

# A Comprehensive Review on Fractional Order Control

Manish Kumar<sup>1</sup>, Devender Sharma<sup>2</sup>, Saurabh Gupta<sup>3</sup>, Anula Khare<sup>4</sup>

<sup>1</sup>Electrical and Electronics engineering & Technocrats Institution Technology & Science, Bhopal <sup>2</sup>Electrical and Electronics engineering & Technocrats Institution Technology & Science, Bhopal <sup>3</sup>Electrical and Electronics engineering & Technocrats Institution Technology & Science, Bhopal

<sup>4</sup>Electrical and Electronics engineering & Technocrats Institution Technology & Science, Bhopal

**Abstract-** Fractional order control is one of the most important enablers of advanced closed loop industrial automation systems. This paper presents a detailed bibliometric study of fractional order control related research published during the past two decades (2000–2022) in the Web of Science (WoS) and Scopus indexed sources. The WoS analysis included yearly publications, publication types and top contributing publishing sources, organizations, researchers, countries and research areas. VoS viewer based full counting methodology was used to analyze citation, co-citation and co-authorship based collaborations among authors, organizations, countries, sources and documents. Co-occurrence, timeline and burst detection analyses of keywords and published articles were also carried out to unravel significant fractional control research trends in the last two decades. The Scopus analysis firstly included annual publication counts, subject areas, document types and top contributing authors, sources, nations, institutions and funding agencies. Secondly, dedicated Scopus searches were carried out for hardware implementation, non integer control and patents. Results indicate that the fractional order controller research has been primarily focused in engineering domain, followed by mathematics, physics, automation control systems and computer science. Materials, mechanics and telecommunications are some of the lesser explored areas. There is a lot of scope of fractional order control research related to patents, hardware implementation, non integer control, anomalous diffusion, chaos and task analysis. Future investigations will also include neural networks based machine learning in fractional order control solutions.

**Key Words:** Fractional order controller, FOPID, Fractional calculus

## 1. INTRODUCTION

Control systems form the basis of all automation. Controllers are the ‘brains’

behind the smooth functioning of all control systems. Therefore, optimum design of controller architectures is of paramount importance while developing any control system. Researchers have explored diverse controller architectures to cater to varied control system applications such as robots, composite material processing, general machining, bolted joints, microgrids and many more [1–4]. With the advent of industry 4.0, more and more system data is becoming available due to a large number and variety of sensors being deployed to capture different plant dynamics. This system data can be used to generate a parametric model of the plant through a process known as system identification. Many researchers have followed this methodology to obtain accurate models of varied physical systems such as machining centers, DC motors and more [5]. Controller architectures can then be designed on top of such system identified models to control the entire plant or an individual process as per user specifications. Different processes require different controller architectures as well as their specific tuning of parameters. Hence, controller selection becomes an important part of the overall control system design. Conventionally, many industries have been employing the proportional, integral and derivative (PID) controllers in almost all kinds of control systems for process automation. The wide scale

prevalence of PID controllers in industries can be ascribed to their inherent design simplicity and ease of installation, tuning, maintenance and low costs [6-10]. However, with the increasing complexity of modern operations which have to face numerous system variations due to both internal and external factors, the relatively simple PID architecture falls short of providing robust and reliable control performance. It is in this scenario that the fractional calculus based controller designs become relevant. Fractional order PID (FOPID) controllers have extra parameters available for tuning, which enable them to be more robust against system disturbances and variations [11-12]. Fractional order controller designs are based on the fractional calculus, which makes it possible for the controller parameters to be tuned in terms of fractional orders [13]. Many researchers have explored a number of fractional order control architectures for varied applications and compared their performance against those of their conventional auto-tuned PID counterparts [14]. In fact, researchers have taken fractional order control to its next stage of evolution in the form of the complex order PID control (COPID), which includes even more parameters for control tuning [15]. These advanced controller configurations have been tried out on numerous systems such as HIV infection resistance [16], robotics [17], solutions of differential equations, control tuning of resonant systems, artificial intelligence algorithms in advanced control of nonlinear systems, DC motor tuning and many more. Others [18] reviewed various aspects of complex and

fractional order modeling and control in different systems. Researchers also compared the stability and time/frequency domain performance aspects of complex, fractional and integral order controllers [19]. The following subsection briefly introduces fractional calculus, its most popular definitions as well as general architectures of PID as well as fractional and complex controllers[20].

