Achievement of C-Root Square Meanness Requirement in the Construction of Several Belt Graphs

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Abstract

A function f is called a C-root square mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f: V(G) \to \{1,2,3,\cdots,q+1\}$ is injective and the induced function $f^*: E(G) \to \{2,3,\ldots,q+1\}$ defined as $f^*(uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ for all $uv \in E(G)$, is bijective. A graph that admits an C-root square mean labeling is called a C-root square mean graph. In this paper, we have discussed the C-root square meanness of the triangular belt graph $TB(\alpha)$, slanting belt graph $SB(\alpha)$, diamond ladder graph Dl_n and the graph $TBL(n,\alpha,k,\beta)$.

Key words: C-root square mean labeling, C-root square mean graph, triangular belt graph, slanting belt graph.

1. Introduction

In this paper, when we refer to a graph, we mean a simple, undirected, finite graph. We use the notations and language from [1-6]. We refer to [7] for a comprehensive overview of graph labeling. Let $S = \{\uparrow,\downarrow\}$ be the symbol representing, the position of the block upward and downward as given in Figure 1.

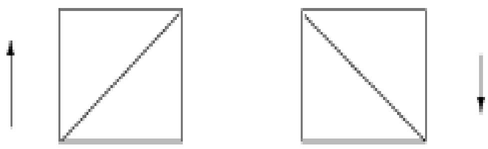

Figure 1. Position of the block upward and downward

Let α be a sequence of n symbols of S, where $\alpha \in S^n$. Construct a graph be tiling n blocks side be side with their positions indicated be α. Let $u_1, u_2, \ldots, u_n, u_{n+1}$ and $v_1, v_2, \ldots, v_n, v_{n+1}$ are the top vertices and bottom vertices of the belt. Denote the resulting graph by $TB(\alpha)$ and refer to it as a triangular belt. For example, the triangular belts corresponding to sequences $\alpha_1 = \{\downarrow\uparrow\uparrow\}, \alpha_2 = \{\downarrow\downarrow\uparrow\downarrow\}$ respectively, are shown in Figure 2.

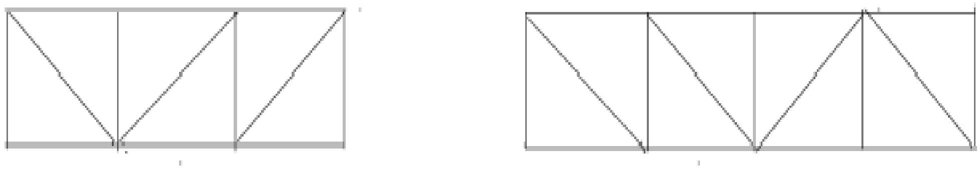

Figure 2. $TB(\uparrow,\downarrow,\downarrow)$ and $TB(\uparrow,\uparrow,\downarrow,\uparrow)$

Consider a class of planar graphs that are formed by amalgamation of triangular belts. For each $n \ge 1$ and α in S^n, n blocks with the first block is $\downarrow$. Take the triangular belt $TB(\alpha)$ and triangular belt $TB(\beta)$, β in S^k where $k > 0$. Rotate $TB(\beta)$ by 90 degrees counter clockwise and amalgamate the last block with the first block of $TB(\alpha)$ by sharing and edge. The resulting graph is denoted by $TBL(n,\alpha,k,\beta)$. Also $u_{1,1}, u_{1,2}, \ldots, u_{1,n+1}, u_{2,1}, u_{2,2}, \ldots, u_{2,n+1}, v_{3,1}, v_{3,2}, \ldots, v_{3,k-1}, v_{4,1}, v_{4,2}, \ldots, v_{4,k-1}$ are the vertices of the graph $TBL(n,\alpha,k,\beta)$. The graph $TBL(n,\alpha,k,\beta)$ has $2(nk+1)$ vertices and $3(n+k)+1$ edges. The concept of C- geometric mean labeling of graphs was introduced by Durai Baskar [12]. Vasuki et al., [11] proved that the graph triangular belt and the graph $TB(n,\alpha,k,\beta)$ admits mean labeling. The F-root square mean graphs were first described by Arockiaraj et al. [8]. Additionally, Muhiuddin et al. [9, 10] examined the classical mean labeling of graphs. Recently Rajesh Kannan et al. developed the classical mean [13] and C-exponential mean [14] for several standard graphs. Motivated by the works of so many authors in the area of graph labeling, we have defined a new type of labeling called C-root square mean labeling. A function f is called a C-root square mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \to \{1,2,3,\cdots,q+1\}$ is injective and the induced function $f^* : E(G) \to \{2,3,\ldots,q+1\}$ defined as $f^*(uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ for all $uv \in E(G)$, is bijective. A graph that admits an C-root square mean labeling is called a C-root square mean graph. In this paper, we have discussed the C-root square meanness of the triangular belt graph $TB(\alpha)$, slanting belt graph $SB(\alpha)$, diamond ladder graph Dl_n and the graph $TBL(n,\alpha,k,\beta)$.

