

# Three valued logic on Complex Logical Variables

R. Malathi

Asst.Prof.of Mathematics, SCSVMV, Kanchipuram

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**Abstract** - In this paper we will discuss about the definitions for these variables for the operations disjunction, conjunction and negation. The values obtained are, corresponding to True (T), True False (TF) and False (F) in the classic propositional logic. The result may be made from the initial assumptions and the results attained, that the imaginary logical variable  $\frac{1}{2} + i\frac{\sqrt{3}}{2}$  introduced hereby is 'true' than the condition 'T' of the classic propositional logic -1 'true false' than the condition 'TF' and  $\frac{1}{2} - i\frac{\sqrt{3}}{2}$  'false' than the condition 'F', respectively. Possibilities for further investigations of this class of complex logical structures are pointed out.

**Key Words:** Complex propositional logic, Imaginary logical variable, Logical equations.

## 1. INTRODUCTION

An approach is proposed in the present work, which provides a possibility to evade the existing difficulties. Due to the specificity of defining the logical operation disjunction and conjunction very often a solution of these equations cannot be found.

Practically, **every attribute by being affirmed and denied according to different aspects may bring about three fundamental propositions true of real subject.**

Due to the limitations of the human mind, it is impossible to consider all aspect of human reality. However, we can consider each aspect at a time. Since, it is a relative approach, each prediction can be confirmed or rejected using three different possibilities.

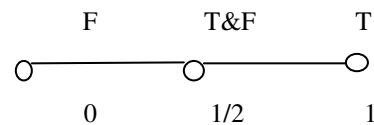
## 2. Imaginary logical variables

The classical two – valued logic can be extended into n-valued logic ( $n \geq 2$ ). Several n – valued logic were, in fact developed in the 1930s. The set  $T_n$  of truth values of an n – valued logic is thus defined as

$$T_n = \left\{ 0 = \frac{0}{n-1}, \frac{1}{n-1}, \frac{2}{n-1}, \dots, \frac{n-2}{n-1}, \frac{n-1}{n-1} = 1 \right\}$$

$$T_3 = \left\{ 0 = \frac{0}{3-1}, \frac{1}{3-1}, \frac{2}{3-1} = 1 \right\} = \left\{ 0, \frac{1}{2}, 1 \right\}$$

Thus, correlating the symbols and the numerical values of three logic we get,



We observe that when we move from left to right, we gradually move from false to truth.

As an example the following equation of the propositional logic may be pointed out:

$$X = T \quad \dots\dots\dots(1) \qquad F \wedge$$

Where X is a logical value which accepts one of three states – True (T), True False (TF) or False (F).

It is evident that in the frame of the classical propositional logic there is no such value of X,  $X \in \{T, TF, F\}$  for which the requirements of equation (1) to be satisfied [12].

An analogy to this condition may be sought in the number theory in which exist equations of the type:

$$x^3 = -1, \quad x = (-1)^{1/3}$$

$$x = [\cos (2m+1) \frac{\pi}{3} + i \sin(2m+1) \frac{\pi}{3}],$$

$$m= 0,1,2 \dots\dots\dots(2)$$

$$m=0, x = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} = \frac{1}{2} + i \frac{\sqrt{3}}{2} \text{ (T)}$$

$$m=1, x = \cos \pi + i \sin \pi = -1 \text{ (T\&F)}$$

$$m= 2, x = \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} = \frac{1}{2} - i \frac{\sqrt{3}}{2} \text{ (F)}$$

This leads to extending the range of the real numbers and to transition to complex ones through the introduction of the imaginary unit  $i$  ( $i^3 = -1$ ). The general appearance of the complex number  $z$  is:

$$z = a+bi \dots\dots\dots(3) \text{ where } a \text{ and } b$$

are real numbers.

Imaginary valued variables are considered in [9], but in an essentially different way, and [10] and [11] concern the classical Aristotelian logic.

It is expedient the same approach to be used, and solution of equation (1) to be found by introducing an imaginary logical variable in the following way:

$$F \wedge (\frac{1}{2} + i \frac{\sqrt{3}}{2}) =$$

$$T \& TF \wedge (\frac{1}{2} + i \frac{\sqrt{3}}{2}) = TF \dots\dots (4)$$

The Imaginary Logical Variable (ILV)  $p$  may be in one of the three states

$$p \in \{ \frac{1}{2} + i \frac{\sqrt{3}}{2}, -1, \frac{1}{2} - i \frac{\sqrt{3}}{2} \} \dots\dots(5)$$

The classical logical variable

$$x \in X = \{T, TF, F\} \dots\dots\dots(6)$$

will be further referred to as Real Logical Variable (RLV).