## **2. BIBLIOMETRIC ANALYSIS AND RELATED STUDIES**

Bibliometric analysis is used to examine the research outputs, trends and developments over a defined period of time in a certain field/area [12-13]. It provides quantitative insights into the various aspects of research carried out in a field, and thus acts as a useful supplement to the conventional review efforts, which include the relational and evaluative methodologies [13-16]. The relational reviews focus on establishing inter-relationships among the various aspects of research investigations, such as the associated authors, their affiliations (organizations and countries) as well as their research outputs [17]. These reviews involve indices such as co-authorship, co-citations, co-occurrences and more. These indices are useful to reveal the various inter-linkages between authors, publications as well as their publication sources across institutions, countries and timescales. The evaluative reviews on the other hand focus on bringing out the absolute impacts of the above mentioned research output metrics/verticals [17]. These reviews involve absolute indices such as total publication outputs across

different verticals and time scales. Past, current and future trending research areas, keywords and topic clusters are also revealed through these reviews. The present study includes both of these approaches to ensure a balanced representation of research conducted in fractional order control over the past two decades. This study also used data visualization tools such as the VoSviewer for analyzing and presenting research collaboration networks and in various aspects mentioned above. CiteSpace was used for citation/reference bursts and time series analyses. A thorough search of literature related to bibliometric studies on fractional order control yielded minimal results. Recently, Abdelfettah et al. conducted a brief bibliometric study of fractional order control research in industry 4.0 domain during 2019–2021. Yang et al. conducted a bibliometric analysis of fractional order control research in China during the last two decades. Shah et al. [18] analyzed research trends in fractional order controller research during 2014–2019 using latent dirichlet allocation based topic modeling approach. The above detailed literature survey shows that there is a wide scope for bibliometric analytical studies in the field of fractional order controllers. The present work aims to fill this gap by focusing on recent research outputs in this field, specifically during 2000 to 2022. The following section gives details of the data collection methodology followed in the present study.

### 3. DATA COLLECTION

In the present work, the Web of Science (WoS) and Scopus repositories repository

were used as the sources of fractional order control related research data. The WoS and Scopus databases are world renowned collections of high quality research publishing sources. The WoS database is composed of the several indices such as the BCI-SSH (Book Citation Index—Social Science and Humanities), BCI-S (Book Citation Index—Science), CPCI-S (Conference Proceedings Citation Index—Science), Conference Proceedings Citation Index—Social Sciences and Humanities (CPCI-SSH), Arts and Humanities Citation Index (AHCI), Social Sciences Citation Index (SSCI), Emerging Science Citation Index (ESCI) and the Science Citation Index Expanded (SCIE). The following fractional order control related search string was used to collect relevant documents from the WoS and Scopus databases: (“fractional order controller”) OR (“fractional calculus” AND (“modeling” OR “model” OR “machine learning” OR “optimization”)) OR (“fractional sliding control system”) OR (“stability of fractional order system”) OR (“CRONE Controller”) OR (“fractional PID controller”) OR (“fractional order compensator”) OR (“optimization of fractional order systems”) OR (“complex order controller”) OR (“fractional robust controller”) OR (“fractional controller”)) from 2000 to 2022. This search yielded a total of 4249 and 5726 research papers from WoS and Scopus respectively, that were analyzed in various ways. Subsequently, two other search strings focusing on ‘hardware implementation’ and ‘non integer control’ were applied to the Scopus database during

the same duration. These search strings were composed as follows: (“fractional order controller”) AND (“hardware implementation”); and (“fractional order controller”) AND (“non integer control”) OR (“non integer order control”). This paper is organized as follows: Sections 2–7 present the WoS based analysis of publication structures, co-authorships, keyword co-occurrences, citations, co-citations and timeline/bursts respectively. Section 8 presents Scopus based results of fractional order control, hardware implementation, non integer control and patents in 8.1, 8.2, 8.3 and 8.4 respectively.

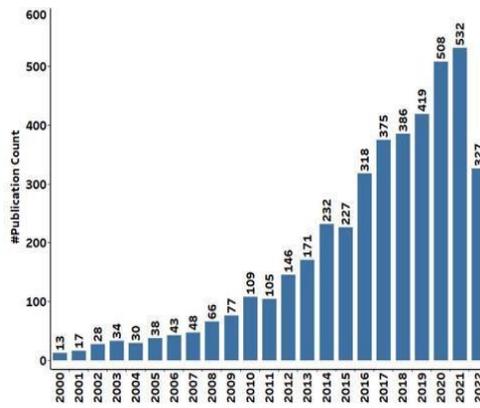


Fig. 1. Yearly fractional order control publications 2000 to 2022.