2 Main Results

Theorem 2.1. The triangular belt $TB(\alpha)$ is a C-root square mean graph, for any α in S^n with the first block is being $\downarrow$ where $S = \{\uparrow,\downarrow\}$ and for all $n \ge 1$.

Proof. Let $u_1, u_2, \ldots, u_n, u_{n+1}$ and $v_1, v_2, \ldots, v_n, v_{n+1}$ are the top vertices and bottom vertices of the triangular belt, respectively. The graph $TB(\alpha)$ has $2n+2$ vertices and $4n+1$ edges.

Define $f : V(TB(\alpha)) \to \{1,2,3,\cdots,4n+2\}$ as follows.

$f(u_i) = 4i-2, \quad for\ 1 \le i \le n+1\ \ and$

$$f(v_i) = \begin{cases} 1, & i = 1 \\ 4i-4, & 2 \le i \le n+1. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$f^*(u_i u_{i+1}) = 4i+1, \ for\ 1 \le i \le n,$

$f^*(v_i v_{i+1}) = 4i-1, \ for\ 1 \le i \le n,$

$f^*(u_i v_i) = 4i-2, \ for\ 1 \le i \le n+1,$

$f^*(u_i v_{i+1}) = 4i, \ 1 \le i \le n\ \ and$

$f^*(u_i v_{i-1}) = 4i-4, \ \ for\ 3 \le i \le n.$

Hence f is a C-root square mean labeling of G. Thus the triangular belt $TB(\alpha)$ is a C-root square mean graph, for any α in S^n with the first block is being $\downarrow$ where $S = \{\uparrow, \downarrow\}$ and for all $n \ge 1$.

Theorem 2.2. The slanting belt $SB(\alpha)$ is a C-root square mean graph, for any α in S^n with the first block is being $\uparrow$ where $S = \{\uparrow, \downarrow\}$ and for all $n \ge 3$.

Proof. Let $u_1, u_2, \ldots, u_n, u_{n+1}$ and $v_1, v_2, \ldots, v_n, v_{n+1}$ are the top vertices and bottom vertices of the slanting belt, respectively. The graph $SB(\alpha)$ has $2n$ vertices and $4n-3$ edges.

Define $f : V(SB(\alpha)) \to \{1, 2, 3, \cdots, 4n+4\}$ as follows.

$$f(u_i) = \begin{cases} 1, & i = 1 \\ 3i-1, & 2 \le i \le 3 \\ 4i-7, & 4 \le i \le n+1, \end{cases}$$

$f(v_i) = 4i-1, \ \ for\ 1 \le i \le n \ \ \ and$

$f(v_{n+1}) = 4n-3.$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i u_{i+1}) = \begin{cases} 2, & i = 1 \\ 4i-4, & 2 \le i \le n \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 4i+2, & 1 \le i \le n-1 \\ 4i, & i = n, \end{cases}$$

$f^*(v_i u_{i+1}) = 4i-1, \ for\ 1 \le i \le n,$

$f^*(u_i v_i) = 4i-3, \ for\ \ 3 \le i \le n\ and$

$f^*(v_i u_{i+2}) = 4i+1, \ \ for\ 1 \le i \le n-1.$

Hence f is a C-root square mean labeling of G. Thus the slanting belt $SB(\alpha)$ is a C-root square mean graph, for any α in S^n with the first block is being $\uparrow$ where $S = \{\uparrow, \downarrow\}$ and for all $n \ge 3$.

