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A Novel Approach for Evaluating the Percentage Shift Between Kinetic Energy and Momentum

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Abstract – This study is built upon the connection between kinetic energy and momentum, aiming to present a novel and more straightforward method to simplify and expedite the calculation of percentage changes in kinetic energy when momentum is known, and vice versa. Grasping these two essential physics concepts is vital for numerous engineering and scientific applications. The new approach introduced in this paper offers a more accessible and efficient way for students and professionals to understand these concepts quickly. By streamlining the calculation process, this paper seeks to enhance learners' and practitioners' comprehension of the relationship between kinetic energy and momentum. This study delves deeper into the intricate relationship between kinetic energy and momentum, two fundamental concepts in physics that are crucial for understanding the behavior of moving objects. By developing a novel and simplified method for calculating percentage changes in kinetic energy based on known momentum values, and vice versa, the research aims to bridge the gap between theoretical understanding and practical application. This innovative approach not only streamlines the calculation process but also provides a more intuitive grasp of how these two quantities are interconnected, potentially revolutionizing the way these concepts are taught and applied in various scientific and engineering fields.

The implications of this study extend beyond academic circles, offering practical benefits for professionals in diverse industries such as aerospace, automotive engineering, and sports science. By providing a more accessible and efficient method for understanding and calculating the relationship between kinetic energy and momentum, the research has the potential to accelerate problem-solving processes, improve design optimization, and enhance overall efficiency in related fields. Furthermore, this simplified approach may serve as a valuable tool for educators, enabling them to convey these complex physics concepts more effectively to students, thereby fostering a deeper understanding and potentially inspiring future innovations in physics and engineering.

Keyword: Kinetic Energy, Momentum, Percentage change, Methodology development, Mathematical formulation, Practical applications, Physics, Engineering.

1. INTRODUCTION

Kinetic energy and momentum are fundamental concepts in physics, especially within classical mechanics, that explain the motion and dynamics of objects. Kinetic energy refers to the energy an object has due to its movement, whereas momentum is defined as the product of an object's mass and velocity. To comprehend how objects move, both kinetic energy and momentum are crucial, as they are vital for describing and forecasting the motion of particles, bodies, and systems. The connection between kinetic energy and momentum offers valuable insights into the dynamics of moving objects and is essential in various scientific and engineering contexts.

The exploration of the link between kinetic energy and momentum is driven by the aim to thoroughly understand the basic principles that dictate object motion. This connection has significant theoretical and practical implications, making it a vital area of research in physics and engineering.

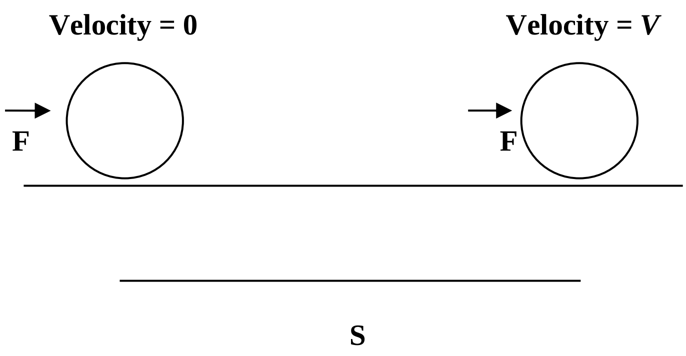
The significance of the relationship between kinetic energy and momentum spans numerous fields, including physics, engineering, and related disciplines. Although these are fundamental concepts, calculating the percentage change in kinetic energy from a given momentum, and vice versa, can often seem complex and challenging. By introducing a more straightforward approach and examining practical examples, the study seeks to provide a clearer understanding of how these critical parameters vary in response to different factors and conditions.

