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Stability of Plates under Uniform In-Plane Loading

Md Mozaffar Masud[1], A. K. L. Srivastava[2]

[1]*Department of Civil Engineering, NIT Jamshedpur, India*
[2] *Department of Civil Engineering, NIT Jamshedpur, India*

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Abstract - The forces are assumed to operate in the plane of the undeformed middle surface of the plate. The equation of motion yields the characteristic equations for natural frequencies, buckling loads, and their corresponding mode shapes. The buckling load parameter for various modes of the stiffened plate with square cutouts subjected to in-plane biaxial loads has been determined for various edge conditions. For a hole to plate size range of 0 to 0.8, numerical results are presented. The plate and stiffeners are treated as separate elements in structural modeling, with compatibility between the two types of elements maintained. A finite element method for the buckling loads on a longitudinally square stiffened plate with a square cutout is investigated under various combinations of biaxial loading at the plate boundary.

Key Words: vibration, cut out, buckling load parameter, in-plane uniform loading.

1.INTRODUCTION

Cutouts are unavoidable in aerospace, civil, mechanical, and marine structures, mainly for practical reasons. In addition, cutouts or openings in a structure that serve as doors and windows are often required by designers. Structures with cutouts, buckling and vibration analysis are a huge challenge that needs to be considered in the structural design. With the addition of stiffeners, the effects of instability are improved.

Olson and Hazell [1] have used finite element method to perform a critical analysis on the clamped integrally stiffened plate. Using the real-time holographic method, the mode shapes and frequencies were determined experimentally. Mukherjee and Mukhopadhyay [2] proposed an eight-noded isoparametric stiffened plate-bending element for free vibration analysis of stiffened plates. Ali and Atwal [3], Shastry and Rao [4], and Reddy [5] have all reported numerical results obtained using the finite element method. Monahan et al. [6] carried out a finite element analysis as well as experiments on clamped thin plates of various cutout sizes. Mundkur et al. [7] used boundary characteristics orthogonal polynomials that satisfied the boundary conditions to investigate the vibration of square plates with square cutouts. The free vibration characteristics of unstiffened and longitudinally stiffened square panels with symmetrically square cutouts are investigated by Sivasubramonian et al. [8] using the finite element method. Lam and Hung [9] studied the vibrations of plates with stiffened openings using the orthogonal polynomials and partitioning method. Natural frequencies of simply supported and fully clamped plates with stiffened openings are presented. To obtain the natural frequencies of a square plate with stiffened square openings, Paramsivam and Sridhar Rao [10] modified the grid framework model. The contribution of the beam element is reflected in all nodes of the plate element that contains the stiffener. Despite the fact that the plate skin and stiffeners are modeled separately, their compatibility is maintained. The present paper investigates the effects of various parameters on the buckling and vibration characteristics of rectangular stiffened plates with cutouts, such as cutout size and location, plate and cutout aspect ratios, different boundary conditions, and stiffener parameters. The vibration and buckling behaviour of stiffened plates with cutouts subjected to in-plane uniform biaxial edge loading at the plate boundary was investigated using finite element analysis. For the stiffened plates with cutouts, a nine-nodded isoparametric quadratic element with the ability to accommodate curved boundaries was selected. The main elegance of formulation lies in the treatment of stiffeners, which allows the stiffener to be placed anywhere within the plate element, allowing for greater flexibility in mesh generation.

2. FINITE ELEMENT FORMULATION

In addition to bending deformations, the effect of in-plane deformations is taken into account. In this study, a nine-noded isoparametric quadratic element with five degrees of freedom (u, v, w, θ_x, and θ_y) per node has been used. The contribution of the plate and that of the stiffener are included in the element matrices of the stiffened plate element. It shows that the contribution of beam element is reflected in all 9 nodes of the plate element, which contains the stiffener. The contribution of a stiffener to a particular node is determined by the proximity of stiffener to that node. With the known stresses, the geometric stiffness matrix is now constructed. From the assembly of those element matrices, the overall elastic stiffness matrix, geometric stiffness matrix, and mass matrix are generated and stored in a single array using the variable bandwidth profile storage scheme. The simultaneous iteration technique proposed by Corr and Jennings is used to solve eigenvalues[11]. The elastic stiffness matrix $[K_p]$, geometric stiffness matrix $[K_{Gp}]$ and mass matrix $[M_p]$ of the plate element may be expressed as follows.

