International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 07 Issue: 06 | June - 2023 SJIF Rating: 8.176 ISSN: 2582-3930

A New Address on Classical Average Assignment of Various Graphs Obtained From Cycle

A. Rajesh Kannan
Department of Mathematics
Mepco Schlenk Engineering College, Sivakasi-626 005, Tamil Nadu, India
Email: rajmaths@gmail.com

Abstract

A function f is called a classical average assignment of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \to \{1,2,3,\cdots,q+1\}$ is injective and the induced function f^* defined by the flooring function of the average of root square, harmonic, geometric and arithmetic means of the vertex labels of the end vertices of each edge, is bijective. A graph that admits a classical average assignment is called an classical average assignment graph. Here, we have discussed the classical average assignment of a cycle, the graph obtained from a path by replacing any of its edges by a cycle, the triangular snake, the alternate triangular snake, the quadrilateral snake, the alternate quadrilateral snake, the tadpoles, the graph and the triangular ladder graph.

Key words: Labeling, cycle graph, classical average assignment, classical average assignment graph, identifying an edge.

1. Introduction

Graph labeling techniques provide economists with a powerful tool to model and analyze complex economic systems, uncovering insights into network structures, dynamics, and interactions that influence economic outcomes. It can be used to model and analyze supply chain networks. By assigning labels to nodes and edges representing suppliers, manufacturers, distributors, and retailers, researchers can analyze the flow of goods, track inventory levels, and identify bottlenecks or inefficiencies in the supply chain. Labeling can also represent attributes such as production capacity, lead times, or quality levels, enabling optimization and decision-making in supply chain management. Also it can be used to analyze international trade networks. By assigning labels to nodes representing countries or regions and labels to edges representing trade flows or trade relationships, researchers can analyze patterns of trade, identify key trading partners, and assess the impact of trade policies or trade agreements. Labeling can also incorporate attributes such as trade volumes, product categories, or trade balances, enabling the analysis of specialization, competitiveness, and the formation of trade clusters or hubs [14-15]. In this paper, when we refer to a graph, we mean a simple, undirected, finite graph. We use the notations and language from [1-6].

We refer to [7] for a comprehensive overview of graph labeling. A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices, respectively. Then the Cartesian product $G_1 \times G_2$ has $p_1 p_2$ vertices with vertex set $\{(u,v): u \in G_1, v \in G_2\}$ and any two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. A triangular snake T_n is obtained from a path by replacing every edge C_3. An Alternate Triangular Snake $A(T_n)$ is obtained from a path by identifying every alternate edge C_3. A Quadrilateral Snake triangular snake Q_n is obtained from a path by identifying every edge of a path by C_4. An Alternate Quadrilateral $A(Q_n)$ is obtained from a path by identifying every alternate edge by C_4. The graph Tadpoles is obtained by identifying a vertex of the cycle C_n to an end vertex of the path P_k. $C_n \circ K_1$ is the graph obtained from C_n by attaching a new pendant vertex to each vertices. Let u_i, v_i, $1 \le i \le n$ be the vertices on the path of length $n-1$ in the ladder $P_2 \times P_n$. the triangular $TL_n, n \ge 2$ is a graph obtained by completing the ladder $P_2 \times P_n$ by adding the edges $u_i v_{i+1}$ for $1 \le i \le n-1$.

The concept of mean labeling was created by Somosundaram and Ponraj [8]. Durai Baskar and Arockiaraj established the F-harmonic mean graph [9]. The F-root square mean graphs were first described by Arockiaraj et al. [10]. Durai Baskar and Arockiaraj [11] defined the F-harmonic mean graphs. Additionally, Muhiuddin et al. [12–13] examined additional notions of classical mean labeling. We investigate a traditional mean labeling of cycle-related graphs-based graphs. A function f is called a classical average assignment of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \to \{1,2,3,\cdots,q+1\}$ is injective and the induced function f^* defined by the flooring function of the average of root square, harmonic, geometric and arithmetic means of the vertex labels of the end vertices of each edges, is bijective. A graph that admits a classical average assignment is called a classical average assignment graph. According to our criteria, Figure 1 assigns the graph a classical average of the graph $K_4 - e$.