2. THEORY

Kinetic Energy:

Kinetic energy is a key concept in physics that explains the energy an object gains due to its movement. When an object is accelerated, specific forces must be applied. This application of force requires work, and once the work is completed, energy is transferred, causing the object to move at a new velocity. This transferred energy is known as kinetic energy and is calculated based on the object's mass and the speed it reaches. In physics, kinetic energy is defined as the measure of the work an object can perform due to its motion. It is crucial to understand that kinetic energy is a scalar quantity, meaning it is characterized only by its magnitude and lacks a directional aspect. In essence, kinetic energy is the energy linked to an object's motion, acquired through work done on it. It relies on both the object's mass and the square of its velocity. Grasping the concept of kinetic energy is essential for understanding various physical phenomena.

2.1.1 Kinetic Energy Mathematical Expression:



Let,
m = Mass of the body
U = 0 = Initial velocity of the body
F = Constant force applied on the body
V = Final velocity of the body
S = Distance covered by the body
a = Acceleration produced in the body in the direction of the force applied

As, $V^2 – U^2 = 2as$

$\therefore V^2 – 0 = 2as$

$\therefore a = \frac{v^2}{2s}$

As the force and displacement are in the same direction, so the work done is,

W = Fs

$= ma \times s \quad \{ \because F = ma \}$

$= (m \times \frac{v2}{2s}) \times s \quad \{ \because a = \frac{v^2}{2s} \}$

$= \frac{1}{2} mv^2$

This work done appears as kinetic energy (KE) of the body, and can be calculated using the formula: [$KE = \frac{1}{2} mv^2$]

Where ‘m’ represents the mass of the object and ‘v’ is its velocity.

2.2 Linear Momentum:

Momentum can be described as "mass in motion." This implies that any object with mass 'm' that is moving will have momentum. The momentum of an object is determined by two factors: the quantity of mass in motion and the speed at which it is moving. To grasp the idea of momentum, let's look at an example with two friends, Sam and Max, who are jogging on a playground. Sam has a mass of 30 kilograms, while Max has a mass of 40 kilograms. They are both running at the same speed and in the same direction. (a) Which of them has more momentum, and why? (b) Who would feel a more significant impact if they hit a wall? The answer to both questions is Max. Even though Sam and Max are moving at the same velocity, their mass influences their momentum. Since Max has a larger mass, he will have more momentum than Sam. Consequently, Max will experience a more significant impact when colliding with a wall due to his higher momentum, which is directly proportional to mass. Essentially, every object contains potential energy within itself. When this object begins to move, it gains kinetic energy. For instance, consider a rock perched on the edge of a cliff. If the rock falls, its potential energy will transform into kinetic energy as it moves. Linear momentum (P) is determined using the formula: [$P = m \times v$] where "m" stands for the object's mass and "v" represents its velocity. Now, a question arises: Is there a connection between an object's kinetic energy and momentum? If so, is there a formula that explains the relationship between these two aspects?

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3. Literature Review

The concepts of kinetic energy and momentum have been extensively studied in classical mechanics. Kinetic energy, defined as $KE=1/2mv^2$, and linear momentum, $P=mv$, are foundational in understanding the motion of objects [1]. Traditional physics education focuses on teaching these formulas in isolation or applying them in basic problem-solving scenarios [2].

Several studies have explored the mathematical relationship between kinetic energy and momentum, showing that kinetic energy is directly proportional to the square of momentum divided by mass, i.e., $KE=P^2/2m$ [3]. However, most of these approaches focus on direct computation and lack methodologies that simplify percentage-based evaluations—particularly important in dynamic analysis, competitive exams, and real-time physics simulations [4].

In practical applications, researchers in fields such as mechanical and aerospace engineering have utilized these relationships for motion analysis and design optimization [5][6]. Yet, little attention has been paid to simplifying the percentage shift evaluation between the two, which could greatly enhance understanding and computational efficiency.

Recent efforts have proposed graphical or tabular techniques to visualize the kinetic-momentum relationship, but these too fall short in terms of providing a universally applicable and easily computable formula [7]. This gap in literature presents the need for a more accessible, efficient, and pedagogically effective method to evaluate percentage changes between kinetic energy and momentum.