#### 4. FRACTIONAL ORDER CONTROL

This section discusses the Scopus results of the overall topic of fractional order control, using the same search criteria that were applied for the Web of Science database. This search yielded a total of 5726 publications as opposed to 4249 documents obtained from a similar search in the Web of Science database. Fig. 24 shows the annual

fractional order publications in Scopus. This figure clearly shows a steeply rising research interest in this field. The annual publications in Scopus are generally higher than those in WoS for the same year (Fig. 1). Fig. 25 shows the percentage of documents published under various subject domains. The majority of fractional order control publications (29.2%) in Scopus focused on general engineering applications, followed closely by mathematical modeling and control structure investigations (22.8%). Computer science emerged as the subject area with the third highest percentage of Scopus publications (17.4%), indicating a moderate proportion of fractional order modeling and control research applications dedicated to the computer science and allied applications. The top two subject domains under Web of Science database were the same as that for Scopus: engineering and mathematics. However, the third highest subject domain as per fractional publications in WoS was 'automation and control', followed by 'computer science' at the fourth position. Physics and astronomical applications attracted 10.4% of Scopus indexed fractional order publications, whereas, materials science and energy received 4.2% and 4.0% documents respectively. Decision sciences, chemical engineering, earth planetary sciences and chemistry obtained 1.7%, 1.4%, 1.2% and 1.3% fractional order publications respectively. These subject areas are likely to witness further growth in research investigations involving fractional order modeling and controllers. Fig. 1 depicts the annual publication trends of the top ten

sources contributing to the field of fractional order control. This figure clearly shows less than 20 articles were published per journal annually till 2019. The sources CHAOS SOLITONS AND FRACTALS, and FRACTALS AND FRACTIONAL started publishing more than 20 annual publications in the last 3 years. The CHAOS SOLITONS AND FRACTALS is the top contributing source in the WoS database as well. Fig. 2 shows the top contributing authors to fractional order control publications in Scopus. This list is topped by Baleanu, D. with 104 Scopus papers. This author also had the most WoS papers (88) in this field. The author with the second highest Scopus publications, Chen Y.Q. was placed third in the top ten WoS authors. On the contrary, the second highest WoS publishing author Tenreiro Machado, J.A. was arrived ninth highest among Scopus contributors. Gomez-Aguilar, J.F. and Muresan, C.I. arrived fourth and fifth respectively in both Scopus and WoS lists. This comparative analysis shows that while some authors might prefer publishing more in sources indexed only in WoS, most of the top researchers publish in sources indexed in both databases. Fig. depicts the top contributing nations to fractional order control studies published in Scopus. This list is mostly similar to that of the top ten nations publishing in WoS, except for Pakistan and Poland, which do not appear in the top Scopus list, wherein Portugal and Italy replace these countries (respectively).

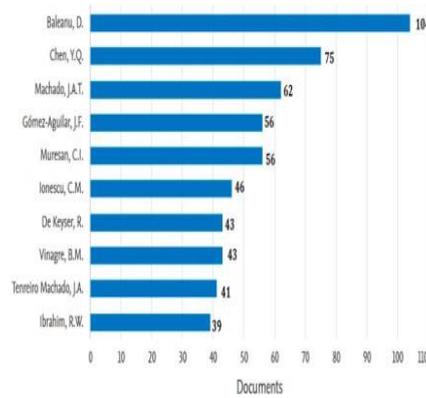


Fig. 2. Scopus documents by top contributing authors.

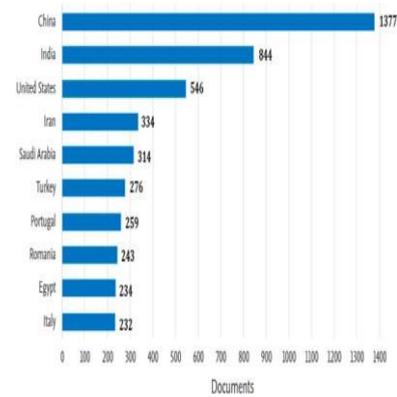


Fig. 3. Scopus documents by top contributing nations.

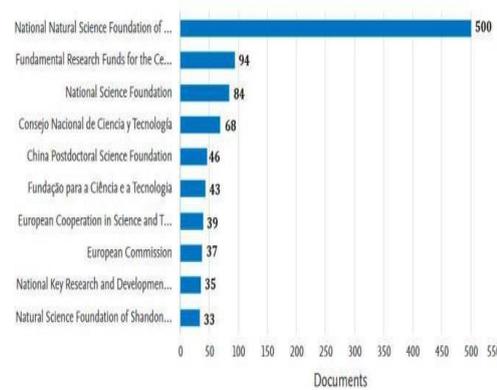


Fig. 4. Scopus documents by top contributing funding agencies.

