On Nano Delta Generalized Closed Sets in Nano Topological Spaces

[1] Dr.N.R.Santhi Maheswari & [2] Ms.P.Subbulakshmi

[1]Associate Professor and Head
PG and Research Department of Mathematics,
G.Venkataswamy Naidu College
Kovilpatti-628 502, Tamil Nadu, India.
Affiliated to Manonmaniam Sundaranar University,

[2] Research Scholar (Reg No: 19222052092003),
PG and Research Department of Mathematics,
G.Venkataswamy Naidu College
Kovilpatti-628 502, Tamil Nadu, India.
Affiliated to Manonmaniam Sundaranar University,

Abstract

The aim of this paper is to introduce a new class of set called nano δ generalized closed sets in nano topological spaces is introduced and some of their basic properties are investigated. Also we investigate its relationship with other types of closed set in nano topological spaces.

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Introduction

Levine [4] Introduced the class of g-closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology. Recently Lellis Thivagar introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and up- per approximation of X. The elements of nano topological space are called nano open sets. He has also defined nano closed sets, nano interior and nano closure of a set. He also introduced the weak forms of nano open sets. Bhuvaneswari [2] introduced nano g-closed sets and obtained some of the basic interesting results. In this paper, we define a new class of sets called nano δ generalized closed and its open sets in nano topological spaces and study the relationships with other nano sets.

Preliminaries

Definition 2.1. [3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belong-ing to the same equivalence class are said to be in discernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

- The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup_{x \in U}\{R(x): R(x) \subseteq X\}$. where R(x) denotes the equivalence class determined by x ϵ U .

- The upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \cup_{x \epsilon U}\{R(x): R(x) \cap X \neq \varphi\}$.

- The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [3] If (U, R) is an approximation space and $X, Y \subseteq U$, then

- $L_R(X) \subseteq X \subseteq U_R(X)$
- $L_R(\varphi) = U_R(\varphi) = \varphi$
- $L_R(U) = U_R(U) = U$
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$
- $U_R(X \cup Y) \supseteq U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$
- $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
- $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3. [3] Let U be the Universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$.
- The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$. We call $(U, \tau_R(X))$ is a nano topological space. The elements of $\tau_R(X)$ are called a open sets and the complement of a nano open set is called nano closed sets.

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R .

Remark 2.4. [3] If $\tau_R(X)$ is the nano topology on U with respect to X . The set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. [3] If $(U, \tau_R(X))$ is a nano topological space with respect to X. Where $X \subseteq G$ and if $A \subseteq G$,then

- The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by N int(A) , N int(A) is the largest nano open subset of A .
- The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A .

Definition 2.6. [3] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq G$. Then A is said to be

(i) nano pre open if $A \subseteq N\ int(Ncl(A))$ and Pre-closed if $Ncl(Nint(A)) \subseteq A$
(ii) nano regular-open if $A = N\ int(Ncl(A))$ and nano regular closed if $A = N\ cl(Nint(A))$.
(iii) nano α -open if $A \subseteq N\ int(Ncl(Nint(A)))$ and α -closed if $Ncl(Nint(Ncl(A))) \subseteq A$.

Definition 2.7. [2] Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly N g closed) if $Ncl(A) \subseteq G$ where $A \subseteq G$ and G is Nano open.

3. Nano delta generalized Closed sets

We introduce the following definitions.

Definition 3.1. The nano δ interior of a subset A of X is the union of all nano regular open set of X contained in A and is denoted by nano-$\delta\ int(A)$. The subset A is called nano- δ open if $A = nano\ \delta\ int(A)$, ie. A set is nano δ open if it is the union of nano regular open sets.

The complement of a nano δ open is called nano δ closed. Alternatively, a set $A \subseteq (U, \tau_R(X))$ is called nano δ closed if $A = nano\ \delta cl(A)$, , where nano $\delta cl(A) = \{N\ int(cl(G)) \cap A \neq \varphi, G \in \tau_R(X)\ and\ x \in G\}$.

Definition 3.2. nano δ generalized closed set (briefly $N\delta g$ closed) if nano $\delta\ cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open in U .

