andso$f^{-1}(A)$ is(1,2)*- $\psi\hat{g}$-openinX. Hencetheinverseimageofevery(1,2)*- $\psi\hat{g}$-open setinYis(1,2)*- $\psi\hat{g}$-openinX.Conversely, assumethattheinverseimageofevery(1,2)*- $\psi\hat{g}$-opensetinYis(1,2)*- $\psi\hat{g}$-openinX. LetA beany(1,2)*- $\psi\hat{g}$-closedinY. ThenA^c (1,2)*- $\psi\hat{g}$-openinY. Byassumption, $f^{-1}(A^c)$ is(1,2)*- $\psi\hat{g}$-open inX. But $f^{-1}(A^c) = X-f^{-1}(A)$ and sof^{-1} (A) is(1,2)*- $\psi\hat{g}$-closedinX. Therefore fis(1,2)*- $\psi\hat{g}$-irresolute.

Theorem3.4: Afunction f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is(1,2)*- $\psi\hat{g}$-irresolute ifandonlyifitis (1,2)*- $\psi\hat{g}$-

continuous.

Proof: Assumethat fis(1,2)*- $\psi\hat{g}$-irresolute .LetFbeany$\sigma_{1,2}$-closedsetinY.As every$\sigma_{1,2}$-closedsetis(1,2)*- $\psi\hat{g}$-closed, Fis(1,2)*- $\psi\hat{g}$-closedinY. Sincefis(1,2)*- $\psi\hat{g}$-irresolute, f^{-1}(F)is(1,2)*- $\psi\hat{g}$-closedinX. Therefore fis(1,2)*- $\psi\hat{g}$-continuous.

Conversely, assume thatfis(1,2)*- $\psi\hat{g}$-continuous. LetF beany $\sigma_{1,2}$-closedset in Y.Asevery $\sigma_{1,2}$-closed set is (1,2)*- $\psi\hat{g}$-closed ,F is (1,2)*- $\psi\hat{g}$-closed in Y. Since fis (1,2)*- $\psi\hat{g}$-continuous, f^{-1}(F)is(1,2)*- $\psi\hat{g}$-closedinX. Therefore fis(1,2)*- $\psi\hat{g}$-irresolute.

Theorem3.5: LetX,YandZbeanybitopologicalspaces. Forany(1,2)*- $\psi\hat{g}$-irresolutefunction f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and any (1,2)*- $\psi\hat{g}$-continuous function g: (Y, σ_1, σ_2) → (Z, η_1, η_2) ,the compositiong o f : (X, τ_1, τ_2) → (Z, η_1, η_2) is(1,2)*- $\psi\hat{g}$-continuous.

Proof:LetFbeany$\eta_{1,2}$-closedsetinZ. Sincegis(1,2)*- $\psi\hat{g}$-continuous, g^{-1}(F)is(1,2)*- $\psi\hat{g}$-closedinY. Sincefis(1,2)*- $\psi\hat{g}$-irresolute, $f^{-1}(g^{-1}(F))$is(1,2)*- $\psi\hat{g}$-closedinX. But $f^{-1}(g^{-1}(F))=(g\circ f)^{-1}(F)$.Therefore $g\circ f$is(1,2)*- $\psi\hat{g}$-continuous.

Theorem 3.6: If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) from a bitopological space X into a bitopological space Y is bijective, open and (1,2)*- $\psi\hat{g}$-continuous then f is (1,2)*- $\psi\hat{g}$-irresolute.

Proof: Let A be a (1,2)*- $\psi\hat{g}$-closed set in Y. Let $f^{-1}(A) \subseteq U$. where U is $\tau_{1,2}$-open in X. Therefore, A $\subseteq$f(U) holds. Since f(U) is $\sigma_{1,2}$-open and A is (1,2)*- $\psi\hat{g}$-closed in Y, $\tau_{1,2}$-cl(A) $\subseteq$f(U) holds and hence $f^{-1}(\tau_{1,2}\text{-cl}(A)) \subseteq U$. Since f is (1,2)*- $\psi\hat{g}$-continuous and cl(A) is closed in Y, $\tau_{1,2}\text{-cl}(f^{-1}(\tau_{1,2}\text{-cl}(A))) \subseteq U$ and so $\tau_{1,2}\text{-cl}(f^{-1}(A)) \subseteq U$. Therefore,

$f^{-1}(A)$ is (1,2)*- $\psi\hat{g}$-closed in X. Hence f is (1,2)*- $\psi\hat{g}$-irresolute.

