

# Advancements in Precision Trajectory Generation: A Synergistic Approach to Design and Implementation for the 5-DOF Alpha-II Robot's Digital and Physical Twins

Sardar Muhammad Ali

Department of Electrical Engineering, (DEE)

Pakistan Institute of Engineering and Applied Sciences,  
Islamabad, Pakistan

[Alisardar0211@gmail.com](mailto:Alisardar0211@gmail.com) [bsee2052@pieas.edu.pk](mailto:bsee2052@pieas.edu.pk)

ORCID iD: <https://orcid.org/0009-0009-8585-4868>

## Abstract

This research paper explores the comprehensive design and verification process of a 5-DOF articulated robotic arm, named the Alpha-II. Leveraging SolidWorks and CoppeliaSim, the digital twin is meticulously developed, with kinematics and working envelope computations verified on CoppeliaSim and MATLAB. The transition to CoppeliaSim for testing involves the implementation of diverse control strategies, such as position control, velocity control, and torque control, to rigorously evaluate the manipulator's movements.

Highlighting the synergy between SolidWorks and CoppeliaSim, this literature emphasizes the versatility of SolidWorks in digital twin design and the effectiveness of CoppeliaSim's virtual environment. The research culminates in the creation of a physical twin, aligning with the meticulously crafted digital counterpart.

This study not only advances the understanding of trajectory generation for 5-DOF robotic arms but also underscores the pivotal role of cohesive digital and physical twin methodologies in cutting-edge robotics research. The rigorous verification, testing, and synthesis process contribute to the development of a refined and thoroughly validated end product, showcasing the significance of integrating simulation tools in the design and analysis of robotic systems.

## 2 Introduction

Articulated robotic arm manipulators, characterized by revolute joints at their major axes, represent a pivotal class of robots capable of achieving diverse and complex tasks. These robots possess the agility to move and position their end effectors across a broad operational range, making them particularly well-suited for intricate applications and tasks requiring access to confined or challenging spaces.

This research focuses on a scaled-down version of a five-axis articulated coordinate robotic arm manipulator, originally equipped with stepper motors as joint actuators. The primary objectives of this study center around the design and testing of a mechanical model using advanced engineering tools. SolidWorks is employed for the meticulous design of the mechanical model, ensuring accuracy and adherence to practical constraints. Subsequently, the design is seamlessly imported into CoppeliaSim for a comprehensive examination of the robot's movements and trajectories.

The intricacies of this research lie in the synthesis of precise design methodologies and advanced simulation technologies, with the ultimate goal of enhancing our understanding of the mechanical and dynamic aspects of the articulated robotic arm. Through this investigation, we aim to contribute valuable insights to the field of robotics, specifically addressing the design, simulation, and performance evaluation of articulated robotic systems. In Figure 1, a concise side view illustrates the structural layout of the Alpha-II Robot, highlighting the exclusive use of 5 rotary joints for articulated motion.

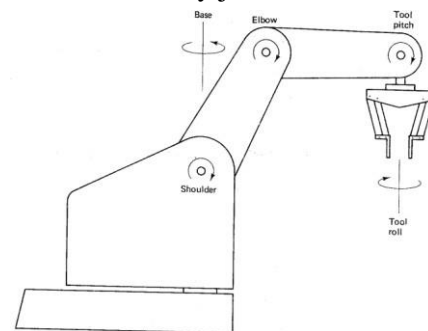


Figure 1: An Illustrative Side View of the Alpha-II Robot

### 3 Advanced Theoretical Framework for Robotic Arm Analysis

Initiating the development of robotic systems requires meticulous mathematical modeling. The preliminary step involves envisioning the robotic arm as a chain of rigid links connected by revolute joints. To ensure consistency across designs, coordinate frames are systematically assigned to each link, providing a standardized orientation. This critical foundation facilitates the derivation of a general arm equation, representing the kinematic motion of the manipulator's links. This theoretical framework sets the stage for an in-depth exploration of advanced concepts in robotic arm analysis, laying the groundwork for precise control system development and spatial maneuvering.

