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On Simultaneous Dual Series equations involving Konhauser Biorthogonal Polynomials

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ABSTRACT: By using Abel integral equations, we solve simultaneous dual series equations involving Konhauser biorthogonal polynomials.

Dual series equations

$$\sum_{n=0}^{\infty} \sum_{j=1}^{\delta} \frac{Anj}{\tau(\delta+\beta+1+kn_j)} Z_{n_j}^{(\delta+2\beta-1)(x\,;\,k)=f_j(x)}$$

$$0 \le x \le y \qquad (1)$$

and

$$\sum_{n=0}^{\infty} \sum_{j=1}^{\delta} \frac{Anj}{\Gamma(\delta+\beta+1+kn_j)} Z_{n_j}^{\delta\,(x\,;k)=g_i(x)}$$

$$y<x<\infty\,,\ (i=1 \text{ to } s) \qquad (2)$$

Where $\left[Z_{n_j}^{\delta}(x\,;k)\right]_{n=0}^{\infty}$ is the Konhauser biorthogonal polynomial set, $\beta = 0, \delta > -1, f_i(x)$ and (g_i(x)are known functions and A_{nj} is unkown constant which is to be determined, have been solved.

We require the biorthogonal properties of the Konhauser biorthogonal polynomials[1]

$$\int_0^{\infty} \exp(-x)x^{\delta} Z_n^{\delta}(x\,;k)\,dx = 0,\ if\ m \neq n$$

$$= \frac{\Gamma\,(1+\delta+kn)}{n\,!} \qquad (3)$$

if m=n

Where $\delta > -1$.

The second formula required is the Wey1 integral given by Karande and Thakare[2]

$$\int_0^{\infty} (\exp(-x)(x-\xi)^{\beta-1} Z_n^{\delta+\beta}(x\,;k)\,dx$$

$$=\Gamma(\beta)\exp(-\xi)\,Z_n^{\delta}\,(\xi;k) \qquad (4)$$

Where $\delta+1>\beta>0$.

The third result that we require is

$$\frac{d}{d\xi}\int_0^{\xi} (\xi-x)^{\beta-1}\,x^{\delta+\beta} Z_n^{\delta+\beta}(x;k)dx$$

$$= \frac{\Gamma(\beta)\Gamma(\delta+\beta+1+kn)}{\Gamma(\delta+2\beta+kn)} Z_n^{\delta+2\beta-1}\,(\xi\,;k) \qquad (5)$$

where $\delta+2\beta>0, \beta>0, \delta>-1$.

We have the Riemann- Liouville fractional integral[3] given by Prabhakar[4]

$$\int_0^{\xi} x^{\delta+\beta}(\xi-x)^{\beta-1}\,Z_n^{\delta+\beta}(x\,;k)\,dx$$

$$= \frac{\Gamma(\delta+\beta+1+kn)\Gamma\,(\beta)\xi^{\delta+2\beta} Z_n^{\delta+2\beta}\,(\xi\,;k)}{\Gamma\,(1+\delta+2\beta+kn)} \qquad (6)$$

where $\beta>0, \delta+1>0$.

If $f\,(\xi)$ and $f'\,(\xi)$are continuous in $0 \le x < \infty$and if $0<\beta<1$, then the solutions of Abel integral equations

$$f_1(\xi) = \int_0^{\xi} \frac{f_1(x)}{(\xi-x)^{\beta}}\,dx \qquad (7)$$

and

$$f_2(\xi) = \int_0^{\infty} \frac{f_2(x)}{(\xi-x)^{\beta}}\,dx \qquad (8)$$

are respectively given by

$$\mathrm{F}_1(x) = \frac{\sin\beta\pi}{\pi}\frac{d}{dx}\int_0^x \frac{f_1(\xi)}{(x-\xi)^{1-\beta}}d\xi \quad (9)$$

and

$$\mathrm{F}_2(x) = -\frac{\sin\beta\pi}{\pi}\frac{d}{dx}\int_x^\infty \frac{f_1(\xi)}{(x-\xi)^{\beta-1}}d\xi$$