$$\left[K_p\right]=\int_{-1}^{+1}\int_{-1}^{+1}\left[B_p\right]^T\left[D_p\right]\left[B_p\right]\left|J_p\right| d\xi\, d\eta \quad (1)$$

$$\left[K_{Gp}\right]=\int_{-1}^{+1}\int_{-1}^{+1}\left[B_{Gp}\right]^T\left[\sigma_p\right]\left[B_{Gp}\right]\left|J_p\right| d\xi\, d\eta \quad (2)$$

$$\left[M_p\right]=\int_{-1}^{+1}\int_{-1}^{+1}\left[N\right]^T\left[m_p\right]\left[N\right]\left|J_p\right| d\xi\, d\eta \quad (3)$$

A stiffener element placed anywhere inside a plate element and oriented in the direction of x has its elastic stiffness matrix $[K_S]$, geometric stiffness matrix $[K_{GS}]$ and mass matrix $[M_S]$ expressed in the same way as the plate element:

$$[K_S] = \int_{-1}^{+1} [B_S]^T [D_S] [B_S] |J_S| \, d\xi \,, \qquad (4)$$

$$[K_{GS}] = \int_{-1}^{+1} [B_{GS}]^T [\sigma_S] [B_{GS}] \; |J_S| \, d\xi \qquad (5)$$

$$[M_S] = \int_{-1}^{+1} [N]^T [m_S] [N] |J_S| \, d\xi \,, \qquad (6)$$

$$[B_P] = [\, [B_P]_1 \quad [B_P]_2 \quad \dots \quad [B_P]_r \quad \dots \quad [B_P]_9 \,] \quad (7)$$

$$[B_{GP}] = [\, [B_{GP}]_1 \quad [B_{GP}]_2 \quad \dots \quad [B_{GP}]_r \quad \dots \quad [B_{GP}]_9 \,] \quad (8)$$

The equilibrium equation for a stiffened plate subjected to in-plane loads is as follows:

$$[M]\{\ddot{q}\} + [\,[K_b] - P[K_G]\,]\{q\} = 0 \quad (9)$$

Equation (9) can be reduced to the governing equations for buckling and vibration problems.

3. RESULTS AND DISCUSSION

The problem considered here consists of a rectangular plate (a x b) with stiffeners having a rectangular cutout of size (g x d) at the center as shown in figure 1. The plate with stiffener subjected to in-plane uniform edge loading at the plate boundary and stiffener cross-section are shown in figure 2.

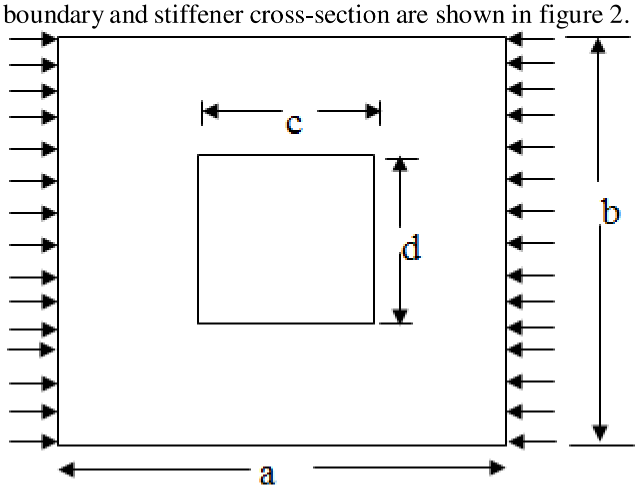
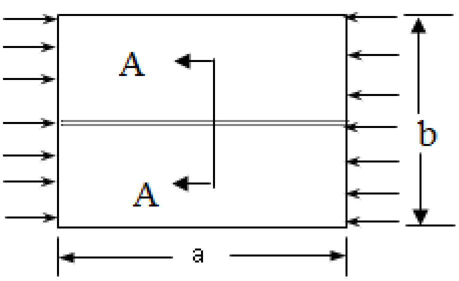
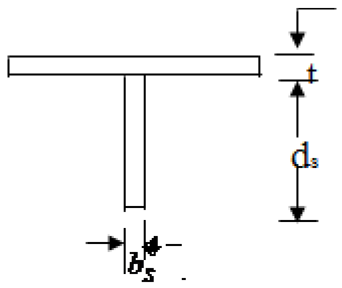


Fig. 1. Stiffened plate with cutout under in plane uniform edge loading at plate boundary





Section A A

Fig. 2. Stiffened plate cross section

The non-dimensionalisation of different parameters like vibration, buckling for stability analysis is taken as given below.