Theorem 2.3. The graph $TBL(n,\alpha,k,\beta)$ is a C-root square mean graph, for any α in S^n with the first and last block are being $\downarrow$ and β in S^k, for all $k>0$.

Proof.
Let $\{u_{1,1},u_{1,2},\ldots,u_{1,n+1},u_{2,1},u_{2,2},\ldots,u_{2,n+1},v_{3,1},v_{3,2},\ldots,v_{3,k-1},v_{4,1},v_{4,2},\ldots,v_{4,k-1}\}$ is the vertex set of the graph $TB(n,\alpha,k,\beta)$.
Define $f:V(TB(n,\alpha,k,\beta))\rightarrow\{1,2,3,\cdots,4(n+k)+2\}$ as follows.

$f(u_{1,i})=4k+4i,\quad for\ 1\le i\le n,$

$f(u_{1,n+1})=4(n+k)+2,$

$f(u_{2,1})=4k,$

$f(u_{2,i})=4k+2+4(i-2),\quad for\ 2\le i\le n+1,$

$f(v_{3,i})=\begin{cases}1, & i=1\\ 4i-4, & 2\le i\le k\ \ and\end{cases}$

$f(u_{4,i})=4i-2,\quad for\ 1\le i\le k.$

Then the induced edge labeling f^* is obtained as follows.

$f^*(u_{1,i}\,u_{1,i+1})=\begin{cases}4k+4i+1, & 1\le i\le n-1\\ 4k+4i+2, & i=n,\end{cases}$

$f^*(u_{2,i}\,u_{2,i+1})=\begin{cases}4k+2, & i=1\\ 4k+4(i-1)+1, & 2\le i\le n,\end{cases}$

$f^*(v_{3,i}\,v_{3,i+1})=4i-1,\quad for\ 1\le i\le k-1,$

$f^*(v_{3,k}\,u_{2,1})=4k-1,$

$f^*(v_{4,i}\,v_{4,i+1})=4i+1,\quad for\ 1\le i\le k-1,$

$f^*(v_{4,k}\,u_{2,2})=4k+1,$

$f^*(v_{3,i}\,v_{4,i})=4i-2,\quad for\ 1\le i\le k,$

$f^*(u_{1,i}\,u_{2,i})=\begin{cases}4k+4i-1, & i=1\\ 4k+4i-2, & 2\le i\le n\\ 4k+4i-3, & i=n+1,\end{cases}$

$f^*(v_{3,2}\,v_{4,1})=4,$

$f^*(v_{3,i}\,v_{4,i-1})=4i-4,\quad for\ 3\le i\le k,$

$f^*(u_{2,1}\,u_{4,k})=4k,$

$f^*(u_{2,2}\,u_{3,k})=4k,$

$f^*(u_{1,i}\,u_{2,i+1})=4k+4i,\quad for\ 1\le i\le n,$

$f^*(v_{3,i}\,v_{4,i+1})=4i,\quad for\ 2\le i\le k-1\ \ and$

$f^*(u_{2,i}\,u_{1,i+1})=4k+4i,\quad for\ 2\le i\le n-1.$

Hence f is a C-root square mean labeling of $TBL(n,\alpha,k,\beta)$. Thus the graph $TBL(n,\alpha,k,\beta)$ is a C-root square mean graph, for any α in S^n with the first and last block are being $\downarrow$ and β in S^k, for all $k>0$.

Theorem 2.4. The diamond ladder graph Dl_n is a C-root square mean graph, for any $n\geq 1$.

Proof. The diamond ladder graph Dl_n is a connected graph with a vertex set
$V(Dl_n)=\{x_i,y_i:1\leq i\leq n\}\cup\{z_i:1\leq i\leq 2n\}$ and an edge set
$E(Dl_n)=\{x_ix_{i+1},y_iy_{i+1}:1\leq i\leq n-1\}\cup\{x_iy_i:1\leq i\leq n\}\cup\{x_iz_{2i-1},x_iz_{2i},y_iz_{2i-1},y_iz_{2i}:1\leq i\leq n\}\cup$
$\{z_{2i}z_{2i+1}:1\leq i\leq n-1\}$.
Thus $|V(Dl_n)|=4n$ and $|E(Dl_n)|=8n-3$.
Define $f:V(Dl_n)\to\{1,2,3,\cdots,8n-2\}$ as follows.