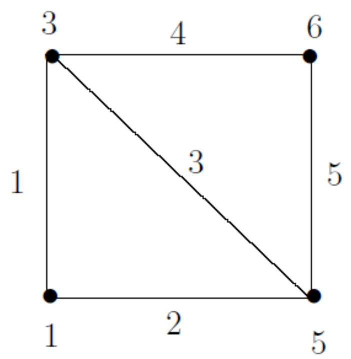

Figure 1. A classical average assignment of $K_4 - e$.

Here, we have discussed the Classical average assignment of a cycle, the graph obtained from a path by replacing any of its edges by a cycle, the triangular snake, the alternate triangular snake, the quadrilateral snake, the alternate quadrilateral snake, the tadpoles, the graph and the triangular ladder graph.

2 Main Results

Theorem 2.1. Every cycle C_n is a classical average assignment graph, for $n \geq 3$.

Proof. Let $u_1, u_2, u_3, \cdots, u_n$ be the vertices of the cycle C_n.

Define $f : V(G) \to \{1,2,3,\cdots,q+1\}$ as follows

$$f(u_i) = \begin{cases} 2i-1, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ n+1, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \\ n-1, & i = \left\lfloor \frac{n}{2} \right\rfloor + 2 \text{ and } n \text{ is odd} \\ n-2, & i = \left\lfloor \frac{n}{2} \right\rfloor + 2 \text{ and } n \text{ is even} \\ 2n-2i+2, & \left\lfloor \frac{n}{2} \right\rfloor + 3 \leq i \leq n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i u_{i+1}) = \begin{cases} 2i, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ n, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ n-1, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 2n-2i+2, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n-1 \text{ and} \end{cases}$$

$f^*(u_n u_1) = 1.$

Hence f is a classical average assignment of the cycle C_n, for $n \geq 3$.

Thus every cycle C_n is a classical average assignment graph, for $n \geq 3$. □

Theorem 2.2. Let G be a graph obtained from a path by identifying any of its edges by an edge of a cycle. Then G is a classical average assignment graph.

Proof. Let $v_1, v_2, v_3, \cdots, v_p$ be the vertices of a path on p vertices. Let m be the number of cycles are placed in path in order to get G and the edge of the j^{th} cycle be identified with the edge (v_{i_j}, v_{i_j+1}) of the path having the length n_j.

For $1 \le j \le m$, the vertices of the j^{th} cycle be $v_{i_j,l}, 1 \le l \le n_j$ where $v_{i_j,1} = v_{i_j}$ and $v_{i_j,n_j} = v_{i_j+1}$.

Define $f : V(G) \to \left\{1,2,3,\cdots,\sum_{j=1}^{m} n_j + p - m\right\}$ as follows.

$$f(v_k) = k, \quad for \ 1 \le k \le i_1,$$

$$f\left(v_{i_j}\right) = i_j + \sum_{k=1}^{j-1} (n_k - 2) + j - 1, \quad for \ 1 \le j \le m,$$

$$f\left(v_{i_j+1}\right) = f\left(v_{i_j}\right) + n_j, \quad for \ 1 \le j \le m,$$

$$f\left(v_{i_j+k}\right) = f\left(v_{i_j+1}\right) + k - 1, \quad for \ 1 \le j \le m-1, \ 2 \le k \le i_{j+1},$$

$$f\left(v_{i_m+k}\right) = f\left(v_{i_m}\right) + k - 1, \quad for \ 2 \le k \le p - i_m \ and$$

for $1 \le j \le m$,

$$f\left(v_{i_j,l}\right) = \begin{cases} f\left(v_{i_j}\right) + l - 1, & 2 \le l \le \left\lfloor \sqrt{\dfrac{f\left(v_{i_j}\right)^2 + f\left(v_{i_j+1}\right)^2}{2}} \right\rfloor - f\left(v_{i_j}\right) + 1 \\ f\left(v_{i_j}\right) + l, & \left\lfloor \sqrt{\dfrac{f\left(v_{i_j}\right)^2 + f\left(v_{i_j+1}\right)^2}{2}} \right\rfloor - f\left(v_{i_j}\right) \le l \le n_j - 1. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(v_k v_{k+1}) = k, \quad for \ 1 \le k \le i_1 - 1,$$