Solution of the Equations:

From (5) and (1), we get

$$\frac{d}{d\xi}\int_0^\xi (\xi - x)^{\beta-1} x^{\delta+\beta} Z_{n_j}^{\delta+\beta}\ (x\,;k)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{\delta} \frac{Ani}{\Gamma\,(\delta+\beta+1kn,)}dx$$

$$= \sum_{n=0}^{\infty}\sum_{j=1}^{\delta} A_{ni}\frac{\Gamma(\beta)}{\Gamma\,(\delta+2\beta+kn,)}\xi^{\delta+2\beta-1}$$

$$\times Z_{n_j}^{\delta+2\beta-1}(\xi;k)$$

$$= \Gamma(\beta)\xi^{\delta+2\beta-1}f_i(\xi).$$

Hence

$$\sum_{n=0}^{\infty}\sum_{j=1}^{\delta} \frac{Ani}{\Gamma\,(\delta+\beta+kn,)}\frac{d}{d\xi}\int_0^\xi (\xi-x)^{\beta-1}$$

$$x^{\delta+\beta}Z_{n_j}^{\delta+\beta}(x;k)dx$$

$$= \Gamma\,(\beta)\,\xi^{\delta+2\beta-1} f_i\quad(\xi)\quad (11)$$

Similarly, from (4) and (2), we get

$$\sum_{n=0}^{\infty}\sum_{j=1}^{\delta} \frac{Ani}{\Gamma\,(\delta+\beta+kn,)}\int_\xi^\xi exp(-x)(x-\xi)^{\beta-1}$$

(12)

$$\times Z_{n_j}^{\delta+\beta}(x\,;k)dx = \Gamma(\beta)\exp(-\xi)\,gi,(\xi)$$

Let

$$f_{1i}(x) = x^{\delta\,x+\beta}pi\ (x) \qquad (13)$$

Where

$$Pi\ (x) = \sum_{n=0}^{\infty}\sum_{j=1}^{\delta} \frac{Anj}{\Gamma\,(\delta+\beta+kn,)} Z_{n_j}^{\delta+\beta-1}(x;k) \qquad (14)$$

Multiplying both sides of (13) by

$(\xi - x)^{\beta-1}$ and integrating with respect to x over $(0,\xi)$ and then differentiating with respect to ξ, we get

$$\frac{d}{d\xi}\int_0^\xi (\xi-x)^{\beta-1}f_{1i}(x)dx$$

$$=$$

$$\frac{d}{d\xi}\int_0^\xi x^{\delta-\beta}(\xi-x)^{\beta-1}\ Pi\ (x)dx$$

Now using (9) and (11), we get

$$\mathrm{F}_{1i}(\xi) = \frac{1}{\pi}\ (\sin\beta\ \pi\ \Gamma(\beta)\xi^{\delta+2\beta-1}\ f_i(\xi)) \qquad (15)$$

Again dividing both sides of (15) by $(x-\xi)^\beta$, integrating with respect to ξ over $(0, x)$ and then using (7), we get

$$\mathrm{f}_{1i}(x) = \ x^{\delta+\beta}\ Pi\ (x)$$

$$= \frac{Sin\ \beta\pi\ \Gamma\ (\beta)}{\pi}\int_0^\infty \frac{\xi^{\delta+3\beta-1}f_i\ (\xi)}{(x-\xi)^\beta}\ d\xi \quad (16)$$

Let

$$f_{2i}\ (x) = \exp(-x)\ \ Pi\ (x) \qquad (17)$$

where Pi (x) is given by (14). (17)

Similarly, Multiplying both sides of (17) by$(x-\xi)^{\beta-1}$ and integrating with respect to x over (ξ,∞) and differentiating with respect of ξ we get by using eqns. (10) and (12).