Therefore, this study proposes a novel formula-based methodology designed to be both time-efficient and intuitive, bridging the theoretical and practical gap observed in prior research.

4. METHODOLOGY:

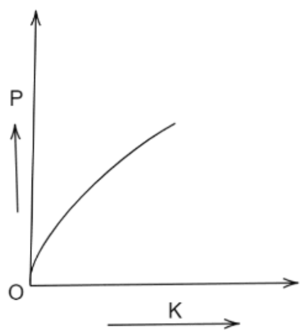
4.1 Traditional Method:

The mathematical relationship between kinetic energy and linear momentum can be established as follows:
Given that linear momentum, P = mv (1),
it follows that $KE = mv^2$.

By multiplying both the numerator and denominator by "m," we have $(m^2v^2) / (mv)^2$. From equation (1),

KE = { P = mv }

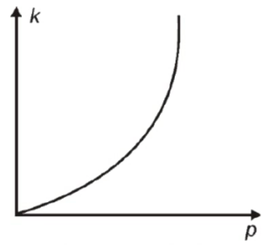
Graphical representation of the relation between kinetic energy and linear momentum:



Again, a question arises: why is the line curved in case of P and not in case of P^2 in a graph between kinetic energy and linear
momentum ?

We know kinetic energy in translation motion

$$k = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \qquad ...(1)$$



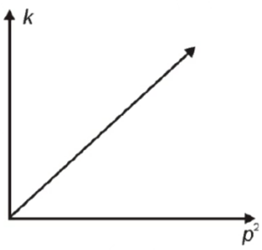
It is in the form of

$y = Ax^2$ (parabolic nature)

When we plot graph between k and p^2

Eq. (1) become in the form of

$y = Cx$



To understand this traditional method, let us consider a few examples involving this relation.

4.1.1 Example:

If a body's linear momentum increases by 20%, what percentage increase will occur in its kinetic energy?

The initial kinetic energy of the body,

$$KE = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{1}{2m}(mv)^2 = \frac{p2}{2m}$$

Increase in momentum,

$= 20\%$ of P $= \frac{20}{100} \times P = \frac{P}{5}$

Final momentum,

$P + \frac{P}{5} = \frac{6P}{5}$

Final kinetic energy of the body,

$K' = \{ \frac{(6P/5)^2}{2m} \} = \frac{36}{25} \times \frac{P^2}{2m}$

$= \frac{36}{25}K \; \{ \because K = \frac{P^2}{2m} \}$

Increase in kinetic energy,

$= (K' - K) = (\frac{36}{25}K - K) = \frac{11}{25}K$

% Increase in kinetic energy,

$= \frac{(K' - K)}{K} \times 100\ \%$

$= \frac{(\frac{11}{25}K)}{K} \times 100\ \%$

$= 44\ \%$

4.1.2 Example:

If the kinetic energy of a body increases by 300%, by what percentage will the linear momentum of the body increase?

Initial kinetic energy,

$KE = \frac{1}{2} mv^2 = \frac{1}{2m}(m^2v^2) = \frac{1}{2m}(mv)^2 = \frac{p2}{2m}$

Initial Momentum,

$P = \sqrt{2mKE}$

Increase in kinetic energy,

$= 300\%$ of $K = 3K$

Final kinetic energy,

$= K' = K + 3K = 4K$

Final momentum,

$P' = \sqrt{2m \times KE'}$

$= \sqrt{2m \times 4KE} = 2\sqrt{2mKE} = 2P$

% Increase in momentum,

$= \frac{(P' - P)}{P} \times 100\ \% = \frac{(2P - P)}{P} \times 100\ \%$

$= 100\ \%$

It is evident that this traditional method can be somewhat time-consuming, especially during competitive exams, often requiring significant time to find a solution. Therefore, a new method is being introduced to simplify learning.