Example 3.3. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{a, d\}\} \subseteq U$ and $\tau_R(X) = \{\varphi, \{a\}, \{a, b, d\}, \{b, d\}, U$. Then $N\delta g$ closed sets are $\{\varphi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}\}$.

Theorem 3.4. Every $N\delta$ closed set is $N\delta g$ closed.

Proof: Let A be a $N\delta$ closed set and U be any nano open set containing A. Since A is $N\delta$ closed, $N\delta cl(A) = A$. Therefore $N\delta cl(A) = A \subseteq U$ and hence A is $N\delta g$ closed.

Remark 3.5. The converse of the above theorem need not be true as shown in the following example.

Example 3.6. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{c\}, \{a, d\}\}$. Let $X = \{a, b\} \subseteq U$ and $\tau_R(X) = \{\varphi, \{b\}, \{a, b, d\}, \{a, d\}, U\}$. Then $N\delta g$ closed sets are {φ, {c},{a, c}, {b, c}, {c, d}, {a, b, c }, {b, c, d}, {a, c, d}, U} and $N\delta$ closed sets are { φ, {c}, {a, c}, {b, c, d}, U} . Here the set {b, c} is $N\delta g$ closed but not $N\delta$ closed set.

Theorem 3.7. Every $N\delta g$ closed set is Ng closed.

Proof. Let A be a $N\delta g$ closed set and U be any nano open set containing A .Since every nano δ closed is nano closed, we have $N\ cl(A) = N\ \delta cl(A) \subseteq U$. Therefore $N\ cl(A) \subseteq U$ and hence A is Ng closed.

Remark 3.8. The converse of the above theorem need not be true as shown in the following example.

Example 3.9. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}$. Let $X = \{a, d\} \subseteq U$ and $\tau_R(X) = \{\varphi, \{a\}, \{a, b, d\}, \{b, d\}, U\}$. Then $N\ \delta g$ closed sets are $\{\varphi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}\}$ and Ng closed sets are $\{\ \varphi, \{b\}, \{c\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$. Here $\{b\}$ is $N\ \delta g$ closed but not $N\ g$ closed.

Theorem 3.10. Union of any two $N\delta g$ -closed subset is $N\delta g$ -closed

Proof. Let A and B be any two $N\delta g$ closed sets in U , such that $A \subseteq U$ and $B \subseteq U$ where G is nano open in U and so $A \cup B \subseteq G$ Since A and are $N\delta g$ - closed, we have $A \subseteq N\ \delta cl(A)$ and $B \subseteq N\ \delta cl(A)$ and hence $A \cup B \subseteq N\ \delta cl(A) \cup N\ \delta cl(B) \subseteq N\ \delta cl(A\ \cup B)$. Thus $A \cup B$ is $N\delta g$ closed in $(U, \tau_R(X))$.

Example 3.11. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{a, c\} \subseteq U$ and $\tau_R(X) = \{\varphi, \{a\}, \{a, c, d\}, \{c, d\}, U\}$ Then $N\ \ \delta g$ closed sets are $\{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, U\}$. Let $A = \{a, b\}$ is $N\delta g$ closed and $B = \{b, d\}$ is $N\delta g$ closed then $A \cup\ B = \{a, b, d\}$ is also $N\delta g$ closed.

Theorem 3.12. Intersection of any two $N\delta g$ -closed subset is $N\delta g$ -closed

Proof. Let A and B be any two $N\delta g$ closed sets in U , such that $A \subseteq U$ and $B \subseteq U$ where G is nano open in U and so $A \cap B \subseteq G$ Since A and are $N\delta g$ - closed, we have $A \subseteq N\ \delta cl(A)$ and $B \subseteq N\ \delta cl(A)$ and hence $A \cap B \subseteq N\ \delta cl(A) \cup N\ \delta cl(B) \subseteq N\ \delta cl(A\ \cap B)$. Thus $A \cap B$ is $N\delta g$ closed in $(U, \tau_R(X))$.

