

A Relative Operation-Based Separation Model for Safe Distances of Virtually Coupled Trains

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Abstract—Virtual coupling is a novel railway transport concept that allows trains to split and join on-the-fly by switching from mechanical to virtual couplers. One of the main challenges in applying virtual coupling in metro railways is to reduce the tracking distance between trains without compromising safety. This article proposes a relative operation-based train separation model to reduce the safe distance between trains. This model applies a fault tolerance principle. The principle is that the preceding train normally operates for a time interval from its last-known state before initiating an emergency brake to stop the train. A difficulty in applying the proposed model is to predict the boundary of all possible time-position trajectories of the preceding train, which is the reachability problem of a hybrid system. To solve this problem, we formalise the operation of the preceding train by a parameterized hybrid automaton. A polytope-based algorithm is then developed for computing an over-approximated reachable set of the automaton. We compare our approach with a state-of-the-art relative braking distance-based train separation model for virtual coupling on a concrete metro line in Chengdu, China, and evaluate the method with several benchmarks. The results demonstrate that the relative operation-based model substantially reduces the safe distances between trains. Compared to conventional approaches, the proposed model provides a considerable 90.7% decrease in unnecessary waiting time at railway stations for virtually coupled trains and a 4.9% increase in the capacity of the given railway lines. **Index Terms**—Virtual coupling, Safe distance, Train separation model, Train control system, Hybrid automata.

I.INTRODUCTION

The ever-increasing need for service improvements led the railway industry to explore the next generation of train control concepts, such as virtual coupling [1]. This concept entails tracked trains virtually coupled via distributed controls and vehicle-to-vehicle communication. The distance between two virtually coupled trains is much shorter than conventional railway systems. On the one hand, virtual coupling expands the railway transportation capacity of existing networks. On the other hand, trains can split and join on-the-fly according to transport demand. Virtual coupling is a promising technique for achieving the zero capacity waste target proposed by the European Rail Research Advisory Council (ERRAC). One of the main challenges in applying virtual coupling in metro railway transportation is reducing the distance between the tracked trains without compromising safety. A long tracked distance can make it difficult for trains to arrive simultaneously at stations, resulting in unnecessary additional waiting time at the station. This shortcoming significantly reduces the transportation capacity and service quality of metro railways. A typical train control system in metro railways adopts an ATP-ATO control scheme, which consists of an automatic train operation (ATO) controller supervised by an automatic train protection (ATP) controller [2]. The ATO is similar to an adaptive cruise controller used in road vehicles. It performs nominal train driving actions like speed regulations, tractions and service brakes. In contrast, the ATP protects a train by computing a safe distance to prevent collisions and initiating an emergency brake whenever a safe distance cannot be guaranteed. A similar control scheme has also been proposed for autonomous vehicles to guarantee safety [3]. Reducing the safe distance between trains is a central problem in virtual coupling because the

safe distance decides the smallest possible tracking distance between the trains under the ATP-ATO control scheme. The safe distance is computed in railways by a so-called train separation model. Conventional train control systems use the absolute brake distance-based train separation (ABS) model (also known as “moving block”), where the safety distance equals the emergency braking distance of the following train plus a safety margin. Adopting that a train is physically impossible to stop instantly, a relative brake distance-based train separation (RBS) model is proposed. In the RBS model, the preceding train is assumed to apply an emergency brake from its last-known state. The safe distance between two tracked trains is decided by ensuring that both trains do not collide under the worst-case stopping scenario [4]. By assuming that the trains always have the same braking performance, the safe distance is simplified to be the difference of the emergency braking distances of the trains plus a safety margin [5]. Unfortunately, the safe distance computed by the RBS model is still too big for virtual coupling in metro railways. For example, considering the worst-case control errors and failures of real-world pneumatic brake systems, the safe distance between two trains is greater than 100 meters at 80 km/h even if the trains have the same braking performance. With such a considerable safe distance, it is difficult for trains to arrive simultaneously at stations. Consequently, gaining actual capacity from the concept of virtual coupling is impacted.