## 5. HARDWARE IMPLEMENTATION

This section presents an analysis of Scopus database results obtained by employing the following search string: (“fractional order

controller”) AND (“hardware implementation”) over the period of past two decades. This search resulted in a corpus of just 68 documents in total. Fig. 32 shows the subject area wise distribution of these 68 documents, depicting a majority of them related to engineering in general (38.9%), computer science (23.8%) and mathematics domains (14.3%). Fig. 3 shows that the annual publication trends of the top ten sources contributing to the sub topic of hardware implementation of fractional order control. This figure clearly shows that even the top contributing journals have been able to publish maximum one paper annually during the last decade! There were no such publications reported in the previous decade! In fact, no Scopus indexed journal has published more than two papers related to this particular sub topic during the entire surveyed period. Fig. 4 depicts the top ten researchers who have tried to promote research in this critical sub domain. Even the top contributing authors do not have more than 5 papers in total, demonstrating the rarity of fractional order hardware implementation research. Fig. 35 shows the top ten nations publishing Scopus documents related to this specific sub topic. The United States, Turkey, Portugal, Romania and Italy do not feature in this list, despite being the top contributors to the overall field of fractional order control (Fig. 28). Hence, the fractional order control research groups of these countries need to focus more on investigations involving hardware implementations. In general, authors from all nations need to promote research in this niche sub domain. In this

regard, the National Natural Science Foundation of China once again tops the list of top funding agencies (Fig. 36), resulting in a total of 7 publications in Scopus indexed sources over the past two decades. Fig. 5 shows the top ten institutions investigating and publishing fractional order control research with hardware implementations. Fig. 4 shows the distribution of the 68 documents’ corpus among articles (58.8%), conference papers (27.9%), conference reviews (7.4%), book chapters (2.9%), books (1.5%) and reviews (1.5%).

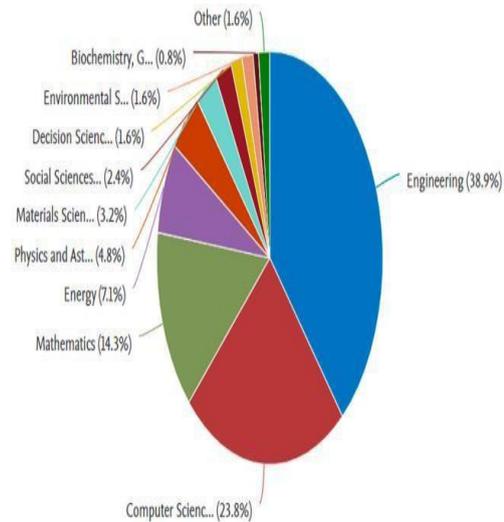


Fig. 5. Hardware implementation: Scopus document distribution as per subject areas.

## CONCLUSION

This paper firstly presented a comprehensive bibliometric study of fractional order controller research published in Web of Science over the past two decades. This study covered a detailed analyses of top contributing authors, organizations, nations, sources as well as annual publications, publication distributions, topical areas, Web of Science indices and subject areas. Full

counting method was adopted for analyzing linkages among nations, institutions, authors, keywords and documents with regards to citations, co-citations and co-authorship. Lastly, citation burst and timeline analyses were performed on top published documents, keywords and topical areas within the field of fractional order control. Secondly, a similar search was carried out on the Scopus database for the same period, to yield the top contributing authors, nations, organizations, sources and funding agencies. Annual publication counts, types of documents published and important subject areas in fractional order control publications were also analyzed. Thirdly, focused searches were carried out on the Scopus database for hardware implementation, non integer control and patent documents published in the field of fractional order control. Following are the main findings of Web of Science based bibliometric analyses:

1. Annually 500 fractional order control related articles are being published in both WoS and Scopus
2. Research papers form the majority of these publications (76% in WoS and 65% in Scopus)
3. The Chaos Solitons Fractals published the most fractional order control articles (indexed in both WoS and Scopus)
4. Among institutions, the Egyptian Knowledge Bank and the Instituto Superior de Engenharia do Porto published maximum articles (182) in WoS (182) and Scopus (146) respectively.

5. Baleanu, Dumitru (Turkey) published maximum articles in WoS (88) and Scopus (104) as well.

6. Since 2010, Chinese researchers published the most articles annually in WoS. China also produced the maximum Scopus publications (1377) in this field in the past two decades.

7. Engineering is most preferred research domain having the most fractional order control publications in WoS (1688) and Scopus (1671). 8. The Science Citation Index Expanded includes the most WoS fractional order control publications (3039)

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