4Completely(1,2)$^*-\psi\hat{g}$-irresolute functions

Definition4.1A functionf : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called **completely (1,2)*- $\psi\hat{g}$-irresolute** if the inverse image of every(1,2)*-$\psi\hat{g}$-closedsubset of Yis(1,2)*-regular closedinX.

Example 4.2 Let X ={a, b, c} =Y , τ_1 = {X, ϕ,{a}} and τ_2 = {X, ϕ,{b,c}}.Then

$\tau_{1,2}$-open sets={X, ϕ,{a},{b,c}} and $\tau_{1,2}$-closed sets= {X, ϕ,{a},{b,c}}.Let σ_1 ={Y, ϕ,{c}} and σ_2 ={Y, ϕ,{b,c}}.Then $\sigma_{1,2}$-open sets = {Y, ϕ,{c},{b,c}} and $\sigma_{1,2}$-closed sets = {Y, ϕ,{a},{a, b}}. Let f :$(X,\tau_1,\tau_2)\to(Y,\sigma_1, \sigma_2)$ bebedefinedbyf(a)=a,f(b)=b,f(c)=b.Then fis completely(1,2)*- $\psi\hat{g}$ -irresolute function.

Theorem4.3 Every completely(1,2)*- $\psi\hat{g}$ -irresolute function is (1,2)*- $\psi\hat{g}$ -irresolute.

Converseofthe abovetheorem neednot betrue as seen fromthe followingexample.

Example4.4 Let X ={a, b, c} =Y , τ_1 = {X, ϕ,{a}} and τ_2 = {X, ϕ}.Then $\tau_{1,2}$-open sets={X, ϕ,{a}} and $\tau_{1,2}$-closed sets= {X,ϕ,{b,c}}.Let σ_1 ={Y, ϕ,{a}} and σ_2 ={Y, ϕ,{b,c}}.Then $\sigma_{1,2}$-open sets = {Y,ϕ,{a},{b,c}} and $\sigma_{1,2}$-closed sets = {Y, ϕ,{a},{b,c}}. Let f :$(X,\tau_1,\tau_2)\to(Y,\sigma_1, \sigma_2)$ bebedefinedbyf(a)=b,f(b)=c,f (c)=a.Then fis(1,2)*- $\psi\hat{g}$ -irresolute function but not completely (1,2)*- $\psi\hat{g}$ -irresolute function,since for the $\sigma_{1,2}$-closed set {a} in Y,f^{-1}({a})={b} is not (1,2)*-regular closed in X.

Theorem4.5A functionf : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is a completely (1,2)*- $\psi\hat{g}$ -irresolute if the inverse image of every (1,2)*- $\psi\hat{g}$ -open set is(1,2)*-regular open in X.f^{-1}(Y-V) is regular closed in X,X-f $^{-1}$(V) is (1,2)*-regular open in X.Then f $^{-1}$(V) is (1,2)*-regular closed in X.Hence f is completely (1,2)*- $\psi\hat{g}$ -irresolute.

Theorem4.6 Iff : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is completely (1,2)*- $\psi\hat{g}$ -irresolutefunction and f(X) is taken with the subspace bitopology then f : X→f(X) is completely (1,2)* $\psi\hat{g}$ -irresolutefunction.

Proof: Iff : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is completely (1,2)*- $\psi\hat{g}$ -irresolutefunction implies f^{-1}(H) is (1,2)*-regular open for every (1,2)*- $\psi\hat{g}$ -open subset H of Y.Nowf $^{-1}$(H$\cap$ f(X))=f $^{-1}$(H)$\cap$X=f $^{-1}$(H) is (1,2)* regular open. Therefore f :X→f(X) is completely (1,2)*- $\psi\hat{g}$ -irresolutefunction.

Definition4.7 A space X is said to be **(1,2)*- $\psi\hat{g}$ -Housdroff (resp,(1,2)*-rT_2)** if for any x,y$\in$X,x$\neq$ y,there exist (1,2)*- $\psi\hat{g}$ -open sets (resp.(1,2)*-regular open)G and H such that x$\in$G,y$\in$H and G $\cap$H=ϕ.

Theorem4.8 Letf : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be injective and completely (1,2)*- $\psi\hat{g}$ -irresolute surjection.IfYis(1,2)*-$\psi\hat{g}$ -Hausdorff space then Xis(1,2)*−rT_2.