II. RELATED WORK

A. Control Approaches for Virtual Coupling

The problem of optimising train operations has a long tradition in the railway community, including optimising operation trajectories, control strategies and timetables. The concept of virtual coupling in railways was first proposed by Bock et al. to improve the capacity of existing railway lines. In this concept, trains are no longer physically coupled; each has individual propulsion and brake systems. An advantage of virtual coupling is that trains can split and join on-the-fly to fulfil transportation needs. Chai et al. considered the time-dependent passenger

demand and train loading capacities in virtual coupling. They proposed a linear programming-based approach for virtual coupling to improve line capacity and reduce congestion in metro railway networks. The distance between virtually coupled trains must be small enough for simultaneous arrival to make the concept practicable in metro railway transportation. In railways, a train control system adopts the ATP-ATO scheme. Both the controllers of the ATO and ATP have been investigated for reducing tracked distances between trains.

B. Predictions of Train Operations

A central problem in applying a train separation model is predicting all possible tracked train operations. Machine learning-based methods that apply data-driven models have been investigated for predicting trajectories of autonomous vehicles. However, as machine learning has an inherent unexplainable problem, a machine learning-based method cannot guarantee to predict the boundaries of train operations. Therefore, it cannot be used to compute the safe distance between trains. Proving the correctness of a train control system with formal methods is an important research direction. Runtime verification is a lightweight formal method that can predict undesired behaviours while the system is running. In the following, we mainly focus on previous work on reachable set-based prediction approaches since this work can guarantee obtaining boundaries of system behaviours. Hybrid automata have been proposed to formalise systems with discrete-continuous state spaces. This formalism is expressive but has considerable difficulties in solving its reachability problem. Girard et al. proposed a zonotope-based approach for overestimating the reachable set of hybrid automata with linear dynamics and guards. Based on those works, Kochdumper et al. proposed an algorithm for computing intersections between nonlinear guards and reachable sets with Taylor models or polynomial zonotopes. Ramdani et al. presented an interval Taylor method-based approach of computing reachable sets of hybrid systems with uncertain nonlinear monotone dynamics.

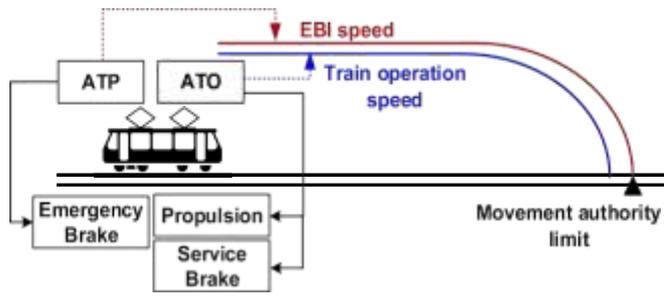


Fig. 1. Typical automatic train control system.

Stursberg et al. used a counterexample-guided verification approach to prove the correctness of a cruise control system, which is modelled as a hybrid automaton. Xu et al. proposed a collision prediction approach for satellites with zonotope-based reachable sets, in which the satellites are simplified as cuboids to compute reachable domains and dangerous domains with uncertain motions. These works focus on proving the correctness of a system. How to compute the safe distance between trains when considering normal operations of the preceding train in virtual coupling is still an ongoing research topic.

III. PRELIMINARIES

A. Automatic Train Control System

Due to unpredictable driving actions and the reaction time of human drivers, virtually coupled trains must be operated by automatic train control (ATC) systems to maintain a safe small tracked distance. An ATC system adopts an automatic train protection-automatic train operation (ATP-ATO) control scheme, as shown in Fig. 1. The ATP provides fail-safe protections with the emergency brake to ensure the tracked trains keep a safe distance. In contrast, the ATO performs automatic driving functions by applying propulsions and service brakes. A safe distance between tracked trains is transferred to a movement authority of the following train, which is the authority for the train to enter and travel through a specific section of track. An EBI speed is the maximal speed that ensures under no circumstances will the train stop at the movement authority limit (i.e., the furthest position of the movement authority) by applying an emergency brake. It is derived from the braking curve of the train with the

guaranteed emergency brake rate. The EBI speed curve is regarded as a “safe envelope” for automatic driving. The ATO shall maintain the train speed below the EBI speed. If the EBI speed at the train location is exceeded, the ATP initiates an immediate emergency brake application.

B. Relative Brake Distance-Based Train Separation Model

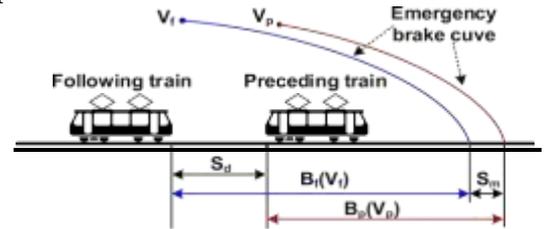


Fig. 2. Relative brake distance train separation model.