Frequency parameter $(\omega) = \varpi b^2 \sqrt{\rho t / D}$ and Buckling parameter $(\lambda) = N_X b^2 / \pi^2 D$

Where D is the plate flexural rigidity, D = $Et^3 / 12(1-\nu^2)$, ρ is the density of the plate material and t is the plate thickness. The stiffener parameter terms and are defined as, assuming a general case of several longitudinal ribs and denoting by EI_S the flexural rigidity of a stiffener at a distance (D_x) from the edge y = 0 as:

$\delta = A_S / bt$ = Ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener. $\gamma = EI_S / bD$ = Ratio of bending stiffness rigidity of stiffener to the plate, where I_S is the moment of inertia of the stiffener cross-section about reference axis. g/a = Ratio of cutout to plate width.

The presence of the cutout in the plate produces stress concentrations and high stress gradients in the neighborhood of the cutout, which calls for an extra fineness of the mesh in this zone in the finite element discretization.

3.1 Buckling studies of stiffened plates with cutout

Linear fundamental frequencies of a simply supported isotropic square plate with various sizes of rectangular cutout (g/a) are computed and compared to [7] in table 1 to validate the results**.** The predicted changes in frequencies for different cutout sizes agree well with results of Mundkur *et al.* [7] given in bracket.

Table 1. Comparison of natural frequency parameter

Natural frequency parameter (ω)				
	SSSS		CCCC	
g/a	Mundkur *et al.* [7]	Present	Mundkur *et al.* [7]	Present
0.167	20.070	19.87	37.425	36.06
0.33	20.9633	20.12	43.867	43.02
0.5	24.2434	24.24	65.715	65.27

The stiffened plates are subjected to uniaxial compressive force N_X for the first case, and biaxial loading with $N_X = N_y$ for the second case study. The corresponding values of N_X and N_y are the buckling loads for the mode shape under consideration.

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The stiffened plate with central square cutout is studied by taking different cutout size ratio g/a. The plate is simply supported at its four edges and the data used for its geometry are a = 100mm, b = 100mm, t = 1mm. The stiffener parameters to be used are as follows: $\delta = 0.1$ and $\gamma = 10$. The other data uses are as: $\nu = 0.30$, E = 3.0 x 10^7 N/mm^2, ρ =7.8 x 10^{-6} Kg/mm^3.

Numerical results for buckling load parameter for stiffened square plate having one central stiffener with square central cutout of different sizes subjected to uniaxial compressive force for various boundary conditions in various modes are presented in figures 3. Figure 3 shows the variation of buckling load parameter (λ) for stiffened plate with one central stiffener subjected to uniaxial load for various boundary conditions, (SSSS, CCCC, CCSS, SSCC). It is observed from figure 3 that buckling load decreases with the increase of cutout sizes for edge conditions SSSS and SSCC. On the other hand, for edge conditions CCCC and CCSS, it tends to increase for g/a > 0.4.

The effect of bi-axial force on buckling load parameter for stiffened square plates for the same dimensions as described above with various cutout sizes for various boundary conditions in different modes are analyzed in figures 4-5 for stiffened plates.

Figure 4 shows the variation of buckling load parameter (λ) for stiffened plate with one central stiffener subjected to biaxial load for various boundary conditions, (SSSS, CCCC, CCSS, SSCC). This variation of buckling load parameter (λ) with cutout size for various boundary conditions, (SSSS, CCCC, CCSS, SSCC) in various modes are shown in figures 5.

It is observed from fig 4-5 here that buckling load decreases with the increase of cutout sizes for SSSS, SSCC and CCSS, but for edge condition CCCC, it increases for cutout size g/a greater than 0.4.

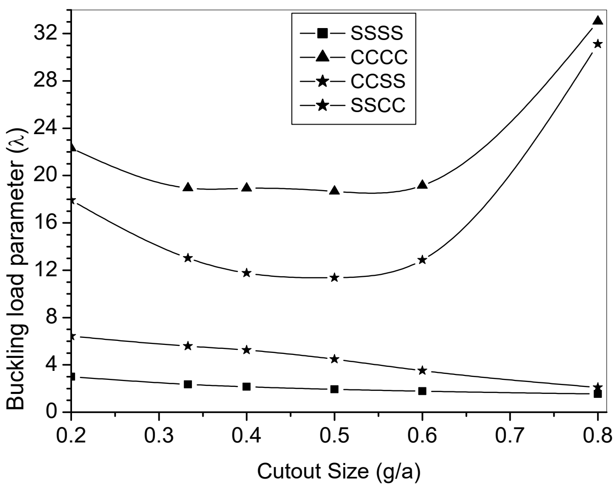

Fig. 3. Buckling load parameter (λ) vs hole/plate ratio (g/a) for uniaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$)

