

Queueing Models for Manufacturing System

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Abstract

Manufacturing systems consist of machines or workstations processing parts or items. Auxiliary essential components are buffers to hold work in process and the material handling system for transporting items from one workstation to another. The tasks at the machines may be performed by human operators or they can be highly automated such as in complex lithography machines. Several research efforts have been directed toward design and performance analysis of manufacturing systems based on queueing theory.

Queueing problem is identified by the presence of groups of units/customers who arrive randomly to receive the service. The units/customers, upon arrival may be attended immediately or may have to wait until the server is free. This methodology is applicable in the field of business, industries, government, transportations, library etc.. Queueing models are basically relevant to service oriented organizations and suggest ways and means to improve the efficiency of service. Queueing

theory is concerned with the mathematical modeling and analysis of systems that provide service to random demands. A queueing model can be developed for an abstract description of a manufacturing system. Typically, a queueing model represents the system's physical configuration, by specifying the number and arrangement of the servers, which provide service to the units/customers and the stochastic (that is, probabilistic or statistical) nature of the demands, by specifying the variability in the arrival process and in the service process.

With a proper interpretation, it is possible to characterize a **queueing system** that is implicitly built into the single stage or multistage manufacturing system. Consequently, the analysis of manufacturing system can be carried out through the analysis of queueing system which can be described as follows. A typical manufacturing system consists of a number of resources that process a variety of products. The parts arriving at the different resources have to wait in the queue if resources are not available. Thus, there are numerous queues in the system at various resources and there are interactions between the queues. The

system is dynamic in the sense that the arrival patterns of products are dependent on the product demand, which is stochastic. The processing rates of the resources are stochastic because resources exhibit a natural variation in their performance. Also, the resources are subject to unforeseen failures. The common characteristic is that a number of physical entities (the arrivals) are attempting to receive service from limited facilities (the server) and as a consequence the arrival must sometimes wait in the line for their turn to be served.

Introduction

The popularity of steady-state analysis of manufacturing systems stems from its computational simplicity. A detailed description of performance models of manufacturing systems via queue theoretic approach has been given by Buzacott and Shanthikumar (1980). Koenigsberg and Mamer (1982) used queueing theory to obtain approximate performance of a production system. O'Grady and Menon (1984) have presented a planning framework which enables the selection of that subset of perspective orders which is compatible with the current configuration of manufacturing system resources. A work station of a flexible manufacturing system was modeled by Yao and Buzacott (1985) as multi server queue with finite waiting room. The proposed model addressed two major issues: improving machine utilization and reducing the work in process. Yao and

Buzacott (1986) developed a closed queueing network model to evaluate the performance of flexible manufacturing system by an exponentialization approach. An ANOVA analysis was adapted by Henneke and Choi (1988) to evaluate the FMS parameters such as part selection, machine centre selection and machine queue capacity. A new kind of queueing problem was analyzed by Coffman et al. (1988) in flexible manufacturing system with input/output dependencies that result because of the same conveyer transport items both to and from the station. Complex production systems were studied by Koster (1988) where a single product is manufactured and production units and buffers may be connected nearly arbitrarily.

Batching is necessary in intermittent production, where the facility is designed to process a variety of parts and the setup cost are significant. The problem of batching in FMS was investigated by Henery et al. (1989). Hsu and Tapiero (1989) provided a conceptual framework for quality control in manufacturing facilities. The need to investigate the complex problem domain of flexible manufacturing system was discussed by Mosier et al. (1990) by developing potential rule structures for manufacturing environment. The effect of reliability indices on performance of a manufacturing system has been explained by Reibman (1990). Queueing delays in complex manufacturing facilities often constitute the major part of manufacturing lead times. A single

facility multi item model was formulated by Karmarkar et al. (1992) that related batching to queueing delays. Cassandras and Hen (1992) have considered the quality control problem of a single automated service facility modeled as a simple queueing system with feed back. Dynamic routing in FMS under unpredicted failure via algorithm approach was described by Younis and Mahmoud (1992).

The performance models of manufacturing systems given by many theoreticians (cf. Viswanadham and Narahari; 1992, Buzacott and Shanthikumar; 1993, Papadopoulos et al.; 1993 and Yao; 1994) can provide insights to system designers and production managers to utilize available resources optimally. Bitran and Sarkar (1994) have attempted to quantify the tradeoffs between capacity and process improvement through variance reduction and throughput. Jang and Liu (1994) have presented a new queueing formula which uses variables easier to estimate than the variance. An iterative solution algorithm is exploited by Sung and Kwon (1994) to approximate system performance measures such as system throughput and machine utilization.