In railways, the safe distance between two tracked trains is computed by a train separation model to guarantee collision-free. A relative brake distance-based separation (RBS) model has been proposed that the safe distance between two trains equals the difference in the braking distances of the trains plus a safety margin. A safety margin is an extra distance to handle the impact of other unknown factors, such as the measurement error of train position and speed and communication delays. Fig. 2 illustrates an RBS model. Let S_d be the safe distance between two tracked trains; $B_p(V_p)$ and $B_f(V_f)$ be the emergency brake distances of the preceding train and the following train starting from their current speed V_p and V_f respectively, and S_m be a safety margin. The RBS model is defined as follows. $S_d = \max((B_f(V_f) - B_p(V_p)), 0) + S_m$ (1) A train separation model guarantees the collision-free property, i.e., two tracked trains are never in the same position simultaneously. The standard RBS model, simplifies the train separation model indicating that the property can be satisfied if the distance between two trains is always greater than the relative emergency braking distances. Unfortunately, this simplification only holds at some ideal conditions. Ning proved that the standard RBS model could prevent collisions only if the braking performance of the preceding train is worse

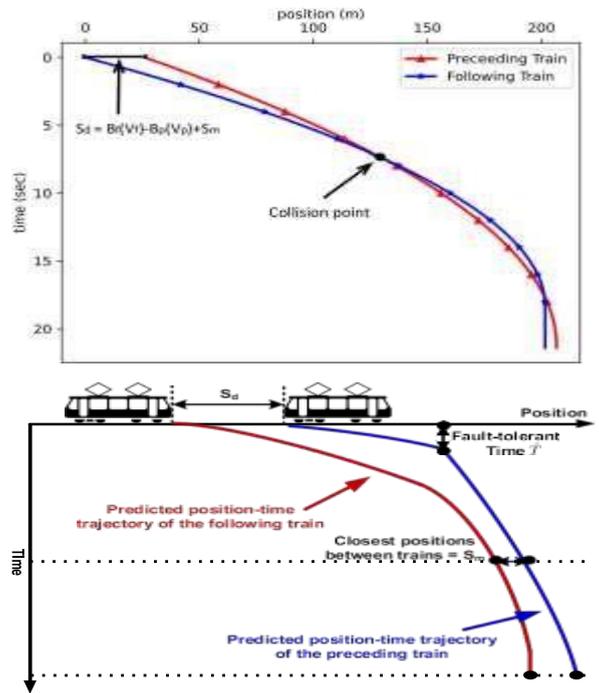
than or equal to the braking performance of the following train. Because the ATP uses the open-looped pneumatic brake system, it is possible that the emergency brake of the following train has a smaller deceleration. This braking rate combination must be considered in real-world train control systems. Therefore, the standard RBS model is insufficient to guarantee collision-free in real-world applications. Consider the following example. Let the initial speeds of two tracked trains be $V_p = 17$ m/sec and $V_f = 22$ m/sec, the emergency brake accelerations be $a_p = -0.8$ m/sec² and $a_f = -1.2$ m/sec². Let the safety margin be $S_m = 5$ m. The safe distance is 26.1 m according to the RBS model. Fig. 3 shows that when the preceding train applies an emergency brake, a collision occurs even if the following train initiates an emergency brake immediately.

IV. COMPUTATIONAL MODEL OF SAFE DISTANCE FOR VIRTUAL COUPLING

A. Relative Operation-Based Train Separation Model

We propose a relative operation-based separation (ROS) model to compute the safe distance for virtual coupling. In the ROS model, the safe distance between two tracked trains is decided by ensuring that when the following train applies an emergency brake, the smallest distance between the trains is greater than or equal to the safety margin with the predicted worst-case operation of the preceding train. According to the ATP-ATO control scheme, as shown in Fig. 1, the ATP computes the EBI speed from the ROS model. After that, the ATO generates its speed constraint concerning the EBI speed. An ATP only initiates an emergency brake with certain failures. Because fault propagation consumes time, if the last-known status of a train does not meet any pre-conditions of such a failure, it is safe to predict that the train will operate normally without triggering an emergency brake for at least a period of fault-tolerant time. According to this assertion, the ROS model applies a T^* fault tolerance (T^* -FT) principle with T^* being an interval of a fault-tolerant time. With this principle, the operation of the preceding train is divided into two

phases. In the first phase, the train operates normally for T^* seconds by the ATO. In the second phase, the train applies an emergency brake strategy that the ATP initiates an emergency brake to stop the train. The T^* -FT principle is formally defined as follows.