$$f^*\left(v_{i_j+k}\, v_{i_j+k+1}\right) = f\left(v_{i_j+k}\right), \quad for \ 1 \le k \le i_{j+1} - i_j - 1 \ and \ 1 \le j \le m-1,$$

$$f^*\left(v_{i_m+k}\, v_{i_m+k+1}\right) = f\left(v_{i_m+k}\right), \quad for \ 1 \le k \le p - i_m - 1,$$

for $1 \le j \le m$,

$$f^*\left(v_{i_j,l}\, v_{i_j,l+1}\right) = \begin{cases} f\left(v_{i_j,l}\right), & 1 \le l \le n_j - 1 \ and \ l \ne \left\lfloor \sqrt{\dfrac{f\left(v_{i_j}\right)^2 + f\left(v_{i_j+1}\right)^2}{2}} \right\rfloor - f\left(v_{i_j}\right) + 1 \\ f\left(v_{i_j,l}\right) + 1, & l = \left\lfloor \sqrt{\dfrac{f\left(v_{i_j}\right)^2 + f\left(v_{i_j+1}\right)^2}{2}} \right\rfloor - f\left(v_{i_j}\right) + 1 \ and \end{cases}$$

$$f^*\left(v_{i_j}\, v_{i_j+1}\right) = \left\lfloor \sqrt{\frac{f\left(v_{i_j}\right)^2 + f\left(v_{i_j+1}\right)^2}{2}} \right\rfloor, \quad for \ 1 \le j \le m.$$

Hence f is a classical average assignment of the graph.
Thus, if G be a graph obtained from a path by identifying any of its edges by an edge of a cycle, then G is a classical average assignment graph.

Corollary 2.3. The triangular snake graph is a classical average assignment graph.

Corollary 2.4. The alternate triangular snake graph is a classical average assignment graph.

Corollary 2.5. The quadrilateral snake graph is a classical average assignment graph.

Corollary 2.6. The alternate quadrilateral snake graph is a classical average assignment graph.

Corollary 2.7. The tadpoles graph is a classical average assignment graph, for $n \geq 3$.

Theorem 2.8. The graph $C_n \circ K_1$ is a classical average assignment graph for $n \geq 4$.
Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the cycle C_n and let u_i be the pendant vertices attached at each v_i for $1 \leq i \leq n$.
Define $f : V(C_n \circ K_1) \to \{1, 2, 3, \cdots, q+1\}$ as follows.

$$f(v_i) = \begin{cases} 4i-2, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ 4i-1, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ 4i-3, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 4n+4-4i, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n-1 \end{cases}$$

$f(v_n) = 3,$

$$f(u_i) = \begin{cases} 4i-3, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ 4i-3, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ 4i-5, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 4n+3-4i, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n-1 \text{ and} \end{cases}$$

$f(u_n) = 4.$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(v_iv_{i+1}) = \begin{cases} 4i, & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1 \\ 4i, & i = \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n \text{ is odd} \\ 4i-1, & i = \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n \text{ is even} \\ 4i-3, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ 4i-6, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 4n+2-4i, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \le i \le n-1 \end{cases}$$

$$f^*(u_iv_i) = \begin{cases} 4i-3, & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ 4i-2, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ 4i-4, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 4n+3-4i, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \le i \le n. \end{cases}$$

Hence f is a classical average assignment of the corona graph $C_n \circ K_1$.
Thus the $C_n \circ K_1$ is a classical average assignment graph, for $n \ge 4$. □

Theorem 2.9. The graph TL_n is a classical average assignment graph, $n \ge 2$.
Proof. Let $\{u_1, u_2, u_3, \cdots, u_n, v_1, v_2, v_3, \cdots, v_n\}$ be the vertex set of TL_n and let $\{u_iu_{i+1}v_iv_{i+1}u_iv_{i+1}; 1 \le i \le n-1\} \cup \{u_iv_i; 1 \le i \le n\}$ be the edge set of TL_n.
Then TL_n have $2n$ vertices and $4n-3$ edges.
Define $f: V(\mathrm{TL}_n) \to \{1, 2, 3, \cdots, 4n-2\}$ as follows.

