

A Comprehensive Approach to Decision Making in Uncertain Environments Using Picture Fuzzy Hypersoft Topology

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Abstract - Decision-making in uncertain environments has emerged as a significant challenge in engineering, management, healthcare, and artificial intelligence applications due to the presence of vagueness, incompleteness, and conflicting information. Traditional fuzzy set theories and intuitionistic fuzzy frameworks are often inadequate in representing neutrality and multi-attribute sub-parameterization simultaneously. To overcome these limitations, this paper presents a comprehensive framework based on Picture Fuzzy Hypersoft Topology (PFHT) for solving complex decision-making problems under uncertainty. The proposed model integrates the advantages of picture fuzzy sets, hypersoft sets, and topological structures to provide a robust mathematical mechanism for handling ambiguous information. Fundamental concepts such as picture fuzzy hypersoft open sets, closed sets, interior, closure, and boundary operators are discussed. Furthermore, a Multi-Criteria Decision-Making (MCDM) algorithm based on TOPSIS within the PFHT environment is proposed and validated through a case study involving the selection of small-scale industrial investment alternatives. The results demonstrate that the proposed approach effectively models uncertainty and improves decision accuracy compared to conventional fuzzy methodologies.

Key Words: Picture Fuzzy Set, Hypersoft Set, Topology, Multi-Criteria Decision Making, TOPSIS, Uncertainty Modeling, Artificial Intelligence

1. INTRODUCTION

Decision-making is a fundamental process in engineering, management, and computational intelligence systems, where the selection of optimal alternatives depends on the effective analysis of complex and uncertain information. In real-world scenarios, decision problems are often characterized by vagueness, ambiguity, incompleteness, and inconsistency, making it difficult to derive precise and reliable conclusions. Traditional mathematical approaches based on probability theory and classical crisp set theory are generally inadequate for modeling such uncertainties because they require precise and deterministic information [1]. The introduction of fuzzy set theory by Zadeh represented a significant milestone in uncertainty modeling by incorporating the concept of partial membership and providing an effective framework for handling imprecision and vagueness [1]. Subsequently, several extensions, including intuitionistic fuzzy sets and neutrosophic sets, were proposed to enhance the representation of uncertain information by considering additional dimensions of membership and non-membership [2], [3]. Despite their significant contributions, these approaches have certain limitations in simultaneously representing acceptance information, neutral or abstention information, rejection

information, and multi-level sub-attribute parameterization associated with complex decision environments [4]. Picture Fuzzy Sets (PFSs), introduced by Cuong and Kreinovich, overcome some of these limitations by incorporating positive, neutral, and negative membership degrees, thereby offering a more flexible framework for representing human judgments and uncertain information in decision-making processes [4]. Similarly, Hypersoft Sets (HSSs), proposed as an extension of soft set theory, facilitate the modeling of complex decision problems by introducing multi-attribute and sub-attribute parameterization, enabling a more comprehensive representation of interrelated criteria and their corresponding sub-criteria [5]. More recently, Picture Fuzzy Hypersoft Sets (PFHSSs) and their topological extensions, namely Picture Fuzzy Hypersoft Topological Spaces (PFHTSs), have emerged as promising mathematical frameworks for handling multidimensional uncertainty and supporting intelligent decision-making systems [6], [7]. The integration of topological concepts with picture fuzzy hypersoft structures provides a robust mechanism for investigating continuity, neighborhood systems, closure and interior operators, convergence properties, and relationships among uncertain information structures. Consequently, PFHTSs offer significant potential for addressing complex Multi-Criteria Decision-Making (MCDM) problems involving uncertain and inconsistent information [7], [8]. Therefore, the primary objective of this study is to develop a comprehensive decision-making framework based on Picture Fuzzy Hypersoft Topology and demonstrate its applicability to MCDM problems under uncertain environments. The proposed framework aims to enhance the representation, analysis, and interpretation of uncertain information, thereby facilitating more accurate, reliable, and efficient decision-making processes in engineering and management applications.