$$f(x_i)=8i-6,\quad for\ 1\leq i\leq n,$$

$$f(y_i)=\begin{cases}9i-5, & 1\leq i\leq 2\\ 8i-4, & 3\leq i\leq n\ \ and\end{cases}$$

$$f(z_i)=\begin{cases}3i-2, & 1\leq i\leq 3\ and\ i\ is\ odd\\ 4i-\left(\dfrac{(-1)^{i+1}+1}{2}\right)-2, & 2\leq i\leq 2n\ and\ i\ is\ even\\ 4i-\left(\dfrac{(-1)^{i+1}+1}{2}\right)-3, & 5\leq i\leq 2n\ and\ i\ is\ odd.\end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(x_ix_{i+1})=\begin{cases}8, & i=1\\ 8i-1, & 2\leq i\leq n-1,\end{cases}$$

$$f^*(y_iy_{i+1})=\begin{cases}10, & i=1\\ 8i+1, & 2\leq i\leq n-1,\end{cases}$$

$$f^*(x_iy_i)=8i-4,\quad for\ 1\leq i\leq n,$$

$$f^*(z_{2i}z_{2i+1})=\begin{cases}7, & i=1\\ 8i, & 2\leq i\leq n-1,\end{cases}$$

$$f^*(x_iz_{2i-1})=\begin{cases}7i-5, & 1\leq i\leq 2\\ 8i-6, & 3\leq i\leq n,\end{cases}$$

$$f^*(x_iz_{2i})=8i-3,\quad for\ 1\leq i\leq n,$$

$$f^*(y_iz_{2i-1})=8i-5,\quad for\ 1\leq i\leq n\ and$$

$$f^*(y_iz_{2i})=8i-2,\quad for\ 1\leq i\leq n.$$

Hence f is a C-root square mean labeling of G. Thus the diamond ladder graph Dl_n is a C-root square mean graph.

Theorem 2.5. Let $\{u_i v_i w_i u_i : 1 \le i \le n\}$ be a collection of n disjoint triangles. Let G be the graph obtained be joining w_i to w_{i+1}, $1 \le i \le n-1$ and joining u_i to u_{i+1} and v_{i+1}, $1 \le i \le n-1$. Then G is a C-root square mean graph.

Proof. The graph has G has $3n$ vertices and $6n-3$ edges respectively.
Define $f : V(G) \to \{1,2,3,\cdots,6n-2\}$ as follows.

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 7i-8, & 2 \le i \le 3 \\ 6i-4, & 4 \le i \le n, \end{cases}$$

$$f(v_i) = \begin{cases} 8i-7, & 1 \le i \le 2 \\ 6i-2, & 3 \le i \le n \quad and \end{cases}$$

$$f(w_i) = \begin{cases} 6i-2, & 1 \le i \le 2 \\ 14, & i = 3 \\ 6i-5, & 4 \le i \le n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i v_i) = \begin{cases} 2, & i = 1 \\ 7i-6, & 2 \le i \le 3 \\ 6i-2, & 4 \le i \le n, \end{cases}$$

$$f^*(v_i w_i) = \begin{cases} 3, & i = 1 \\ 6i-2, & 2 \le i \le 3 \\ 6i-3, & 4 \le i \le n, \end{cases}$$

$$f^*(u_i w_i) = \begin{cases} 5i-1, & 1 \le i \le 2 \\ 6i-4, & 3 \le i \le n, \end{cases}$$

$$f^*(w_i u_{i+1}) = \begin{cases} 6i, & 1 \le i \le 3 \\ 6i-1, & 4 \le i \le n-1, \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} 6i-1, & 1 \le i \le 3 \\ 6i, & 4 \le i \le n-1 \; and \end{cases}$$

$$f^*(u_i v_{i+1}) = 6i+1, \quad for\ 1 \le i \le n-1.$$

Hence f is a C-root square mean labeling of G.

3. Conclusion

In this paper, the C- root square meanness property of the triangular belt graph, slanting belt graph, diamond ladder graph and the graph $TBL(n, \alpha, k, \beta)$ are discussed. It is possible to investigate the C-root square meanness for other graphs.

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