#### 1. Link Co-Ordinate Diagram:

In the preliminary stages of our design process, we adhered to the prescribed methodology by meticulously constructing a link coordinate diagram for the Alpha-II Robot. With a primary focus on rotary motion along its 5 degrees of freedom (5-DOF), the design exclusively incorporated 5 rotary joints, omitting prismatic joints from the system. The resultant link coordinate diagram, illustrated in Figure 2 below, encapsulates the foundational structure of the Alpha-II Robot, providing a visual representation of the interconnected rigid links and revolute joints critical for its articulated motion.

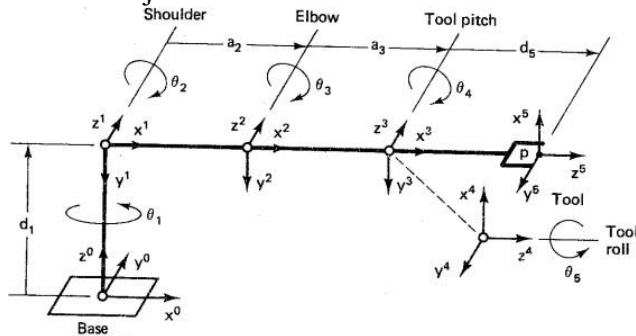


Figure 2: Link Co-Ordinate Diagram of Alpha-II Robot

#### 2. DH Parameters:

Following the construction of the link coordinate diagram for the Alpha-II Robot, our next step involved the systematic calculation of Denavit-Hartenberg (DH) parameters, crucial for the kinematic analysis of the robot. The tabular representation below details these parameters, capturing the spatial relationships between consecutive links and joints.

Our endeavor to enhance the original Alpha-II Robot design was guided by three pivotal constraints, instigating novel modifications in its structure and functionality:

- **Size Optimization:** Deliberately downsizing the robot by a factor of 3/2, key Denavit-Hartenberg (D-H) length parameters were adjusted to enhance overall dimensions and operational efficiency.
- **Revolutionary Actuation Mechanism:** Our design introduced a groundbreaking actuation mechanism, replacing direct motor placement with belts and pulleys. This strategic shift streamlined the design, ensuring improved precision and operational fluidity.
- **Targeted Functional Enhancement:** A pivotal focus in refinement was aligning the modified design with a specific functional requirement—proficiently picking up a 1cm x 1cm x 1cm cube. This enhancement showcases innovative strides, tailoring the Alpha-II Robot for specific and advanced operational objectives.

Axis	$\theta$	d (mm)	a(mm)	$\alpha$ (rad)	Home(rad)
1	q1	144	0	$-\pi/2$	0
2	q2	0	120	0	0
3	q3	0	120	0	0
4	q4	0	0	$-\pi/2$	$-\pi/2$
5	q5	85	0	0	0

Table 1: DH Parameters Table with Constraints

#### 3. Arm Matrix:

In response to constraints driving the refinement of our Alpha-II Robot, we innovatively modified the Denavit-Hartenberg table, birthing a new arm matrix. This recalibrated matrix, a direct result of our deliberate size reduction and groundbreaking actuation mechanism, mirrors the precision-driven enhancements in our design. The distinctly tailored entries underscore the transformative impact of our innovative approach, marking a significant leap forward in the kinematic framework of the Alpha-II Robot.

$$\begin{bmatrix} (S1 \cdot S5) + (C1 \cdot C5 \cdot C234) & (S1 \cdot C5) - (C1 \cdot S5 \cdot C234) & -C1 \cdot S234 & (0.12 \cdot C1 \cdot C2) + (0.12 \cdot C1 \cdot C23) - (0.085 \cdot C1 \cdot S234) \\ (C1 \cdot S5) + (S1 \cdot C5 \cdot C234) & (-C1 \cdot S5) - (S1 \cdot S5 \cdot C234) & -S1 \cdot S234 & (0.12 \cdot C2 \cdot S1) + (0.12 \cdot S1 \cdot C23) - (0.085 \cdot S1 \cdot S234) \\ -C5 \cdot S234 & S5 \cdot S234 & -C234 & 0.144 - (0.12 \cdot S2) - (0.120 \cdot S23) - (0.085 \cdot C234) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation 1: Transformative Arm Matrix Equation

## 4 Cutting-Edge Digital Twin Design in SolidWorks

### 1. Integrated Engineering Design Framework:

(Technological Synergy for Advanced Digital Twin Sub-Assemblies Development)

#### ❖ Base:

Before crafting the base, considerations were made to counteract the torque exerted by the links' weight. A metal base, featuring a wide area, ensures stability as it accommodates all link weight vectors, reducing the risk of toppling. The cylindrical housing in the center houses bearings and the base shaft for actuation purposes. A D-cut at the shaft's top end secures the waist link.

