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QUASI CLIQUE DOMINATING SET

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Abstract: Let G be a simple graph with at least three vertices. We call a vertex set $D \subseteq V(G)$ as ζQuasi clique dominating set if D is a dominating set and for every v in D, $deg_D(v) \geqslant \lceil \zeta(p-1) \rceil$, where $\zeta \in (0,1)$ and p is the number of vertices in D. The minimum cardinality of ζ quasi clique dominating set is called as ζ quasi clique domination number and it is denoted by $\gamma_{qcl}(G)$. In this paper we introduce new parameter ζ quasi clique dominating set and we discuss the properties of ζ quasi clique dominating number. Minimal ζ quasi clique dominating set is also defined and it is studied.

Keywords: Dominating set, ζ Quasi clique dominating set, Minimal ζ quasi clique dominating set, upper ζ quasi clique dominating number.

I.Introduction

Let us consider a simple graphs with at least 3 vertices. Let $G = (V, E)$ be a graph with $|V| = k$ and $E = t$. Let $\Delta(G)$ denote the maximum degree of a graph G and $\delta(G)$ denote the minimum degree of a graph G. The neighborhood of a vertex x is that the set $N(x)$ consisting of all vertices y which are adjacent with x. The closed neighborhood is $N[x] = N(x) \cup \{x\}$. The boundary of D, denoted $B(D)$, is $N(D) \setminus D$. The sub graph induced by D is denoted by $G[D]$. In a graph vertices with degree one are called as pendent vertices and the vertices adjacent to pendent vertices are called as support vertices.

In this paper we have defined the quasi clique dominating set and derived bounds for a ζ quasi clique dominating number γ_{qcl} and minimal ζ quasi clique dominating set and upper ζ quasi clique dominating number are discussed.

2.Preliminaries

Definition 2.1.[1]

A set D of vertices in a graph G is a dominating set of G, if every vertex in $V - D$ is adjacent to some vertex in D. The dominating number $\gamma(G)$ is the minimum cardinality of a dominating set.

Definition 2.2.[6]

The diameter (d) of a graph is the length of longest shortest path between any two vertices of a graph.

Definition 2.3.

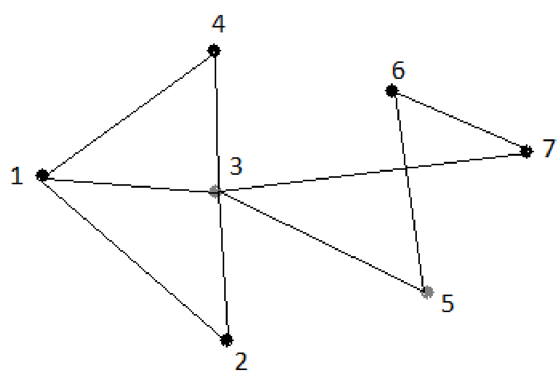
A vertex cover of a graph is a set of vertices that includes at least one endpoint of every edge of the graph.

Definition 2.4.
The vertex connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.
Definition 2.5.
The edge connectivity $\lambda(G)$ of a graph G is the minimum number of edges whose removal results in a disconnected or trivial graph.
Definition 2.6.
The complement of a graph G is a graph $\bar{G}$ on the same vertices such that two distinct vertices of $\bar{G}$ are adjacent iff they are not adjacent in G.
Definition 2.7.
A vertex cut or an articulation point is a vertex whose deletion along with the incident edges results in a graph with more components than the original graph.
Definition 2.8.[6]
The girth $g(G)$ of an undirected graph G is the length of a shortest cycle contained in the graph G.
Definition 2.9.[7]
For parameter $0 < \zeta < 1$, a vertex-induced sub graph Q of G is called a ζ-quasi-clique if Q is connected and for every vertex $v \in V(Q)$, $deg_Q(v) \geqslant [\zeta(p-1)]$.