On the base of relation (4) and by the logical operations disjunction ( $\vee$ ), conjunction ( $\wedge$ ) and negation ( $\neg$ ) the complex logical variable (CLV) will be introduced, which is of the type:

$$g_1 = x_1 \vee p_1; g_2 = x_2 \wedge p_2 = x_2 \vee p_2;$$

$$g_3 = x_3 \wedge p_3 \dots\dots\dots(7)$$

where  $x_1 \cdot x_2$  and  $x_3$  are RLV or their negations, and  $p_1, p_2$  and  $p_3$  – ILV or their negations. The set of all possible complex logical variables will be denoted by

$$G = \{ g_1; g_2; \dots g_i \dots \} \dots\dots\dots(8)$$

The following relation is true for the complex logical variables, like for the real ones:

$$g_3 = g_1 \sim g_2 = (g_1 \wedge g_2) \vee (\neg g_1 \wedge g_2) \dots\dots(9)$$

### 3. CONCLUSIONS

It is evident that a series of results may be obtained in the complex propositional logic, which have analogs in the classical propositional logic. In paper the notion ‘complex variable’ is used as an equivalence to describe elements of various algebraic structures in which propositional logic may be described.

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### REFERENCES

1. Kleene,S.C., Mathematical Logic, J.Wiley & Sons, N.Y., 1967.
2. Mendelson,E, Introduction to Mathematical Logic, D.von Nostrand Com. Inc., Princeton, 1975.
3. Kowalski,R., Logic for Problem Solving, Elsevier North Holland Inc., 1979.
4. Johnson,B., Topics in Universal Algebra, Springer-Verlag, N.Y., 1972.
5. Ionov, A.S., G.A. Petrov, Quaternion Logic as a Base of New Computational Logic,<http://zhurnal.ape.relarn.ru/articles/2007/047.pdf> (in Russian).
6. Hung T. Nguyen, Vladik Kreinovich, Valery Shekhter, On the Possibility of Using Complex Values in Fuzzy Logic For Representing Inconsistencies, <http://www.cs.utep.edu/vladik/1996/tr96-7b.pdf>

7. D.E. Tamir, A. Kandel, Axiomatic Theory of Complex Fuzzy Logic and Complex Fuzzy Classes, Int. J. of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844, Vol. VI (2011), No. 3 (September), pp. 562-576
8. Richard G. Shoup, A Complex Logic for Computation with Simple Interpretations for Physics, Interval Research Palo Alto, CA 94304, <http://www.boundarymath.org/papers/CompLogic.pdf>
9. Igor Aizenberg, Complex-Valued Neural Networks with Multi-Valued Neurons, Springer, 2011,
10. Chris Lucas, A Logic of Complex Values, Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics, 1-3 December 2001, University of New Mexico, pp. 121-138. ISBN 1-931233-55-
11. Louis H. Kauffman, Virtual Logic — The Flagg Resolution, Cybernetics & Human Knowing, Vol.6, no.1, 1999, pp. 87–96
12. An Essay on Complex Valued Propositional Logic by V.Sgurev, Print ISSN: 1312- 2622;online ISSN:2367-5357.

## BIOGRAPHY



**Dr.R.Malathi**, M.Sc., M.Phil., Ph.D in Mathematics. Published 23 papers and Presented 25 papers in both National and International Level. Editorial Board Member in IJESIRD, JECET and GJRF. Have acted as Book Editor, Associate Editor, Judge, Chairperson, Deputy Warden, Resource Person, Organizing Secretary, etc. Membership in AMTI, ISIAM and IAENG. Have published 3 books with ISBN and 5 books without ISBN in National and International level. Have received 7 awards in which 2 are International. Research Work entitled, “Design of knowledge discovery and decision support system using Jaina logic”. Guided 28+ M.Phil and M.Sc students. Participated in Various Activities actively like thesis scrutinizer, reviewer of International Conference, ICACM, Answer Script Evaluator for PG/UG, Research co-ordinator, Anti-Ragging and hospitality committee, ICC, etc.