#### ❖ Waist Link:

The waist link seamlessly fits onto the base shaft, featuring a D-cut profile to match the base shaft. Aluminum base with acrylic side plates, easy to cut with laser technology, ensures structural integrity. Bearing housings and spacers maintain side plate alignment, mitigating the brittleness of acrylic.

#### ❖ Shoulder Link:

The shoulder link, comprising two acrylic side plates and a specially designed D-shaft coupler, cleverly optimizes space. Bearing housings and couplers facing away from each other enhance structural efficiency, accommodating the shaft of the elbow link.

#### ❖ Elbow Link:

Similar to the shoulder link, the elbow link employs elongated side plates matching the D-H parameter. It houses the tool pitch shaft and bearing housings through M3 screws and nuts.

#### ❖ Tool Roll:

A challenging design due to the integration of the SG90 servo, the tool roll assembly features an intricate coupling of the 8mm shaft and the SG90 servo via a market-found coupler. Metal gear variant MG90 servo resolves potential deformation issues. Acrylic plates house and fix the actuator, and a shaft passes through bearings for actuation.

#### ❖ The Gripper:

The most complex part of the project, the gripper integrates an SG90 servo for actuation. The design features claws with a geared profile meshed with an extra gear actuated by the servo. While challenging, the design ensures functional precision. A modified coupler connects the gripper to the tool roll shaft, completing the intricate Digital Twin sub-assemblies.

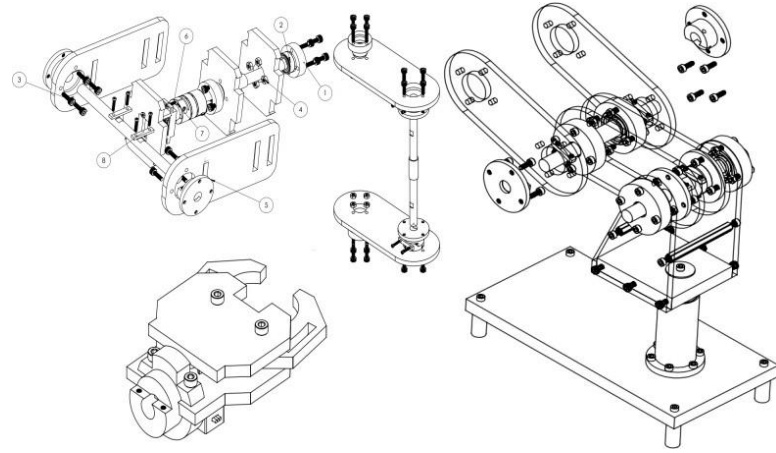


Figure 3: Integrated Engineering Design Framework - Digital Twin Sub-Assemblies

### 2. Finalized Assembly Design of Digital Twin Model:

(Optimized Precision through Refined Digital Twin Assembly)

The culmination of the Digital Twin design involved meticulous assembly, where each sub-assembly was imported and intricately pieced together. Precision optimization was the focal point during this phase, demanding careful consideration in mate selection. The chosen mates ensured that the links exclusively rotated about a fixed axis concerning the other links, eliminating any undesired movements.

This assembly process not only integrated the individual components seamlessly but also upheld the structural integrity and kinematic principles of the Alpha-II Robot. The resulting assembly stands as a testament to the precision achieved through the Digital Twin approach.