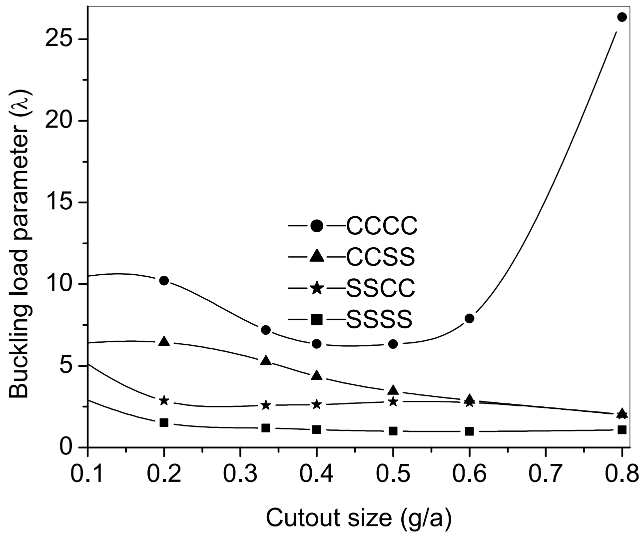

Fig.4. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxial loaded stiffened plate with one central stiffener (δ = 0.1 and γ = 10).

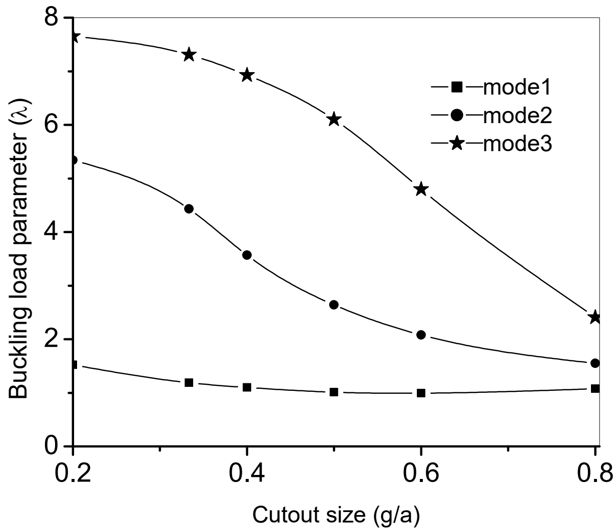

Fig.5. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxial loaded simply supported stiffened plate with one central stiffener (δ = 0.1 and γ = 10) in various modes.

4. CONCLUSIONS

The vibration frequencies increase for higher modes due to increased complexity in the mode shapes. The variation of the fundamental frequencies with increased in-plane forces is the same as that of uniaxial force in various modes. The curves for the uniaxial and biaxial loadings are identical for normalized compressive forces.

For SSSS, SSCC, and CCSS, the buckling load decreases when the cutout size increases, Shear deformation has a greater effect on clamped plates than on plates that are simply supported. Vibration frequencies increase as the restraint at the edges increases.

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NOTATIONS

a	Plate dimension in longitudinal direction
b	Plate dimension in the transverse direction
t	Plate thickness
E, G	Young's and shear moduli for the plate material
υ	Poisson's ratio
b_s, d_s	web thickness and depth of a x-stiffener
ξ, η	Non –dimensional element coordinate
A_S	Cross sectional area of the stiffener
I_S	Moment of inertia of the stiffener cross-section about reference axis
$\{q\}_r$	Vector of nodal displacement a r^{th} node
$[D_P]$	Rigidity matrix of plate
$[D_S]$	Rigidity matrix of stiffener
$[K_e]$	Elastic stiffness matrix of plate
$[K_S]$	Elastic stiffness matrix of stiffener
$\lfloor M_p \rfloor, \lfloor M_S \rfloor$	Consistent mass matrix of plate, stiffener
$[K_G]$	Geometric stiffness matrix
$[N]_r$	Matrix of a shape function of a node r
P_{cr}	Critical buckling load
g	Cutout length
d	Cutout width
g/d	Cutout width ratio