B. The Time-Position Space of Preceding Train

With the T^* -FT principle, the ATO behaviour of the preceding train must be considered when predicting the time-position space of the train. An ATO system can choose different control strategies by considering operation efficiency, energy savings, passenger comfort, etc. This article considers a typical strategy in which a train operation process between two stations is divided into three phases: departure, cruising and arrival. The ATO target speed in the first two phases is according to the EBI speed, whereas in the third phase, it is computed according to the intended stopping position. During the departure phase, the train accelerates with its maximum propulsion until it reaches the target speed. During cruising, the ATO system ensures that the train operates at the target speed. During the arrival phase, the ATO applies a programmed stopping

process. An ATO system applies the following adjustment inhibition strategy (AIS) to avoid frequent control adjustments. Once the ATO system is in either the propulsion or brake status, the system stays in that status for a time interval before switching to the other status. Let a control $u \in R$ be the acceleration of a train. A control trace u is defined as a sequence of controls, i.e., $u = (u_0, \dots, u_N)$. Given an integer K , the set R^K contains all control traces with length K over R . Given integers K_1 and K_2 , we denote by T_Δ and $[T_{\Delta K_1}, T_{\Delta K_2}]$ the control cycle and the time interval of the AIS, respectively. An ATO control space contains all possible control traces concerning the ATC operation logic. Definition 3 (ATO control space): Let V_i be the ATO target speed at the i th position in a control trace, and AP and AB be the range of accelerations of propulsion and service brake, respectively. The ATO control space UO is the set of all possible ATO control traces as follows.

$$UO = \{u \mid u = u_A \cdot u_B\},$$

where :

$u_A = (u_1, \dots, u_n)$ represents the departure phase: $\forall i \in [1, n] : u_i = \max AP$
 $u_B = (u_1 \cdot \dots \cdot u_n)$ is the cruising and arrival phases: $\forall i \in [1, n] : \left(\begin{array}{l} h_{vi} < V_i \\ \wedge \\ h_{vi} \geq V_i \end{array} \right)$
 $i \in [1, n] \Rightarrow u_i \in [K_1, K_2] \text{ Ak } P$
 $i \in [1, n] \Rightarrow u_i \in [K_1, K_2] \text{ Ak } B$
 If we define the solution of the model $f(D(T), u)$ as the TPS at time $T + T_\Delta$ from $D(T)$ under a control u , then $f(D(T_0), u)$ is a TPT (i.e., a sequence of time-position states) starting from $D(T_0)$ under a control trace u such that

$$f(D(T_0), u) = (D(T_0), D(T_1), \dots, D(T_n)) \quad (4)$$

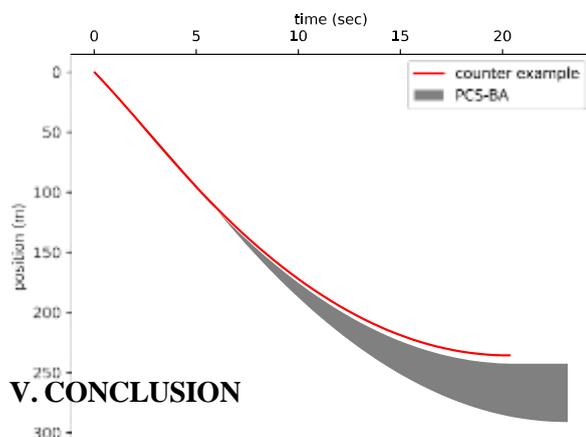
$$\text{where } \forall i \in [1, n] : D(T_i) = f(D(T_{i-1}), u_{i-1}).$$