Several models for performance evaluation of automated manufacturing systems (AMS) have been proposed recently. Benjaafar (1994) has specified some models for performance evaluation of flexibility in manufacturing systems. Ram and Viswanadham (1994) applied this framework to evaluate the

performance of an AMS. Narahari and Viswanadham (1994) discussed several manufacturing scenarios, where transient analysis is of vital importance. Bretthauer (1995) discussed a manufacturing model that includes upper limit on production throughput times and working process in the system. A book on advance models for manufacturing systems management has been written by Brandimarte and Villa (1996). A classification of models for production and transfer lines was described by Papadopoulos and Heavey (1996). An integrated multi item production inventory system with stochastic demands and capacitated production was studied by Risa (1996). Rao et al. (1998) provided the literature on the application of waiting line models in manufacturing system. The decomposition, isolation and expansion methodology were used by Kavusturucu and Gupta (1999) to calculate the throughput of the system for manufacturing systems with finite buffers under N-policy. Iterative methods for queueing and manufacturing systems have been proposed by Ching (2001a).

Flexible manufacturing system generally provides the possibility to use different process plans to manufacture a part owing to versatile machines. In recent times considerable attention has been paid to flexibility optimization. Pereira and Paulre (2001) proposed that flexibility in manufacturing system is not independent of the environment with which it interacts. Diallo et al. (2001) projected a new approach for designing

manufacturing cells when these process plans can be changed because of machine breakdowns. Norman (2002) has analyzed the choice between flexible and designated manufacturing technologies when firms can choose the flexibility of their manufacturing systems. Sharafali et al. (2004) has considered the problem of production scheduling in a flexible manufacturing system with stochastic demand. A strategic planning model was developed by Chandra et al. (2005) that determines the overall business value of flexible manufacturing systems. A stochastic model to determine the performance of a flexible manufacturing cell was presented by Aldaihani and Savsar (2005). Llorens et al. (2005) established an analytical approximation that considered how the determinants of manufacturing flexibility at the system level affect the desired strategic change in the organizations. Machine loading problem of a flexible manufacturing system was extended by Kumar et al. (2005) which encompasses various types of flexibility aspects pertaining to part selection and operation. A programming approach was proposed by Nagarjuna et al. (2006) to solve the loading problem in random type FMS.

Jain et al. (2008) have developed a queueing model for the performance prediction of flexible manufacturing system with a multiple discrete materials handling devices. Pradhan et al. (2008) have presented analytical approximation to estimate the performance

measures of a manufacturing system with multiple product classes and job circulation. A concept that described how companies can manage their international operations so as to facilitate the coordination of their manufacturing networks was given by Rudberg and West (2008), which was focused on the blending of cost competitiveness, flexibility and innovativeness. A generalized economic manufacturing quantity model for an unreliable production system was proposed by Chakraborty et al. (2008). This model was formulated by assuming that the time to machine breakdown, corrective and preventive repair times follow arbitrary probability distribution.

Machine Repair Problems

There are multiple situations in **unreliable manufacturing system** where machines execute certain jobs. During the working phase it may be possible that some machines are not available to perform their activities during some time interval; this temporal absence of availability is known as breakdowns. The availability of the machine can be improved through a corrective maintenance strategy by the repairman. If at any time more than one machines need repairman's consideration, queue is developed. If the number of machines broken down exceeds the number of repairmen then the excess number of machines will have to wait until repairmen are available, in the case of several repairmen. The problem of

machine failures in the context of queueing system has been studied by many authors. Palm (1947) developed a model for machine repair based on the assumption that running time and repair time of a machine were exponentially distributed. Another approach to the machine repair problem was considered by Takacs (1957). Since then machine repair models have captivated the interest of many authors (cf. Hillard, 1976; Ascher and Feingold, 1984; Kijima, 1989).