Example 3.13. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{a, c\} \subseteq U$ and $\tau_R(X) = \{\varphi, \{a\}, \{a, c, d\}, \{c, d\}, U\}$ Then $N\ \ \delta g$ closed sets are $\{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, U\}$. Let $A = \{a, b\}$ is $N\delta g$ closed and $B = \{b, d\}$ is $N\delta g$ closed then $A\ \cap B = \{b\}$ is also $N\delta g$ closed.

Theorem 3.14. If A is $N\delta g$ closed subset of U such that $A \subseteq B \subseteq N\ \delta cl(A)$, then B is a $N\ \delta cl(A)$ closed set in U .

Proof. Let A be a $N\delta g$ closed set of U such that $A \subseteq B \subseteq N\,\delta cl(A)$. Let G be a nano open set of U such that $B \subseteq G$, then $A \subseteq G$. Since A is $N\delta g$ closed. We have $N\,\delta cl(A) \subseteq G$. Now $N\,\delta cl(B) \subseteq N\,\delta cl(N\,\delta cl(A)) \subseteq G$. Therefore B is $N\delta g$ closed set in U . ∎

Theorem 3.15. A subset A of $(U, \tau_R(X))$ is $N\delta g$ closed if $N\,\delta cl(A) - A$ contains no nonempty $N\delta g$ closed set.

Proof. Suppose if A is $N\delta g$ closed. Then $N\,\delta cl(A) \subseteq G$ where $A \subseteq G$ and G is nano open. Let Y be nano closed subset of $N\,\delta cl(A) - A$. Then $A \subseteq Y^c$ and Y^c is nano open. Since A is $N\,\delta g$ closed $N\,\delta cl(A) \subseteq Y^c$ or $Y = [N\,\delta cl(A)]^c$. That is $Y \subseteq N\,\delta cl(A) - A$. Therefore $Y = [N\,\delta cl(A)]^c \cap [N\,\delta cl(A)] = \varphi$. Hence $Y = \varphi$.

Definition 3.16. A subset A of a nano topological space $(U, \tau_R(X))$ is called nano δ generalized open if A^c is $N\delta g$ closed.

Theorem 3.17. A subset $A \subseteq U$ is $N\delta g$ open iff $F \subseteq N\,int(A)$ whenever F is a nano closed set and $F \subseteq A$.

Proof. Let A be $N\delta g$ open. Suppose $F \subseteq N\,int(A)$ whenever F is a nano closed and $F \subseteq A$. Let $A \subseteq G$ where $G = F^c$ is nano open. Then $G^c \subseteq A$ and $G^c \subseteq N\,int(A)$. Then we have A^c is $N\delta g$ closed. Hence A is $N\delta g$ open. Conversely, If A is $N\delta g$ open, $F \subseteq A$ and F is nano closed. Then F^c is nano open and $A^c \subseteq F$. Therefore $Ncl(A^c) \subseteq (F^c)$. But $Ncl(A^c) = (N\ int(A))^c$. Hence $F \subseteq N\,int(A)$.

Theorem 3.18. If $N\,int(A) \subseteq B \subseteq A$ and if A is $N\delta g$ open, then B is $N\delta g$ open.

Proof. Let $N\,int(A) \subseteq B \subseteq A$, then A^c is $N\delta g$ closed and hence B^C is also $N\delta g$ closed by above theorem. Therefore B is $N\delta g$ open.

4. Nano δg -Interior and Nano δg Closure

Definition 4.1. Let $(U, \tau_R(X))$. be a nano topological space and $A \subseteq U$ then Nano δg -interior is defined as $N\,\delta g - int(A) = \cup \{ B : B \text{ is nano } \delta g - \text{open}, B \subseteq A \}$. Clearly $N\delta g - int(A)$ is the largest nano δg -open set over U which is contained in A .

Definition 4.2. Let $(U, \tau_R(X))$. be a nano topological space and $A \subseteq U$ then Nano δg -closure is defined as $N\,\delta g - cl(A) = \cap \{ F : F \text{ is nano } \delta g \text{ closed}, A \subseteq F \}$. Clearly $N\delta g - cl(A)$ is the smallest nano δg -closed set over U which is contains A .

