# Algorithm for Generation of all the Prime Numbers between $n^2$ and $(n+2)^2$ where n is an Odd Number

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**Abstract:**

Vijay Vilas Jadhav (means I) have invented new algorithm for generation of all the prime numbers between $n^2$ and $(n+2)^2$. This algorithm successfully generates all the prime numbers for any odd positive integer n except the value of n as 1.

**Keywords:** Prime numbers, modulo operation, even numbers, odd numbers, primality check

**Introduction:**

This algorithm is based on the basic simple method of checking primality of any odd number. We take square root of a given number and round it off to the nearest integer and divide the given number by all the odd numbers (except 1) up to the square root of the given number.
For example: To check the primality of number 33.
First, we take the square root of the 33 which is 5.74456 (approximately)
Now we divide 33 by all the odd numbers less than or equal to 5 (we rounded off 5.744 to the nearest positive odd integer) one by one, except the odd number 1.
This is how we perform the basic simple primality check of any odd number.

**Concept and Method:**

This algorithm uses above mentioned basic primality checking method along with modulo operation to generate the prime numbers.

**Algorithm:**

**Step1:**

Take any odd number 'n' (except value of n as 1). For example:
n=5;
therefore $n^2$=25

**Step2:**

Now do the modulo operation on $n^2$ by all the odd numbers (except odd number 1) till n+2, one byone.
For example:
In this example for n = 525 mod 3= 1
25 mod 5=0
25 mod 7= 4

**Step3:**

Now if we subtracted the result of the respective modulo operation from the respective divisor and this result of subtraction is added to the $n^2$ then the number formed is completely divisible by the divisor of the respective modulo operation.
**[$n^2$ + (divisor of respective mod operation - result of respective mod operation)]** ……equation(1) equation (1) is completely divisible by the divisor of the respective mod operation.

For example: In case of above example 25 mod

3=1

then,
3-1=2 ….(3 is divisor and 1 is result of respective modulo operation)

Here,
2 is the result of subtraction and it is the first number we generated.

Therefore,
From equation (1), we have25+2=27
which completely divisible by 3 (divisor of the respective modulo operation).

Now, if we add the divisor to the first generated number again, we will get the second number as 5 and when we will add this second number to the $n^2$, the number formed is completely divisible by the divisor.
For example: 25 +5=30 …… (completely divisible by divisor that is 3 in this case)
Here 5 is the second number generated by adding the divisor to the first number. Similarly, we can generate the third number by adding divisor to the second generated number and generate other next numbers by same process.

Thus, when we add the second number 5 to the $n^2$, the result is completely divisible by divisor. Again, we add 3 (that is divisor) to the second number and generate the third number as 8 and when we add this third number (that is 8) to the $n^2$ the result is completely divisible by the divisor (that is 3).

Keep repeating this process and we have to keep the track of the all the newly generated numbers till only the value of the generated numbers is less than or equal to the value of **$((n+2)^2-n^2)$.**

In this example, we have taken n as 5, So, the value of $((n+2)^2-n^2)$ is
49-25=24

For example:
2; …..(the first number) 2+3=5;
…..(the second number) 5+3=8;
…..(the third number) 8+3=11;
…..(the fourth number) 11+3=14;
…..(the fifth number) 14+3=17; …..(the sixth number) 17+3=20; …..(the seventh number)
20+3=23; …..(the eighth number)
( note that 23 is the last number in the track of the numbers for 3 as divisor since the value of $((n+2)^2-n^2)$ is 24 for this example)

Track of the numbers: 2,5,8,11,14,17,20,23

**Step4:**

Repeat the Step3 for all other modulo operations for other consecutive odd numbers (as divisor) less than or equal to (n+2) and keep the track of the only even numbers from the track of the numbers generated.

Track of the generated numbers for 25 mod 3 is
2,5,8,11,14,17,20,23

Out of these numbers
Track of the only even numbers is 2,8,14,20

Similarly, for25 mod 5=0
Track of generated numbers is 5,10,15,20 Track of the only even numbers is 10,20

Similarly, for25 mod 7=4
Track of generated numbers is 3,10,17,24 Track of the only even numbers is 10,24

**Step5:**

Now write down all the even numbers till the value of the $((n+2)^2-n^2)$ For example:

Here, in our example for n=5
$(5+2)^2-5^2=24$

All the even numbers till 24 except 0 are 2,4,6,8,10,12,14,16,18,20,22,24

**Step6:**

Now, we will eliminate all the common even numbers from this list of all even numbers till the value of $((n+2)^2-n^2)$ by comparing the even numbers in our lists of tracks of even numbers for each modulo operation.

For example:
In this example
Track of even numbers for 25 mod 3: 2,8,14,20
Track of even numbers for 25 mod 5: 10,20
Track of even numbers for 25 mod 7: 10,24

Now we will eliminate the common even numbers from the track of even numbers till the value of $((n+2)^2-n^2)$:
~~**2**~~,4,6,~~**8**~~,~~**10**~~,12,~~**14**~~,16,18,~~**20**~~,22,~~**24**~~

Remaining numbers:
4,6,12,16,18,22

**Step7:**

When you add each Remaining number to the $n^2$ and generate a new number the generated number is a definite prime number.

For example:
In this example:

25+4=**29**
25+6=**31**
25+12=**37**
25+16=**41**
25+18=**43**

25+22=**47**

Thus, we have successfully generated all the prime numbers for n=5

**Results:**

By using basic simple primality check method for any odd positive integer and modulo operation concept this algorithm successfully generates all the prime numbers between $n^2$ and $(n+2)^2$ where n is an any odd number except 1.

**Conclusion:**

This algorithm will help in generation of all the definite prime numbers in particular range or window. We can increase the window by adding even number greater than 2 to the n that means we can find all the prime numbers between $n^2$ and $(n + m)^2$ where m is an even number and n is an odd number (except 1).

**References:**

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