

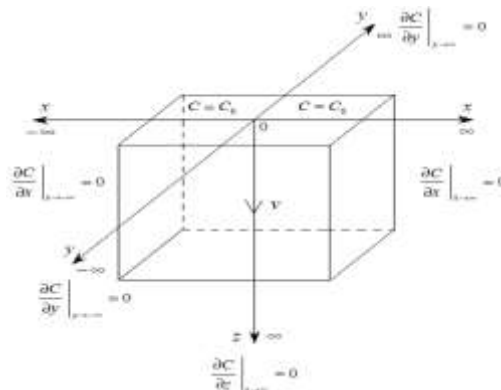
An Analytical Solution of Three-Dimensional Transport of Contaminants in Unsaturated Porous Media

Nataraja G R

Assistant Professor, Department of Mathematics,
Governement First Grade College, Davangere, Karnataka

The main objective of this chapter is to derive analytical solutions for the three-dimensional advection-dispersion equation for one-dimensional flow in a three-dimensional semi-infinite medium using a straight forward solution procedure. The transport equation consists of terms describing linear equilibrium adsorption, zero-order production and first and third-order decay. The advection-dispersion equation is solved subject to both first and third-type inlet boundary conditions, and assuming arbitrary initial and inlet concentration distributions. Analytical solutions to the partial differential equations are obtained using integral transform techniques. The approach is shown to be applicable to a broad class of transport problems. General solutions for five specific combinations of initial and boundary concentration profiles.

In a unidirectional flow field it is convenient to consider dispersion occurring either in the direction of the net flow (longitudinal dispersion) or perpendicular to the flow (transverse dispersion). Mechanical dispersion and diffusion may contribute to both longitudinal and transverse dispersion, so that the dispersion coefficients consist of a mechanical dispersion term (which is velocity dependent) and a diffusion term.



Dispersion is studied by introducing a radioactive tracer into the soil solution and following its distribution in time and space. The tracer distribution in a homogeneous porous medium with steady, uniform water flow in the z direction is described by the dispersion equations. The assumption is made so that the transport of a solute is adequately described by the advection-dispersion equation. The transport in a homogeneous and isotropic medium during one-dimensional steady state flow with three-dimensional dispersion is of the form

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial z^2} - v \frac{\partial C}{\partial z} + D_T \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - \lambda C$$

The flow is assumed strictly vertical and consequently the same restriction applies to the convective transport of the dissolved solutes and the model allows for both longitudinal and transverse dispersion. The model in its present form can simulate dispersion for area and line sources.

A common dispersion experiment is that in which there is a step-change in concentration at the boundary of a semi-infinite system at time $t = 0$, in which the initial concentration of tracer is zero. The appropriate initial and boundary conditions for this problem are

$$C(x, y, z, 0) = 0 \quad 0 \leq z < \infty, \quad -\infty < x < \infty, \quad -\infty < y < \infty$$

$$C(x, y, 0, t) = g(x, y) = C_0 \quad -\infty < x < \infty, \quad -\infty < y < \infty, \quad t > 0$$

$$\left. \frac{\partial C}{\partial x} \right|_{x \rightarrow \pm \infty} = 0, \quad 0 \leq z < \infty, \quad -\infty < y < \infty \quad t > 0$$