Definition 4 (T^\wedge -time-position space): Let UKO be the subset of the ATO control space UO containing all control traces, i.e., $UKO \subset UO$, each of which has a length K with $K = T^\wedge / T_\Delta$. The T^\wedge -time-position space $DT^\wedge(D(T_0))$ is the set of all possible TPTs starting from $D(T_0)$ within T^\wedge seconds such that $DT^\wedge(D(T_0)) = \{f(D(T_0), u) \mid u \in UKO\}$ (5) We denote by D the last TPS of $D(D(T_0), \dots, D(T_n))$, i.e., $D = D(T_n)$. Given a control trace $u = (a_{P1}, \dots, a_{PT})$ with a_{FT} being the emergency brake

acceleration of the preceding train and the last-known TPS $D(T_0)$ of the preceding train, the time-position space D_p of the preceding train in the ROS model with the T^\wedge -FT principle is as follows. $D_p = \{(D_1 \cdot D_2) \mid D_1 \in DT^\wedge(D(T_0)), D_2 = f(D_1, u)\}$ (6) Intuitively, the subsequence D_1 specifies the normal operation phase, controlled by ATO, with the T^\wedge -FT principle, whereas D_2 represents the emergency brake phase. Due to unmodelled dynamics and mismatched parameters, the ROS model contains uncertainties in parameters and control traces. When the time value T^\wedge of the T^\wedge -FT principle is greater than

0 sec, the boundaries of the time-position space cannot be computed with boundary values of accelerations. Because the ATC operations follow specific logical rules, the possible accelerations at each time point are multi-variant. They are challenging to obtain. We prove that using the global acceleration boundaries of an ATO cannot cover all possible TPTs of a train. Proposition 2: Let D_1, \dots, D_N be the complete TPTs of the preceding train in the ROS model obtained by simulations with the boundary values of accelerations. If $T^\wedge > 0$, then there exists a complete TPT $D_o \in D_p$ and a point of time T , it holds that $D(T)_o < \min(D(T)_1, \dots, D(T)_N)$, where $D(T)_o \in D_o$ and $D(T)_i \in D_i$ with $i \in [1, N]$. Proof: According to the T^\wedge -FT principle, the train operation trajectory is as follows. The ATO system controls the train for T^\wedge seconds. After T^\wedge seconds, the ATP system immediately initiates the emergency brake, and then the train moves with its maximum emergency brake acceleration until it fully stops. Without loss of generality, we use the following parameters in the proof: The ATO target speed is 20 m/sec. The upper and lower boundaries of the propulsion acceleration are 1.0 m/sec² and 0.4 m/sec², respectively. The upper and lower boundaries of the service brake acceleration are -0.3 m/sec² and -0.6 m/sec², respectively. The time duration of the adjustment inhibition strategy is set to be between 5 and 12 control cycles; The control period is 0.2 sec. The acceleration of the emergency brake is -1.2 m/sec². The time duration of the T^\wedge -FT principle is 6 seconds. Let the initial train speed be 18.5 m/s.

Simulations according to the boundaries of the parameters (PCS-BA) suggest that the complete time-position trajectories are shown as the grey area in Fig. 5. However, there are possible operations where the train stops faster than the simulation results. For example, one counterexample is that the propulsion acceleration is 0.75 m/sec^2 , the service brake acceleration is -0.5 m/sec^2 , and the time duration of the adjustment inhibition strategy is 7 control cycles.



V. CONCLUSION

This article presented a relative operation-based train separation (ROS) model for virtual coupling. The model applied a T^{\wedge} -FT principle, according to which the preceding train normally operated for T^{\wedge} seconds before initiating an emergency brake. The reachable set-based method was applied to predict the boundary of time-position trajectories of the preceding train in the ROS model. The train operation was formalized with a parameterized hybrid automaton, with the train accelerations and control switching conditions specified by parameters. A polytope-based algorithm was developed for computing the reachable set of the parameterized hybrid train operation model. Various simulations were designed, and the results of different train separation models were compared. The results showed that larger values of T^{\wedge} allowed higher EBI speeds and significantly shorter distances between trains. This result validated that the ROS model significantly reduced unnecessary waiting time when virtually coupled trains arrived at a station and improved the capacity of railway lines. Several interesting topics can be investigated in future work. First, a bigger

value of fault-tolerant time increases the risk of train operations. As the value of T^{\wedge} is significant for improving the virtual coupling performance, reducing risks with a long prediction time without activating the emergency brake is an important issue. Secondly, when considering a convoy with more than two trains, the normal operation phases of the preceding trains are more complicated. It is worth investigating how to model their behaviours and compute the boundaries of time-position trajectories. Finally, local and string stabilities of virtual coupling with advanced control methods, such as MPC and its extensions, using the ROS model is still an ongoing research topic.

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