$f(v_1) = 1,$
$f(v_i) = 4i-4, \quad for \; 2 \le i \le n \; and$
$f(u_i) = 4i-2, \quad for \; 1 \le i \le n.$

Then the induced edge labeling f^* is obtained as follows.

$f^*(u_i v_{i+1}) = 4i,\ for\ 1 \le i \le n-1,$

$f^*(u_i v_i) = 4i-3,\ for\ 1 \le i \le n,$

$f^*(u_i v_{i+1}) = 4i-1,\ for\ 1 \le i \le n$

$and\ f^*(v_i v_{i+1}) = 4i-2,\ for\ 1 \le i \le n-1.$

Hence f is a classical average assignment of the triangular ladder graph TL_n.

Thus the triangular ladder graph TL_n is a classical average assignment graph, for $n \ge 2$.

□

3. Conclusion

The classical average assignment of graph created by replacing any of a path's edges with a cycle, the triangular snake, the alternate triangular snake, the quadrilateral snake, the alternate quadrilateral snake, the tadpoles, the graph, and the triangular snake have all been addressed. Analyzing the classical meanness of various graphs would be quite fascinating. Future study should look into investigating classical mean labeling of other classes of graphs.

References

[1] F. Harary, Graph Theory, Narosa Publishing House Reading, New Delhi, India, 1988.

[2] N. L. Biggs, E. K. Lloyd, and R. J. Wilson, Graph Theory, pp. 1736–1936, Clarendon Press, Oxford, UK, 1976.

[3] O. Ore and American Mathematical Society, Theory of Graphs, Providence: American Mathematical Society, Providence, Rhode Island, 1962.

[4] J. Gross and J. Yellen, Graph Theory and its Applications, CRC Press, London, UK, 1999.

[5] G. Chartrand, L. Lesniak, and P. Zhang, Graphs and Digraphs, CRC Press, Boca Raton, FL, USA, 6th ed., 2016.

[6] S. G. Penrice, "Some new graph labeling problems: a preliminary report," DIMACS Technical Report 95-29, S. U. N. Y. College at Cortland, New York, NY, USA, 1995.

[7] J. A. Gallian, "A dynamic survey of graph labeling," Electronic Journal of Combinatorics, vol. 17, 2019.

[8] S. Somosundaram and R. Ponraj, "Mean labeling of some graphs," National Academy Science Letters, vol. 26, pp. 210–213, 2003.

[9] A. Durai Baskar and S. Arockiaraj, "F-harmonic mean graphs," International Journal of Mathematical Archive, vol. 5, no. 8, pp. 92–108, 2014.

[10] S. Arockiaraj, A. Durai Baskar, and A. Rajesh Kannan, "F-root square mean labeling of graphs obtained from paths," International Journal of Mathematical Combinatorics, vol. 2, pp. 92–104, 2017.

[11] A. Durai Baskar and S. Arockiaraj, "F-geometric mean labeling of graphs obtained by duplicating any edge of some graphs," SUT Journal of Mathematics, vol. 53, no. 2, pp. 107–125, 2017.

[12] G. Muhiuddin, A. M. Alanazi, A. R. Kannan, and V. Govindan, “Preservation of the classical meanness property of some graphs based on line graph operation,” Journal of Mathematics, vol. 2021, Article ID 4068265, 10 pages, 2021.

[13] A. M. Alanazi, G. Muhiuddin, A. R. Kannan, and V. Govindan, “New perspectives on classical meanness of some ladder graphs,” Journal of Mathematics, vol. 2021,
Article ID 9926350, 14 pages, 2021.

[14] A.Barrat, M.Barthelemy, R.Pastor-Satorras and A. Vespignani “The architecture of complex weighted networks” Proceedings of the National Academy of Sciences, vol. 101(11), 3747-3752.

[15] G.Caldarelli, Scale-Free Networks: Complex Webs in Nature and Technology, Oxford University Press (2007).