Lemma 4.3. Let A and B be any two subsets of U in a nano topological spaces $(U, \tau_R(X))$ and the following are true

(i) $N\,\delta g - int(A) \subset A$
(ii) $A \subset B \Rightarrow N\,\delta g - int(B) \subset N\,\delta g - int(B)$
(iii) $N\,\delta g - int(A) \cup N\,\delta g - int(B) \subset N\,\delta g - int(A \cup B)$
(iv) $N\,\delta g - int(A) \cap N\,\delta g - int(B) \subset N\,\delta g - int(A \cap B)$

Lemma 4.4. For a subset A of U .

(i) $N\,\delta g - cl(A) \subset N\,cl(A)$
(ii) $N\,\delta g - int(A) \subset N\,\delta g - cl(A)$

Lemma 4.5. A subset A of U is nano δ generalized closed if and only if $A = N\,\delta cl(A)$

Lemma 4.6. Let A and B be two subsets of nano topological space $(U, \tau_R(X))$. Then

(i) $N\delta g - int(U) = U$ and $N\delta g - int(\Phi) = \Phi$.
(ii) $N\delta g - int(A) \subset A$
(iii) If B is any $N\delta g$ - open set contained in A , then $B \subset N\delta g - int(A)$
(iv) If $A \subset B$, then $N\delta g - int(A) \subset N\delta g - int(B)$
(v) $N\delta g - int(N\delta g - int(A)) = N\delta g - int(A)$

Proof.

(i) Let U and φ are $N\delta g$ open sets. $N\delta g - int(U) = \cup \{B : B \text{ is a } N\delta g - open, B \subset U\} = U \cup all\ N\delta g - open\ sets = U$. (ie) $N\delta g - int(U) = U$. Since φ is the only $N\delta g$ -open set contained in φ , $N\delta g - int(\Phi) = \Phi$.

(ii) Let $x \in N\delta g - int(A) \Rightarrow x$ is a interior point of A.
$\Rightarrow$ A is a neighbourhood of x.
$\Rightarrow x \in A$.
Thus, (ie) $x \in N\delta g - int(A) \Rightarrow x \in A$. Hence $N\delta g - int(A) \subset A$.

(iii) Let B be any N δg -open sets such that $B \subset A$. Let $x \in B$. Since B is a N δg -open set contained in A . x is a N δg - interior point of A. (ie) $x \in N\delta g - int(A)$. Hence $B \in N\delta g - int(A)$.

(iv) Let A and B be subsets of $(U, \tau_R(X))$ such that $A \subset B$. Let $x \in N\delta g - int(A)$. Then x is a N δg-interior point of A and so A is a N δg-neighbourhood of x. Since $A \subset B$, B is also N δ - neighbourhood of x. This implies $x \in N\delta g - int(B)$. Thus we have shown that $N\delta g - int(A) \subset N\delta g - int(B)$.

Theorem 4.7. If a subset A of space $(U, \tau_R(X))$ is $N\delta g$ - open, then $N\delta gint(A) = A$.

Proof. Let A be $N\delta g$ -open subset of $(U, \tau_R(X))$. We know that $N\delta g - int(A) \subset A$. Also, A is $N\delta g$ -open set contained in A. From above theorem 4.7, $A \subset N\delta g - int(A)$. Hence $N\delta g - int(A) = A$.

Theorem 4.8. Let A be a subset of a nano topological space $(U, \tau_R(X))$. Then

(i) $(N\delta g - cl(A))^c = N\delta g - int(A^c)$
$\{F: F \text{ is } N\delta g \text{ closed}, A \subset F\}$

(ii) $(N\delta g - int(A))^c = N\delta g - cl(A^c)$

Proof.

(i) $(N\delta g - cl(A))^c =$
$(\cap \{F: F \text{ is } N\delta g \text{ closed}, A \subset F\})^c$

$= \cup \{F^c: F^c \text{ is } N\delta g \text{ open}, F^c \subset A^c\}$
$= N\delta g\ int(A^c)$

(ii) $(N\delta g - int(A))^c =$
$(\cup \{B: B \text{ is } N\delta g \text{ open}, B \subset A\})^c$

$= \cap \{B^c: B^c \text{ is } N\delta g \text{ closed}, A^c \subset B^c\}$
$= N\delta g\ cl(A^c)$

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