$$\mathrm{F}_{2i}(\xi) = -\frac{\sin\beta\pi\Gamma\,(\beta)}{\pi}\frac{d}{d\xi}(\exp(-\xi)g,(\xi)) \quad (18)$$

Dividing both sides of (1δ) by $(\xi - x)^{\beta}$, integrating with respect to ξ over (x, ∞) and then using (δ), we get

$$f_{2i}(x) = \exp(-x)\ pi\ (x)$$

$$= -\frac{Sin\ \beta\pi\ \Gamma\ (\beta)}{\pi} \int_x^{\infty} \frac{\left(\frac{d}{d\xi}\right) \exp(-\xi g f \xi)}{(x-\xi)^{\beta}}\ d\xi \quad (19)$$

from (16) and (19), we write respectively

$$p_i(x) = -\frac{\exp(x) \sin(\beta\pi)\Gamma}{\pi} \int_0^x \frac{\xi^{\delta+2\beta-1} f_i(\xi)}{(x-\xi)^{\beta}}\ d\xi$$

$$0 < x < y \qquad (20)$$

and

$$p_i(x) = -\frac{\exp(x) \sin(\beta\pi)\Gamma\ (\beta)}{(\pi)}$$

$$\times \int_x^{\infty} \frac{\left(\frac{d}{d\xi}\right) \exp(-\xi) g_i(\xi) d\xi}{(\xi-x)^{\beta}} \qquad (21)$$

$$y < x < \infty$$

The L.H.S. of (20) and (21) are identical, hence multiplying both by $x^{\delta+\beta} \exp(-x) Y_{m_j}^{\delta+\beta}(x;k)$, integraing (20) with respect to x over (0, y), integrating (21) with respect to x over (y, ∞); adding and using the orthogonality relation (3), we get, with the help of (14), the solution of the dual series eqn. (1) and (2) in the form

$$\sum_{j=1}^{\delta} A_{nj} = \frac{1}{\pi} [((n_j)! \sin(\beta\pi)\Gamma(\beta)] \int_0^y \exp(-x)$$

$$\times Y_{nj}^{\delta+\beta}(x;k) \left\{ \int_0^x \frac{\xi^{\delta+2\beta-1} f_i(\xi)}{(x-\xi)^{\beta}}\ d\xi \right\} dx -$$

$$-(n_j)! \sin(\beta\pi)\Gamma\ (\beta) \int_y^{\infty} x^{\delta+\beta} y_{nj}^{\delta+\beta}(x\,;k).$$

$$\times \left\{ \int_x^{\infty} \frac{\left(\frac{d}{d\xi}\right) \exp(-\xi) gi\ (\xi)}{(\xi-x)^{\beta}} d\ \xi \right\} dx] \quad (22)$$

or

$$\sum_{j=1}^{\delta} A_{nj} = \frac{1}{\pi} [\sin(\beta\pi)\Gamma(\beta\pi)](n_j)! \int_0^y \exp(-x)$$

$$\times Y_{nj}^{\delta+\beta}(x;k) f_i^*(x) dx - \int_y^{\infty} x^{\delta+\beta}\ Y_{nj}^{\delta+\beta}\ (x;k)$$

$$\times g_i^*(x) dx\}] - \qquad (23)$$

with $\delta + 1 > 0, \beta > 0$, where

$$f_i^*(x) = \int_x^x \frac{\xi^{(\delta+2\beta+1)} f_i\ (\xi)}{(\xi - x)^{\beta}}\ d\xi$$

$$g_i^*(x) = \int_x^{\infty} \frac{\left(\frac{d}{d\xi}\right) \exp(-\xi) gi\ (\xi)}{(\xi - x)^{\beta}}\ d\xi$$

Particular case : If was set s=1 in eqn. (1) and (2) then reduce to dual series equations involving Konhauser biorthogonal polynomials and our solution (23) is in complete agreement with that of Patil and Thakare[5]

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