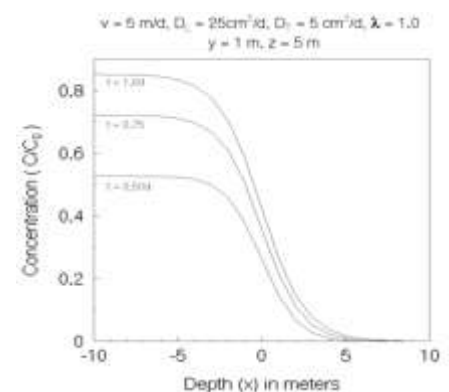
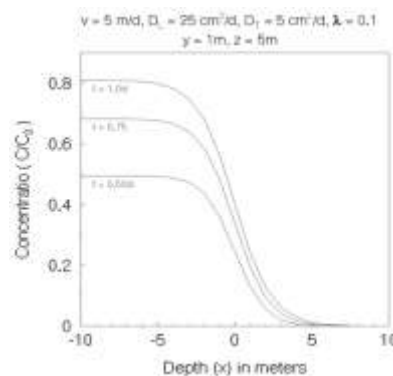
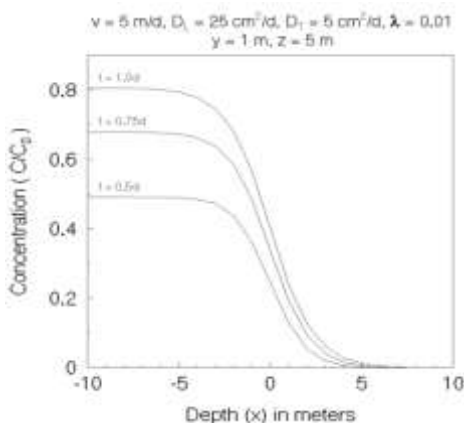
$$\left. \frac{\partial C}{\partial y} \right|_{y \rightarrow \pm \infty} = 0, \quad 0 \leq z < \infty, \quad -\infty < x < \infty, \quad t > 0$$

$$\left. \frac{\partial C}{\partial z} \right|_{z \rightarrow \infty} = 0, \quad -\infty < x < \infty, \quad -\infty < y < \infty \quad t > 0$$

By applying Laplace transform on Left and Fourier Transform on Right side of the Advection-dispersion equation to get the solution of the form

$$C(x, y, z, t) = \frac{z C_0}{\sqrt{4\pi D_L}} \int_0^t \tau^{-3/2} \cdot \exp \left[-\frac{(z - v\tau)^2}{4D_L \tau} \right] \exp [\lambda \tau] \left\{ \operatorname{erfc} \left[\frac{x}{\sqrt{4D_T \tau}} \right] \cdot \operatorname{erfc} \left[\frac{y}{\sqrt{4D_T \tau}} \right] \right\} d\tau.$$

$$C(x, y, z, t) = \frac{z C_0}{\sqrt{4\pi D_L}} \int_0^t \tau^{-3/2} \cdot \operatorname{erfc} \left[\frac{x}{\sqrt{4D_T \tau}} \right] \cdot \operatorname{erfc} \left[\frac{y}{\sqrt{4D_T \tau}} \right] \exp \left[-\frac{(z - v\tau)^2}{4D_L \tau} + \lambda \tau \right] d\tau.$$



Result and Discussions

For the validation, we assumed that a porous medium with the following arbitrary transport parameters: $D_L=25\text{cm}^2/\text{d}$, $D_T=5\text{cm}^2/\text{d}$, and $v=50\text{cm}/\text{d}$. All concentrations were conveniently expressed as dimensionless quantities C/C_0 , with C_0 being equal to unity.

Figures shows the solution for various radioactive decay λ and time according to equation, we plot a graph for C/C_0 vs transverse direction (x) for different values of other depths i.e., $z = 50\text{m}$, $y = 1, 2\text{m}$, decay constant $\lambda = 0.01, 0.1, 1.0$ and time $t = 1, 2$ years. These figures show concentration profiles obtained with solution and as well as the results obtained with the half space solutions for various times. The results are almost identical to the steady state solution given above equation.

Figures illustrate solute movement as a result of diffusion from the buried solute source. In figure the concentration profile is plotted for various depths below the soil surface (z) and the transverse direction (x), whereas figure contains solute concentration profiles in the xz plane. Because the source is located close to the surface, a nonsymmetrical solute profile develops in the z direction.

Figures shows the solutions for various times and decay according to equation. The solution for large times is clearly identical to the steady state. The invasion of the solute into the medium at $x > 0$ and the subsequent flattening of the solute front in all the three directions can clearly be observed and figure shows concentration profiles in the transverse x direction for various values of z at $t = 1$ for a first and third type boundary condition.

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