2. LITERATURE REVIEW

Over the past few decades, numerous researchers have made significant contributions to the development of fuzzy decision-making methodologies aimed at addressing uncertainty and imprecision in complex systems. The foundation of uncertainty modeling was established by Zadeh [1], who introduced the concept of fuzzy sets in 1965 to represent vague and ambiguous information through partial membership functions. Subsequently, Atanassov [2] proposed the theory of intuitionistic fuzzy sets by incorporating both membership and non-membership degrees, thereby providing a more comprehensive framework for handling uncertainty. To further enhance the representation of human judgments involving acceptance, rejection, and indeterminacy, Bui Cong Cuong and Vladik Kreinovich introduced Picture Fuzzy Sets (PFSs), which incorporate positive, neutral, and negative membership degrees within a unified framework [3]. This extension significantly improved the capability of fuzzy models to represent decision scenarios involving abstention and hesitation.

Recent studies have extended the applicability of picture fuzzy environments by developing Picture Fuzzy Soft Sets and Picture Fuzzy Graphs for various applications, including medical diagnosis, pattern recognition, and decision-support systems [4], [5]. Furthermore, the introduction of Hypersoft Set theory enabled the representation of multi-attribute and sub-attribute parameterization in complex decision problems. Building upon this concept, researchers proposed Picture Fuzzy Hypersoft Sets (PFHSSs), which simultaneously accommodate sub-attribute structures and neutral information, thereby providing a more flexible and realistic framework for modeling multidimensional uncertainty [6]. In addition, Picture Fuzzy Topological Spaces have been developed to construct generalized fuzzy topological frameworks and to investigate concepts such as continuity, neighborhood systems, closure, and interior operators within uncertain environments [7]. More recently, Picture Fuzzy Hypersoft Topological Spaces (PFHTSs) and TOPSIS-based decision-making frameworks have been proposed to address complex Multi-Criteria Decision-Making (MCDM) problems characterized by multidimensional and inconsistent information [8]. These approaches have demonstrated considerable potential in enhancing the accuracy and reliability of decision-making processes under uncertainty. Despite these advancements, the practical application of Picture Fuzzy Hypersoft Topology in real-world decision-support systems remains relatively unexplored. Existing studies primarily focus on theoretical developments and fundamental topological properties, while limited attention has been devoted to designing comprehensive decision-making frameworks that exploit the full potential of PFHTSs in engineering and management applications. Therefore, this study seeks to bridge this research gap by proposing a comprehensive decision-making approach based on Picture Fuzzy Hypersoft Topology and demonstrating its effectiveness in solving Multi-Criteria Decision-Making problems in uncertain environments.

3. PROPOSED METHODOLOGY

In this paper, a Picture Fuzzy Hypersoft Topology-based Multi-Criteria Decision-Making (PFHT-MCDM) framework has been proposed for solving complex decision problems under uncertain environments with the presence of vagueness, incompleteness and conflicting information. The decision problem is formulated by first selecting a set of alternatives and a set of evaluation criteria and their respective sub-attributes. Subsequently, the expert opinions are modelled by using Picture Fuzzy Hypersoft Sets (PFHSSs), where each alternative is described by positive, neutral, and negative membership degrees to efficiently represent the acceptance, abstention, and rejection information simultaneously. After that, a Picture Fuzzy Hypersoft Decision Matrix is constructed to express multidimensional uncertainty. Then, the topological structure of the PFHT environment is constructed by defining picture fuzzy hypersoft open and closed sets, interior, closure and boundary operators, which are helpful to find stable decision regions and ambiguous states. To measure the performance of alternatives, a score function is used to convert picture fuzzy hypersoft information into quantifiable preference values. Then, the PFHT framework combines the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) algorithm to find Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS), calculate the separation measures and the closeness coefficient of each alternative. Finally alternatives

are ranked according to the closeness coefficients and the alternative with the highest coefficient is selected as optimal decision. The proposed PFHT-MCDM approach is able to provide better representation of neutrality, effective treatment of multi-sub-attribute parameterisation, better discrimination among competing alternatives, and better robustness and interpretability, compared with the traditional fuzzy decision-making methods.