Proof: Letx and y be any two disjoint points of X.Since f is injective,f(x) ≠ f(y).Since Y is (1,2)*-$\psi\hat{g}$ -Hausdorff space there exist disjoint (1,2)*-$\psi\hat{g}$ -open sets G and H such that f(x) ∈ G and f (y) ∈ H.Since f is a completely (1,2)*- $\psi\hat{g}$ -irresolute function f^{-1}(G), f(H) are disjoint function,f^{-1}(G), f(H) are disjoint (1,2)*-regular open sets containing x and y respectively.Hence X is (1,2)*-rT_2.

Theorem4.9 Iff : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is completely (1,2)*-continnuousandg:(Y,σ_1,σ_2)→(Z,η_1,η_2) is completely (1,2)*- $\psi\hat{g}$ -irresolute then g ∘ f:(X, τ_1,τ_2)→(Z,η_1,η_2)is a completely (1,2)*- $\psi\hat{g}$ -irresolute function.

Proof: LetGbeany(1,2)*- $\psi\hat{g}$ -closedsetinZ. Sincegis completely (1,2)*- $\psi\hat{g}$ -irresolute, g^{-1} (G)is(1,2)*- regular closedinY.Since(1,2)*-regular closed sets is $\sigma_{1,2}$ -closed and fis completely (1,2)*-continuous f^{-1} (g^{-1}(G))is (1,2)*-regular closed inX.Since every (1,2)*-regular closed set is (1,2)*- $\psi\hat{g}$ - closed,g∘f is completely(1,2)*- $\psi\hat{g}$ -irresolute function.

Theorem4.10 Iff : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is completely (1,2)*- $\psi\hat{g}$ -irresolute andg :(Y,σ_1,σ_2)→(Z,η_1,η_2) is (1,2)*- $\psi\hat{g}$ -continuous then g ∘ f:(X, τ_1,τ_2)→(Z,η_1,η_2)is a completely (1,2)*-continuous function.

Proof: LetGbeany$\eta_{1,2}$-closedsetinZ. Sincegis(1,2)*- $\psi\hat{g}$ -continuous, g^{-1}(G)is$\sigma_{1,2}$ -closedinY.Since fis completely (1,2)*- $\psi\hat{g}$ -irresolute,f^{-1} (g^{-1}(G))is(1,2)*-regular closed inX. But f^{-1}(g^{-1}(G))=$(g∘f)^{-1}$(G).Thereforeg∘fis completely(1,2)*-continuous function.

Theorem4.11 Iff : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is completely (1,2)*- $\psi\hat{g}$ -irresolute andg :(Y,σ_1,σ_2)→(Z,η_1,η_2) is (1,2)*- $\psi\hat{g}$ -irresolute then g ∘ f:(X, τ_1,τ_2)→(Z,η_1,η_2)is a completely (1,2)*-irresolute function.

Proof: LetGbeany(1,2)*- $\psi\hat{g}$ -closedsetinZ. Sincegis(1,2)*- $\psi\hat{g}$ -irresolute,

g^{-1}(G)is(1,2)*- $\psi\hat{g}$ -closedin Y.Since fis completely (1,2)*- $\psi\hat{g}$ -irresolute, $f^{-1}(g^{-1}(G))$is(1,2)*-regular closed inX. But $f^{-1}(g^{-1}(G))=(g\circ f)^{-1}(G)$.Thereforeg∘fis completely(1,2)*-irresolute function.

Theorem4.12 Iff : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is completely (1,2)*-continuous and g :$(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ is completely (1,2)*- $\psi\hat{g}$ -irresolute then g∘f:$(X, \tau_1,\tau_2)\rightarrow(Z,\eta_1,\eta_2)$is a completely (1,2)*- $\psi\hat{g}$ -irresolute function.

Proof: LetGbeany(1,2)*- $\psi\hat{g}$ -closedsetinZ. Sinceg is completely (1,2)*- $\psi\hat{g}$ -irresolute, g^{-1}(G)is(1,2)*-regular closedin Y.Since fis (1,2)*-continuous,$f^{-1}(g^{-1}(G))$is(1,2)*-regular closed inX. But $f^{-1}(g^{-1}(G)) =(g\circ f)^{-1}(G)$.Thereforeg∘fis completely(1,2)*-irresolute function.

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