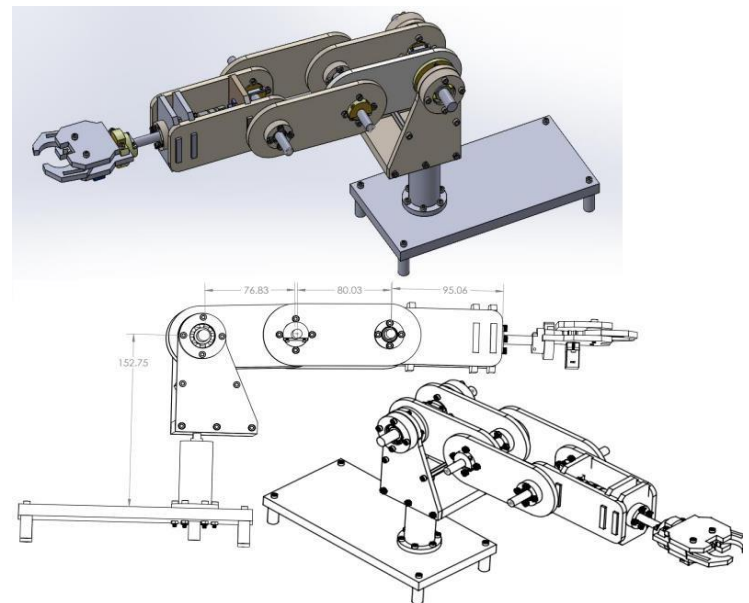


Figure 4: Finalized Assembly Design of Alpha-II Digital Twin Model



### 3. Advanced Kinematic Analysis:

(Evaluating Kinematics through Forward and Inverse Modeling)

#### i) Forward Kinematics (MATLAB & CoppeliaSim):

To validate the forward kinematics of our Alpha-II Robot, a comprehensive analysis was conducted using both Matlab and CoppeliaSim. The process commenced by determining the end effector's position for specific joint angles through the CoppeliaSim translation tool. This critical step laid the groundwork for subsequent assessments, ensuring the accuracy and reliability of the forward kinematics model.

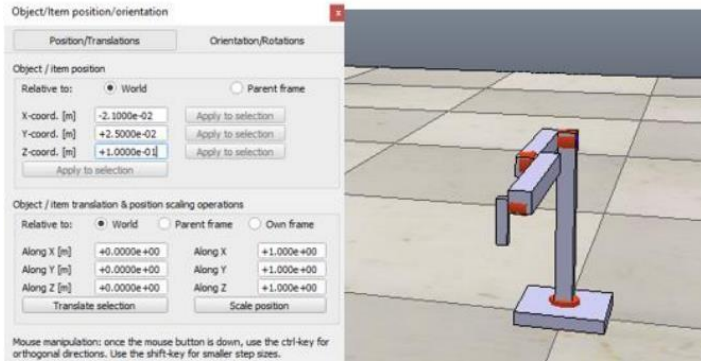


Figure 5: Spatial Configuration of Alpha-II Robot

Continuing the validation process, the fourth column of the arm equation played a pivotal role. The obtained joint angles from CoppeliaSim were seamlessly integrated into the equation, allowing for the verification of the end position. This meticulous step served as a crucial check, affirming the alignment between the calculated and observed end effector positions in our Alpha-II Robot.

$$\begin{bmatrix} (0.12 * C1 * C2) + (0.12 * C1 * C23) - (0.085 * C1 * S234) \\ (0.12 * C2 * S1) + (0.12 * S1 * C23) - (0.085 * S1 * S234) \\ 0.144 - (0.12 * S2) - (0.120 * S23) - (0.085 * C234) \end{bmatrix}$$

Equation 2: Arm Equation's Fourth Column

The outcomes of the MATLAB code closely align with the position determined in CoppeliaSim, providing a robust validation of our robot's forward kinematics. The meticulous comparison between MATLAB calculations and CoppeliaSim results establishes the credibility of our implemented model. Emphasizing the precision of tool tip position, it is crucial to acknowledge the impact of accurate DH parameters and joint angle measurements. The calibration of the robot and the use of precise measurement techniques are pivotal in ensuring the reliability of the obtained results.

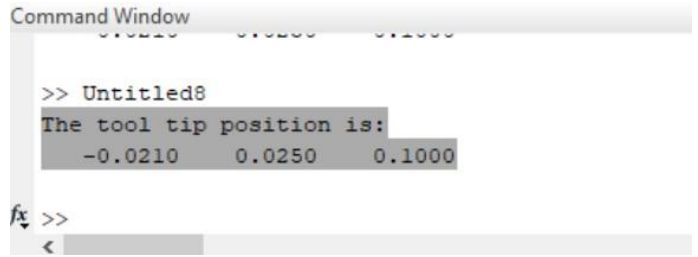


Figure 6: MATLAB Verification of Forward Kinematics

ii) **Inverse Kinematics:** The realm of mathematical modeling for a robotic arm manipulator necessitates a profound understanding of inverse kinematics, an indispensable aspect for control systems in trajectory planning. In the trajectory planning process, the focus shifts to tool tip space, and the inverse kinematic equations play a pivotal role.