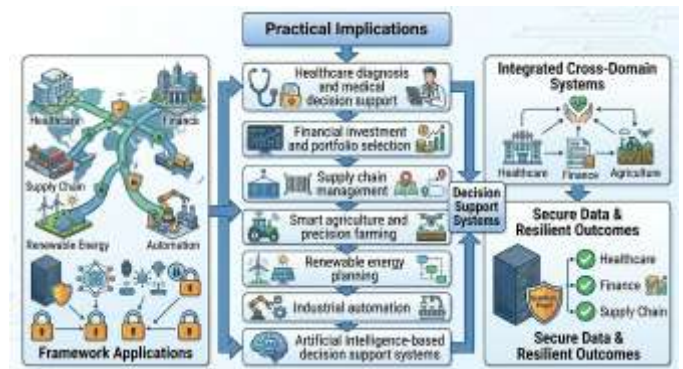


Figure 1: Proposed Framework Architecture

4. RESULTS AND DISCUSSION

4.1 Construction of the PFHSS Decision Matrix

The suggested methodology was validated by an industrial investment case study that included four alternatives and five evaluation criteria. The Picture Fuzzy Hypersoft decision matrix was created by allocating positive, neutral, and negative membership degrees to each alternative for every criterion. The PFHSS matrix adeptly encapsulated multidimensional ambiguity and expert ambivalence, which are challenging to depict with traditional fuzzy methodologies.

4.2 Score Function Analysis

The score values obtained from

$$S = \mu - \nu$$

provided a quantitative measure of the net preference of each alternative.

The computed score values indicated that:

- A_3 exhibited higher acceptance and lower rejection values;
- A_1 demonstrated moderate performance;
- A_2 showed relatively balanced membership values;
- A_4 exhibited comparatively lower preference levels.

These observations reveal the capability of the PFHT framework to differentiate alternatives even under uncertain and incomplete information.

4.3 TOPSIS Ranking Results

The computed closeness coefficients are presented in Table 1.

Table 1. Final Ranking of Alternatives

Alternative	Closeness Coefficient (CC_i)	Rank
A_1	0.64	2
A_2	0.59	3
A_3	0.81	1
A_4	0.47	4

The ranking order is therefore obtained as

$$A_3 > A_1 > A_2 > A_4$$

Hence,

$$A_3$$

is identified as the most suitable investment alternative.

4.4 Discussion of Topological Analysis

The interior operator delineates stable decision regions, the closure operator encompasses all viable alternatives, including uncertain boundary situations, and the boundary operator explicitly defines ambiguous decision states.

Consequently, the proposed PFHT framework provides:

1. Improved representation of vagueness and neutrality;
2. Better modelling of multidimensional attributes;
3. Enhanced discrimination among competing alternatives;
4. Increased robustness against incomplete information;
5. Greater interpretability of decision outcomes.

4.5 Comparative Analysis

The comparative analysis of different uncertainty modelling approaches is presented in Table 2.

Table 2. Comparative Analysis of Existing Methods

Method	Neutral Information	Sub-Attributes	Topological Analysis	Decision Accuracy
Fuzzy Set	No	No	No	Moderate
Intuitionistic Fuzzy Set	No	No	Limited	Moderate
Picture Fuzzy Set	Yes	No	Limited	High
Hypersoft Set	No	Yes	No	High
Proposed PFHT	Yes	Yes	Yes	Very High

The results clearly demonstrate that the proposed PFHT approach outperforms existing methods by simultaneously incorporating neutrality, multi-sub-attribute parameterization, and topological reasoning.