3. ζ Quasi clique dominating set

Definition 3.1.
ζ Quasi clique dominating set
Let G be a simple graph with $k \geqslant 3$ vertices. A set $D \subseteq V(G)$ is called as ζ quasi clique dominating set if D is a dominating set and for every v in $D, deg_D(v) \geqslant [\zeta(p-1)]$, where $\zeta \in (0,1)$ and p is the number of vertices in D. The minimum cardinality of quasi clique dominating set is called as ζ quasi clique domination number and it is denoted by $\gamma_{qcl}(G)$.

Example 3.2.



For the above graph $\{3,6,7\}$ is a dominating set and for every v in $D, deg_D(v) \geqslant [\zeta(p-1)]$, where $\zeta \in (0,1)$. Therefore $D = \{3,6,7\}$ is a ζ quasi clique dominating set with cardinality three. Since ζ quasi clique dominating set $D = \{3,6,7\}$ has the minimum cardinality for the above graph, hence $\gamma_{qcl}(G) = 3$.

Now we characterize the bounds for ζ quasi clique dominating number.

Theorem 3.3.

For a graph G with diameter $d = 2$ and $\delta(G) > 1$, $\gamma_{qcl}(G) \geqslant \delta(G) + 1$.

Proof: Let $a \in V(G)$ and $deg(a) = \delta(G)$. Since $d(G) = 2$, we get $N(a)$ is a dominating set of a graph G. Hence $D = N(a) \cup \{a\}$ is a ζ quasi clique dominating set of G and hence $\gamma_{qcl}(G) \geqslant \delta(G) + 1$.

Theorem 3.4.

For any connected graph G, $\gamma_{qcl}(G) \leqslant 2t - k + 2$.

Proof: Let G be any connected graph. Then $t \geqslant k - 1$. Since $\gamma_{qcl}(G) \leqslant k$, we get $\gamma_{qcl}(G) \leqslant 2(k-1) - k + 2$ which can be written as $\gamma_{qcl}(G) \leqslant 2t - k + 2$.

Theorem 3.5.

Let G be a graph with no isolated vertices then $\frac{2k}{\Delta(G)+1} \leqslant \gamma_{qcl}(G)$.

Proof: Let D be a ζ quasi clique dominating set of a graph G with no isolated vertices. Since $deg(v) \leqslant \Delta(G)$ for every $v \in D$. Each vertex in$D$ is adjacent to at least one member of $V - D$ and for every $v \in D$ $deg_D(v) \geqslant \lceil \zeta(p-1) \rceil$. Therefore $t \leqslant (\Delta(G) - 1)|D| = (\Delta(G) - 1)\gamma_{qcl}(G)$. Since vertex in $V - D$ has an adjacency or will be an isolated vertex, So that $t \leqslant 2|V - D| = 2[k - \gamma_{qcl}(G)]$. Hence $\Delta(G)\gamma_{qcl}(G) - \gamma_{qcl}(G) \geqslant 2[k - \gamma_{qcl}(G)]$ which can be written as $\frac{2k}{\Delta(G)+1} \leqslant \gamma_{qcl}(G)$.

Theorem 3.6.

For any graph G, $\gamma_{qcl}(G) \leqslant \beta(G) + 1$.

Proof: Let D be the vertex cover of a graph G. If $|D| = \frac{k}{2}$, then D is a ζ quasi clique dominating set of a graph G. Hence $\gamma_{qcl}(G) \leqslant \beta(G) + 1$. If $|D| < \frac{k}{2}$, then for any vertex $u \in V - D, D \cup \{u\}$ is a ζ quasi clique dominating set of a graph G. Hence $\gamma_{qcl}(G) \leqslant \beta(G) + 1$.

Proposition 3.7.

Let G be any graph, then $\gamma_{qcl}(G) \leqslant k - \delta(G) + 1$.

Proof: Let D be the vertex cover of a graph G, then $\gamma_{qcl}(G) \leqslant \beta(G) + 1$. Since $\beta(G) \leqslant k - \delta(G)$. Hence $\gamma_{qcl}(G) \leqslant k - \delta(G) + 1$.

Proposition 3.8.

If G is any graph, then $\gamma_{qcl}(G) \leqslant k - \kappa(G) + 1$.

Proof: Since $\gamma_{qcl}(G) \leqslant \beta(G) + 1$ and $\beta(G) \leqslant k - \kappa(G)$. So we can conclude that $\gamma_{qcl}(G) \leqslant k - \kappa(G) + 1$.