The machine interference problems have been solved by Bunday and Scratron (1980) in which N automatic machines are maintained by a team of r operatives. Two types of numerical methods of computing transient probabilities of finite Markovian queues models for machine repair problem have been proposed by Asrham et al. (1983). An approximate method based on the diffusion process was presented by Haryono and Sivazlian (1985) for analyzing a non Markovian machine repair problem. Posafalvi and Sztrik (1987) have developed three models for heterogeneous machine interference problems where the occurrence of ancillary duty makes the operator non available for repair activities. Posafalvi and Sztrik (1989) gave a computationally tractable method for the solution of steady state equations for heterogeneous machine interference model where priority machines have preemptive priority over the ordinary ones. A comparison between the approximate results with some of

the exact existing results was made by Wang and Sivazlian (1990), by using approximate formulas for the probability density function of the number of failed machines in $G/G/R$ machines repair system. Sung and Ock (1992) have considered a single product, single machine production problem where machine is setup for production operation over an interval of time, the operation quality inspection is periodically carried out at equal time intervals.

Sztrik and Bundy (1993) presented a queueing model that can be used to analyze the asymptotic behavior of the machine interference problem composed of N heterogeneous machines and n operative. Mehrnaz et al. (1993) formulated a minimization problem in order to optimize logistics decisions under which static equilibrium environment and organizations conditions are set. Several system characteristics were evaluated by Wang and Hsu (1995) for the machine repair problem with N operating machines and R non reliable service stations under steady state conditions. Lee et al. (1995) obtained approximate solution to the probability distribution of the number of inoperative machines for the boundary policies with special emphasis on the machine repair problems. The problem of lot sizing in production facility in the face of machine breakdowns has been examined by Dagpunar (1997). The machine repair problem in which N identical automatic machines are maintained by a single non reliable service station has been studied by Wang and

Kuo (1997). A profit model was developed to determine the optimum number of machines to be assigned to the service station in order to maximize the total expected profit per machine per unit time. Continuous dynamic programming formulation of the scheduling of a manufacturing system was given by Bai and Elhafsi (1997). Frostig (1999) has considered a production system with an unreliable machine which was maintained by a single repairman. The general model for two stage lot sizing problem analyzed by Lee and Rung (2000); the analytic formulas were also derived for special production structure and for multi stages serial production system with machine prone to failure.

Motivated by the problem commonly found in the surface mount technology of electronic assemble line, Lee and Lin (2001) studied some scheduling problems involving repair and maintenance rate modifying activities. Kenne et al. (2003) presented an analysis of the optimal production control and corrective maintenance planning problem for a failure prone manufacturing system consisting of several identical machines. Optimal flow control for a one machine, two product manufacturing system subject to random failures and repairs was presented by Kenne and Gharbi (2004). Kenne and Gharbi (2004) considered a production control problem in manufacturing system with failure prone machines and a constant demand rate. The optimal lot size

policies were suggested by Giri and Yan (2005) for an economic manufacturing quantify problem for an unreliable manufacturing system where the machines were subject to random failures and at most two failures can occur in a production cycle. The basic economic manufacturing quantity (EMQ) model was formulated by Giri et al. (2005) under general repair time distribution. The optimal production policy was also derived for specific failure and repair time distributions. A unified framework was developed by Charlet et al. (2007) for a production control problem for a manufacturing system subject to random failures and repairs. Pardo and Fuente (2008) proposed the analysis, development and designing of a fuzzy queueing model with finite input sources in which the arrival pattern as well as service pattern follow an exponential distribution under certain parameters.

Machine Repair Problem with Spare

Machine interference problem is inevitable incident of the manufacturing systems and the replacement of the whole machinery is often cost efficient. This problem can be cope up with the help of standby units. The backup components (**spares**) should be included with the machines to be used when failures exceed the built-in redundancy level. The provision of spares is also recommended to ensure the proper functioning of the system. Also in many real life situations such as in exploration, mining, rescue, and defense etc., re-supply of machines are often difficult or impossible. In anticipation of this situation, the machines often

include redundant parts to allow for some component failures in the field that do not eliminate a machine from further use. Spares are classified as cold, warm and hot, their failure rates are taken as zero, less than operating unit and equal to that of regular operating units, respectively.