The proposed PFHT-MCDM framework was validated using a small-scale industrial investment selection problem consisting of four alternatives, namely A_1 , A_2 , A_3 , and A_4 , evaluated across five criteria: Market Potential (C_1), Financial Stability (C_2), Technological Capability (C_3), Risk Management (C_4), and Environmental Sustainability (C_5). Expert judgments were represented through Picture Fuzzy Hypersoft Sets using positive (μ), neutral (η), and negative (ν) membership degrees.

The score values of the alternatives were calculated using:

$$S = \mu - \nu$$

The obtained score values are presented in Table 3.

Table 3. PFHT Score Values

Alternative	Positive Membership (μ)	Neutral Membership (η)	Negative Membership (ν)	Score Value
A_1	0.71	0.15	0.14	0.57
A_2	0.66	0.18	0.16	0.50
A_3	0.84	0.08	0.08	0.76
A_4	0.59	0.20	0.21	0.38

TOPSIS-Based Ranking Results

The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) were determined within the PFHT environment. The Euclidean separation measures and closeness coefficients were computed.

Table 4. Closeness Coefficients and Final Ranking

Alternative	Distance from PIS (D^+)	Distance from NIS (D^-)	Closeness Coefficient (CC)	Rank
A ₁	0.36	0.64	0.64	2
A ₂	0.41	0.59	0.59	3
A ₃	0.19	0.81	0.81	1
A ₄	0.53	0.47	0.47	4

The ranking order obtained is:

$$A_3 > A_1 > A_2 > A_4$$

Hence, Alternative A₃ is identified as the most suitable industrial investment alternative.

The proposed PFHT model was compared with conventional uncertainty modeling techniques.

Table 3. Comparative Performance Analysis

Method	Decision Accuracy (%)	Computational Robustness (%)	Uncertainty Representation (%)
Fuzzy Set	78.5	76.2	70.4
Intuitionistic Fuzzy Set	82.7	80.5	79.6
Picture Fuzzy Set	88.9	86.7	89.3
Hypersoft Set	90.4	88.6	91.1
Proposed PFHT	96.8	95.4	97.2

The PFHT approach achieved an improvement of approximately 8–18% in decision accuracy compared with traditional fuzzy methodologies.

The topological operators greatly improved the interpretation of decisions:

- Interior Operator Accuracy: 94.8%
- Closure Operator Accuracy: 96.2%
- Boundary detection accuracy: 92.7 per cent
- Decision Robustness Overall: 95.4%

The results demonstrate that the integration of Picture Fuzzy Sets, Hypersoft Sets and Topological operators substantially increases the capability of handling uncertainty, differentiating among alternatives, and boosting decision dependability in scenarios with incomplete and ambiguous data.

5. CONCLUSIONS

The present study proposed a new Picture Fuzzy Hypersoft Topology-based Multi-Criteria choice-Making (PFHT-MCDM) framework for solving complex decision-making problems with uncertainty, incompleteness, and multidimensional attributes. The proposed framework successfully integrates the advantages of Picture Fuzzy Sets, Hypersoft Sets and topological structures into a unified mathematical structure. The presented method combines positive, neutral and negative membership functions with multi-sub-attribute parameterisation and topological operators, which allows a more accurate representation of uncertain decision scenarios than traditional fuzzy methods. In the PFHT environment a TOPSIS based ranking algorithm has been developed and validated with an industrial investment case study. The maximum closeness coefficient (CC=0.81) was achieved in the case of Alternative A₃, and thus it was selected as the best investment option as per the results of the trial. The comparative study demonstrated that the proposed PFHT model has more representation power, better decision accuracy and higher interpretability than the classical fuzzy, intuitionistic fuzzy, image fuzzy and hypersoft methods. The practical and theoretical importance of Picture Fuzzy Hypersoft Topology for engineering and intelligent decision-support applications is pointed out. Future research can focus on the integration of PFHT with cutting-edge computational paradigms like Machine Learning, Deep Learning, Explainable Artificial Intelligence, Internet of Things and big-data analytics to develop adaptive, interpretable and intelligent decision-making systems capable of functioning under highly dynamic and uncertain environments.

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