In the context of the Alpha-II robot, these equations transform Cartesian points in space and tool orientation into the required joint angles for achieving the desired tool tip position. This intricate process ensures precision in trajectory planning, forming the backbone of the mathematical framework for our robot's control system. The subsequent section outlines the specific inverse kinematic equations tailored for the Alpha-II robot.

$$q1 = \tan^{-1}\left(\frac{w2}{w1}\right)$$

$$q5 = \pi \ln \sqrt{w4^2 + w5^2 + w6^2}$$

$$q234 = \tan^{-1}\left(\frac{-(w4 * C1) - (w5 * C1)}{-w6}\right)$$

$$q3 = \cos^{-1}\left(\frac{b1^2 + b2^2 - a_2^2 - a_3^2}{2a_2a_3}\right)$$

Equation 3: Inverse Kinematic Equations for Alpha-II Robot

Where,  $b1 = (C1 * w1) + (S1 * w2) + (d5 * S234)$  and  $b2 = -w3 + d1 - (d5 * C234)$  are known.

The derived inverse kinematic equations for the Alpha-II robot serve as foundational elements in the trajectory planning process. These equations, meticulously designed to map Cartesian points and tool orientation to joint angles, are seamlessly integrated with polynomials. This symbiotic fusion enables the comprehensive planning of the end effector's trajectory within tool tip space. The incorporation of polynomials adds a dynamic layer to the trajectory, allowing for precise & fluid movements,

essential for the Alpha-II robot's operational efficiency. This sophisticated combination forms the backbone of the trajectory planning mechanism, ensuring a harmonized & controlled motion of the robot's end effector.

$$\theta(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2t + 3a_3t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3t$$

Equation 4: Time-Dependent Joint Angle Trajectory Formulation

## 5 Optimized Digital Twin Trajectory Generation in CoppeliaSim & MATLAB

### 1. Digital Twin Model Import and Assimilation:

Following the meticulous design of the Digital Twin, the next crucial step involves the seamless importation and assimilation of this model into CoppeliaSim's digital virtual environment. This integration ensures a dynamic representation of the Alpha-II robot within the simulation platform.

To achieve this, the Digital Twin model undergoes conversion into a URDF (Unified Robot Description Format) file. This process involves establishing a robust parent-child hierarchy, a pivotal step for accurately reflecting the structural relationships within the robot. The resulting URDF file serves as the digital blueprint, allowing CoppeliaSim to emulate the intricate design and kinematics of the Alpha-II Digital Twin within its virtual realm.

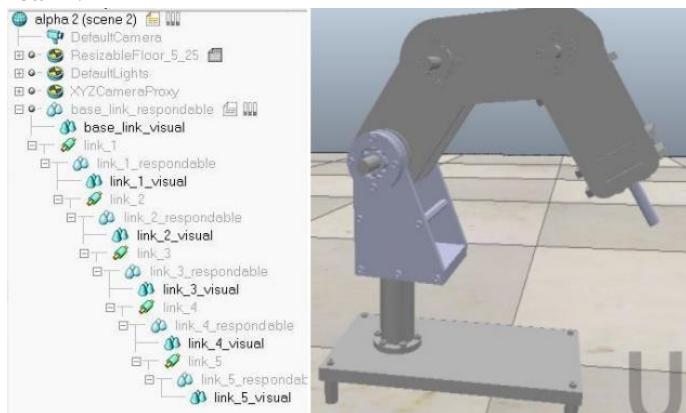


Figure 7: Imported Digital Twin Model in CoppeliaSim with Scene Hierarchy

### 2. Jacobian Matrix Application:

The Jacobian matrix emerges as a fundamental tool in our exploration of dynamic trajectory planning within the Digital Twin framework. In the context of the Alpha-II robot, the Jacobian matrix serves as a mathematical apparatus for mapping the relationship between joint velocities and end-effector velocities.

Employing matrix calculations, the Jacobian matrix becomes instrumental in understanding how changes in joint configurations impact the speed and direction of the end effector's motion. This dynamic insight is paramount for trajectory planning, allowing us to optimize and control the robot's movements within the Digital Twin environment. As we delve into matrix-based computations, the application of the Jacobian matrix stands as a cornerstone in enhancing the efficiency and precision of the Alpha-II robot's trajectory planning in the digital realm.