Proposition 3.9.

If G is any graph, then $\gamma_{qcl}(G) \leqslant k - \lambda(G) + 1$.

Proof: We know that $\gamma_{qcl}(G) \leqslant \beta(G) + 1$ and $\beta(G) \leqslant k - \lambda(G)$. Hence we get $\gamma_{qcl}(G) \leqslant k - \lambda + 1$.

Proposition 3.10.

For a connected graph G with at least 2 pendent vertices , then $\gamma_{qcl}(G) + \gamma_{qcl}(\bar{G}) \leqslant k + 3$.

Proof: Since $3 \leqslant \gamma_{qcl}(G) \leqslant k$ and $\gamma_{qcl}(\bar{G}) = 3$, we get $\gamma_{qcl}(G) + \gamma_{qcl}(\bar{G}) \leqslant k + 3$.

Theorem 3.11.

Let G be a graph , then $\gamma_{qcl}(G) + \kappa(G) \leqslant k + \Delta(G)$.

Proof: For a graph $G, \gamma_{qcl}(G) \leqslant k$ and $\kappa(G) \leqslant \Delta(G)$. Hence $\gamma_{qcl}(G) + \kappa(G) \leqslant k + \Delta(G)$.

Theorem 3.18.

For any connected graph G, $3 \leqslant \gamma_{qcl}(G)\gamma_{qcl}(\bar{G}) \leqslant (k - \delta(G) + 1)(1 + \Delta(G))$.

Proof: Since $\gamma_{qcl}(G) \leqslant k-\delta(G)+1$. Therefore $\gamma_{qcl}(G)\gamma_{qcl}(\bar{G}) \leqslant (k-\delta(G)+1)(k-\delta(\bar{G})+1) \leqslant (k-\delta(G)+1)(k-(k-1-\Delta(G))) \leqslant (k-\delta(G)+1)(1+\Delta(G))$.

4 . Minimal Quasi Clique Dominating Set

Definition 4.1.

A ζ quasi clique dominating set D is a minimal ζ quasi clique dominating set if for all vertices $v \in D$, the set $D-\{v\}$ is not a ζ quasi clique dominating set of G.

Definition 4.2.

The upper ζ quasi clique dominating number of G denoted as Γ_{qcl}, is the maximum cardinality of a minimal ζ quasi clique dominating set in G.

Lemma 4.3.

For a graph with no isolated vertices. A vertex v of G belongs to every minimal ζ quasi clique dominating set of G iff v is a support vertex of G.

Proof: Let $v \in V(G)$. If v is not a support vertex of G, then $V(G)-v$ is a ζ quasi clique dominating set of G and so it contains a minimal ζ quasi clique dominating set of G. Hence there exists a minimal ζ quasi clique dominating set excluding $\{v\}$. So that every support vertex must belong to any minimal ζ quasi clique dominating set.

REFERENCES

[1]D. Archdeacon, J. Ellis-Monaghan, D. Froncek, P. Lam, B. Wei and R. Yuster,Some Remarks on Domination, Journal of Graph Theory.

[2]E. J. Cockayne, R. M. Dawe and S. T. Hedetniemi, Total domination graphs, Networks 10 (1980), 211-219.

[3]O. Favaron, M. Henning, C. M. Mynhart and J. Puech, Total domination in graphs with minimum degree three, Journal of Graph Theory 34 (2000), 9-19.

[4]L. Sun, An upper bound for total domination number, J. Beijing Inst. Technol., Engl. Ed. 4 (1995), 111-114.

[5] Peter Che Bor Lam, Bing Wei, On the total domination number of graphs, Utilitas Mathematica · March 2007.

[6] Maryam Atapour and Nasrin Soltankhah, On total dominating sets in graphs, arXiv:0810.4667v1 [math.CO] 26 Oct 2008.

[7] J. Abello, M.G.C. Resende, S. Sudarsky, Massive quasi-clique detection, S. Rajsbaum (Ed.), LATIN 2002: Theoretical Informatics, Springer-Verlag, London (2002), pp. 598-612