4.2 Proposed Method:

The algorithmic representation of the proposed methodology is as follows:

Since we know, linear momentum is directly proportional to the square root of kinetic energy, and kinetic energy is directly proportional to the square of linear momentum. Therefore, we conclude that both fundamental quantities will increase or decrease simultaneously.

$$\left[\left\{ \frac{(100 + x)}{100} \right\}^n - 1 \right] \times 100$$

(This equation is to be used when the increase in percentage is mentioned)

$$\left[1 - \left\{ \frac{(100 - x)}{100} \right\}^n \right] \times 100$$

(This equation is to be used when the decrease in percentage is mentioned)

Where, x = % increased or decreased in momentum or kinetic energy is mentioned.

For Power 'n'

- If we have to find kinetic energy when a percentage increase or decrease in momentum is given, then, n = 2

  Because, $KE \propto P^2 \quad [\because KE = \frac{p^2}{2m}]$

- If we have to find momentum when a percentage increase or decrease in kinetic energy is given, then, $n = \frac{1}{2}$

  Because, $P \propto \sqrt{KE} \quad [\because P = \sqrt{2mKE}]$

5. RESULTS:

Validation of the developed methodology through some analysis of examples using the new proposed method.

5.1 Example:

If the linear momentum of a body increases by 20%, what will be the % increase in the kinetic energy of the body?

$= [\{\frac{(100 + x)}{100}\}^{n} - 1] \times 100$

$= [\{\frac{(100 + 20)}{100}\}^{2} - 1] \times 100$

$= [\{\frac{(12)}{10}\}^{2} - 1] \times 100$

$= (\frac{144}{100} - 1) \times 100$

$= (\frac{144-100}{100}) \times 100$

$= 44\ \%$

5.2 Example:

When the momentum of a body is increased 'N' times, how much does its kinetic energy increases?

$= [\{\frac{(100 + x)}{100}\}^{n} - 1] \times 100$

$= [\{\frac{(100 + N)}{100}\}^{2} - 1] \times 100$

$= [\{\frac{(100N)}{100}\}^{2} - 1] \times 100$

$= (\frac{10000N^2}{10000} - 1) \times 100$

$= (\frac{10000N^2-10000}{10000}) \times 100$

$= N^2\ \%$

5.3 Example:

If the kinetic energy of a body becomes twenty-one times its initial value, then what will be the increase in percentage of the momentum?

$= [\{\frac{(100 + x)}{100}\}^{n} - 1] \times 100$

$= [\{\frac{(100 + 21)}{100}\}^{1/2} - 1] \times 100$

$= [\{\frac{(121)}{100}\}^{1/2} - 1] \times 100$

$= (\frac{11}{10} - 1) \times 100$

$= (\frac{11-10}{10}) \times 100$

$= 10\ \%$

5.4 Example:

If the kinetic energy of a body is increased by 800%, then what will be the percentage change in its linear momentum?

$= [\{\frac{(100 + x)}{100}\}^{n} - 1] \times 100$

$= [\{\frac{(100 + 800}{100}\}^{\frac{1}{2}} - 1] \times 100$

$= [\{\frac{(900)}{100}\}^{\frac{1}{2}} - 1] \times 100$

$= (\frac{30}{10} - 1) \times 100$

$= (\frac{30-10}{10}) \times 100$

$= 200\ \%$

5.5 Example:

When the momentum of a body decrease by 10%, how much does its kinetic energy decreases?