The reliability characteristics of a system with M operating machines, S warm standby spares and R repairmen in the repair facility has been obtained by Wang and Sivazlian (1989). Approximate formulae for the probability density function of the number of failed machines in the system were utilized by Sivazlian and Wang (1989) to compute the system characteristics of $G/G/R$ machines repair problem with warm standbys. A cost model for $M/M/R$ machine repair problem consisting of M operating machines with S spares and R repairmen was developed by Wang and Sivazlian (1992) in order to determine the optimal number of repairmen and spares simultaneously. A Markovian approach was used by Singh and Sharma (1993) to solve the $M^x/E_k/1$ spare machine repair problem under steady state conditions. A diffusion approximation technique has been developed by Jain (1993) for (m,M) machine repair system containing M identical machines under the care of r repairmen. The mixed standby model was considered by Wang (1993) to study the $M/M/R$ machine repair problem consisting of M operating machines along with cold and warm standbys under steady state conditions. The probability density function of the number of failed machines in the system was obtained by Wang (1994) via diffusion approximation for $M/E_k/1$ machine repair problem with spares. A direct search algorithm was used by Wang (1995) to determine the optimal values of the number of two types of spares and the number of servers to maintain a specified

level of system availability at minimum cost. The reliability characteristics of a repairable system with M operating machines and S spares and a removable repairman in the repair facility have been studied by Hsieh and Wang (1995). The truncated Poissonian queue with balking, heterogeneous server, spares, was considered by Al- Seedy (1995). A recursive method was developed by Gupta and Rao (1996) to obtain the steady state probability distribution of the number of down machines at arbitrary time epoch of a machine interference problem with spares. Gupta (1997) provided a new transform free closed form expression for the probability distribution of the number of machines in the repair facility and the performance measures for machine interference problem with warm spares and server vacations. By using reflecting boundaries, the approximate formulae for some performance measures have obtained by Jain (1997) for machine repair problem with spares and state dependent rates. An effective recursive algorithm was developed by Rao and Gupta (2000), to obtain the distributions of the number of down machines at arbitrary, departure and pre arrival epoch. Jain et al. (2004) used Laplace transformation technique to derive the explicit expressions for the reliability function and the mean time to system failure for machine repair system with spares and reneging. A profit model of the $M/M/R$ machine repair problem was developed by Wang et al. (2007) in order to determine the optimal values of the number of spares and number of repairmen. Ke and Wang (2007) studied the machine repair problem consisting of operating machines with two types of spare machines and multi servers who leave for vacation of random length when there are no failed machines queueing up for repair in the repair facility. Ke and Lin (2007) modeled a manufacturing system

with M operating machines and S spares under the supervision of a group of technicians in repair facility.

Machine Repair Problems with Multi-Mode Failure

So far, research on machine repair problems have focused mainly for single mode of failure. However to analyze more robust repairable machining systems of the physical world, some attempts have also been made to investigate **the two mode failure** models as well as multiple mode failure. Goyal and Sharma (1980) gave the stochastic analysis of a two standby system with two failure modes and slow switch. Gopalan and Ramesh (1986) dealt with the cost benefit analysis of a one server two unit systems subject to two modes of failure, namely, shock and degradation. A repairable system operated by a human operator and subject to two modes of failures was considered by Kumar and Garg (1991). The optimization of parallel system subject to two modes of failure and repair provision was investigated by Reddy and Rao (1993). The cold standby machine repair problem considered by Wang (1994) dealt with two failure modes under steady state condition. The cost analysis of the M/M/R machine repair problem with two modes of failure was done by Wang and Wu (1995). Hsieh (1997) described a procedure to determine the optimal number of machines which should be assigned to a repairman for multiple machine repair problem, each machine was subject to two modes of failure, minor and major. Wang and Lee (1998) studied the cold standby machine repair problem consisting of M operating machines with S cold standby machines and R repairmen where machines have K failure

modes under steady state conditions. The M/M/C/K/N machine repair problem with additional repairmen and two modes of failure has been investigated by Jain et al. (2000). Levantesi et al. (2003) presented an approximate analytical method for the performance evaluation of asynchronous production line with deterministic processing time, multiple failure modes and finite buffer capacity. Jain et al. (2008) examined a multi component repairable system with state dependent rates by assuming that the units may fail in two modes.

Conclusion

The research work carried out is mainly concentrated on the formulation and analysis of queueing and inventory models of manufacturing systems in different frameworks. The models developed can be applied to realistic systems that can be fed up by congestion situations, yielding accurate performance predictions.

Queueing models are very helpful for determining how to operate a manufacturing system in the most effective way. Providing too much service capacity to operate the system involves excessive costs, but not providing enough service capacity results in excessive waiting.

The models studied for manufacturing system may enable the production managers in finding an appropriate balance between the cost of manufacturing and the amount of waiting.

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