### 3. Digital Twin Workspace Envelope Exploration: (MATLAB)

Navigating the intricate workspace envelope of the Alpha-II robot within the Digital Twin domain is a nuanced endeavor. The inherent complexity of this envelope, resembling a donut-sphere with certain sections cut out, makes visualization and description challenging. In response, the power of MATLAB was leveraged to develop a dedicated code.

This MATLAB code serves as a computational solution to unveil the spatial limits of the workspace envelope. By systematically analyzing and calculating the dimensions, the code provides a comprehensive understanding of the operational reach of the Alpha-II robot within its Digital Twin domain. This exploration not only enhances our comprehension of the robot's capabilities but also contributes to refining its trajectory planning and overall performance in the digital environment.

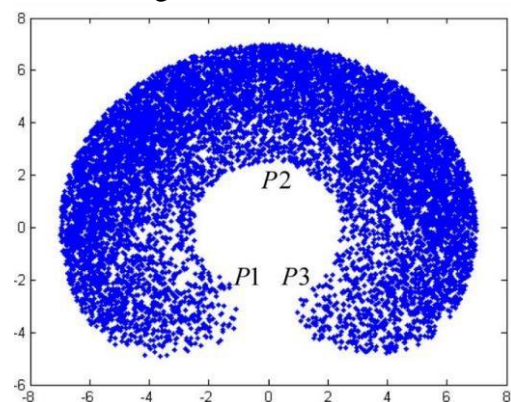


Figure 8: Top View of Alpha-II Digital Twin Workspace Envelope

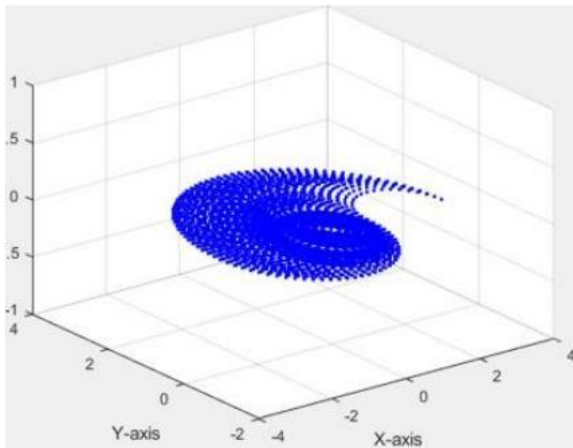


Figure 9: Isometric View of Alpha-II Digital Twin Workspace Envelope

#### 4. Advanced Digital Twin Precision Trajectory Generation:

In this research endeavor, we embark on a journey to elevate precision trajectory generation within the digital twin framework for the Alpha-II robot. Through the integration of cutting-edge methodologies, we delve into the intricacies of path planning, employing both CoppeliaSim and MATLAB.

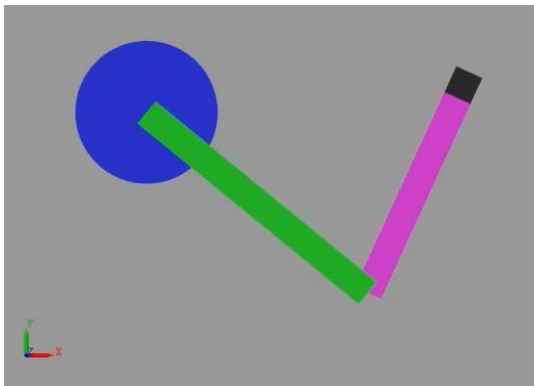


Figure 10: Trajectory Generation in CoppeliaSim: Dynamics of Alpha-II Robot's Path Planning in Digital Environment

Our exploration focuses on refining trajectory generation techniques, ensuring not only precision but also dynamism in the robot's movements. The fusion of innovative approaches in CoppeliaSim and MATLAB opens new avenues for enhanced control and adaptability. Join us as we navigate the convergence of advanced digital twin technologies, unveiling novel pathways for precise and dynamic trajectory generation in the field of robotics.