$= [1 - \{\frac{(100 - x)}{100}\}^{n}] \times 100$

$= [1 - \{\frac{(100 - 10)}{100}\}^{2}] \times 100$

$= [1 - \{\frac{(90)}{100}\}^{2}] \times 100$

$= \left(1 - \frac{8100}{10000}\right) \times 100$

$= \left(\frac{10000 - 8100}{10000}\right) \times 100$

$= 19\ \%$

5.6 Example:
When the Kinetic energy of a body decreases by 36 %, what percentage will the linear momentum decrease by?

$$[\,1 - \{\frac{(100-x)}{100}\}^{n}\,] \times 100$$

$$= [1 - \{\frac{(100-36)}{100}\}^{\frac{1}{2}}\,] \times 100$$

$$= [\,1 - \{\frac{(64)}{100}\}^{\frac{1}{2}}\,] \times 100$$

$$= (1 - \frac{8}{10}) \times 100$$

$$= \left(\frac{10-8}{10}\right) \times 100$$

= 20 %

5.7 Example:
When the Kinetic energy of a body decreases by 51 %, what percentage will the linear momentum decrease by?

$$[\,1 - \{\frac{(100-x)}{100}\}^{n}\,] \times 100$$

$$= [1 - \{\frac{(100-51)}{100}\}^{\frac{1}{2}}\,] \times 100$$

$$= [\,1 - \{\frac{(49)}{100}\}^{\frac{1}{2}}\,] \times 100$$

$$= (1 - \frac{7}{10}) \times 100$$

$$= \left(\frac{10-7}{10}\right) \times 100$$

= 30 %

6. APPLICATIONS

The suggested approach can accelerate computations involving numerous conversions between kinetic energy and momentum percentages. This is particularly relevant for extensive scientific experiments and intricate engineering projects, which demand a solid grasp of basic principles and adept problem-solving abilities. The suggested approach offers significant benefits in scenarios requiring frequent conversions between kinetic energy and momentum percentages. By streamlining these calculations, researchers and engineers can save valuable time and reduce the potential for errors in complex analyses. This efficiency gain is especially crucial in large-scale scientific experiments, where even small inaccuracies can compound and lead to significant deviations in results. Similarly, in intricate engineering projects, such as aerospace design or particle physics simulations, the ability to quickly and accurately perform these conversions can greatly enhance the overall productivity and precision of the work.

Moreover, this approach underscores the importance of a strong foundation in fundamental principles and advanced problem-solving skills. As scientific and engineering challenges become increasingly complex, professionals must not only understand the basic concepts but also be able to apply them creatively and efficiently. The ability to swiftly navigate between different representations of energy and momentum allows for more dynamic and adaptive problem-solving, enabling researchers and engineers to explore various scenarios and optimize their designs or experimental setups more effectively. This enhanced capability can lead to more innovative solutions and potentially groundbreaking discoveries in fields ranging from astrophysics to renewable energy technologies.

7. CONCLUSION

The use of these simplified techniques can greatly assist students, educators, and professionals in various fields by fostering a more profound and intuitive understanding of the relationship between kinetic energy and momentum. It is crucial to keep investigating innovative and more accessible methods for teaching and applying basic physics principles, and the strategies outlined in this paper contribute to this ongoing effort. Expanding on the simplified techniques for understanding kinetic energy and momentum can lead to significant improvements in physics education and practical applications. By providing students with more intuitive and accessible methods, educators can help bridge the gap between abstract concepts and real-world phenomena. These techniques may include visual representations, interactive simulations, or hands-on experiments that demonstrate the relationship between kinetic energy and momentum in tangible ways. Such approaches can enhance retention and promote critical thinking skills, enabling students to apply these fundamental principles to more complex problems in their future academic and professional endeavors.

Furthermore, the benefits of these simplified techniques extend beyond the classroom. Professionals in fields such as engineering, sports science, and accident reconstruction can utilize these methods to make quick, accurate assessments in their work. For instance, engineers designing safety systems for vehicles could employ these techniques to estimate impact forces more efficiently, while sports scientists might use them to analyze and improve athletic performance. By continually refining and expanding upon these simplified approaches, researchers and educators can contribute to a more widespread understanding of physics principles, potentially leading to innovations and advancements across various industries that rely on the application of kinetic energy and momentum concepts.

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