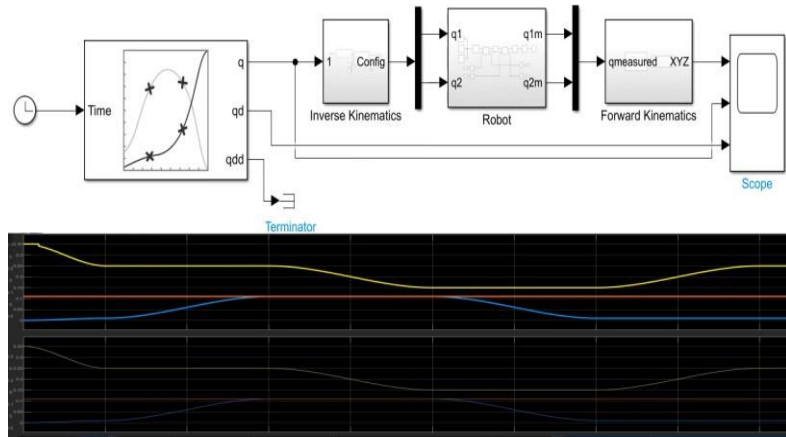


Figure 11: Polynomial Trajectory Block Implementation in MATLAB with Measured End Effector Position & Generated Trajectory Graph

## 6 Physical Twin Realization: Precision Alignment with the Crafted Digital Counterpart

The construction of the physical twin involved a meticulous selection of materials and components to ensure optimal performance and cost-effectiveness. The side walls of the links were crafted from a 5mm thick sheet of acrylic, chosen for its affordability and ease of precision cutting using a low-power laser cutter.

Aluminum emerged as the preferred material for the base, bearing housings, and shafts due to its lightweight nature and ease of machinability. Brass, an alloy of copper and zinc, was employed for the coupler, offering superior malleability and corrosion resistance.

Critical to the design, 628/8-2Z bearings were chosen for the 8mm shaft, renowned for their excellent load rating. Actuators, in the form of 9g servos, were selected for their suitability in managing the miniature scale of the model without compromising torque requirements. Throughout the assembly, M3 screws and nuts were utilized, chosen for their prevalence in the market. However, for mounting the 9g servo on acrylic plates, M2 screws were incorporated, addressing the specific constraints imposed by the servo's structural design. This comprehensive selection process ensures a harmonious integration of components, laying the foundation for the precise and efficient functioning of the physical twin.





Figure 12: Physical Twin Realization

## 7 Conclusion

In culmination, this research endeavors to push the boundaries of precision trajectory generation for the Alpha-II robot, seamlessly integrating cutting-edge digital twin methodologies. Through the meticulous design and simulation phases in CoppeliaSim and MATLAB, we achieve a refined digital twin that serves as the foundation for our physical twin realization.

The digital-to-physical transition involves careful material and component selection, ensuring the physical twin aligns precisely with its digital counterpart. This fusion of computational accuracy and tangible execution underscores the significance of cohesive digital and physical twin methodologies in robotics research.

Our exploration extends beyond theoretical frameworks, addressing practical challenges in component choices, construction, and alignment. The successful realization of a physical twin stands as a testament to the viability and efficacy of our approach.

As the robotic landscape advances, this research contributes not only to the understanding of trajectory generation for 5-DOF robotic arms but also emphasizes the pivotal role of integrated digital and physical twin strategies in shaping the future of robotics research and development.

## 8 References

- [1] <https://ieeexplore.ieee.org/document/5761886>
- [2] <https://ieeexplore.ieee.org/document/9649858>
- [3] <https://ieeexplore.ieee.org/document/8925317>
- [4] <https://ieeexplore.ieee.org/abstract/document/8311944>
- [5] [\[2306.13617\] CIDGIKc: Distance-Geometric Inverse Kinematics for Continuum Robots](#) ([arxiv.org](https://arxiv.org))
- [6] [Hybrid Trajectory Planning for Autonomous Driving in Highly Constrained Environments | IEEE Journals & Magazine | IEEE Xplore](#)
- [7] [Mobile Manipulation in Unknown Environments with Differential Inverse Kinematics Control | IEEE Conference Publication | IEEE Xplore](#)
- [8] [A Real-Time Interactive Augmented Reality Depth Estimation Technique for Surgical Robotics | IEEE Conference Publication | IEEE Xplore](#)
- [9] [\(PDF\) Line-of-Sight Based 3D Localization of Parallel Kinematic Mechanisms \(researchgate.net\)](#)
- [10] <https://ieeexplore.ieee.org/document/10171749>