An Application of Discrete Two Parameter Singular Perturbation Method

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Abstract: Discrete Two Parameter Singular Perturbation Method is extended up to second-order approximation. A fifth order discrete power system model with three time scales is considered. It is modeled as a two parameter singularly perturbed system. Then an Initial value Problem (IVP) is studied using this singular perturbation method (SPM). SPM consists of an outer series solution and two boundary layer correction (BLC) solutions. Boundary layer corrections are required to recover the initial conditions lost in the process of degeneration. SPM is carried out up to second-order approximate solution and these results are compared with the exact solution. The results substantiate the application.

Index Terms: Time scales, discrete two parameter system, singular perturbation method, Initial value Problem

1. Introduction

The dynamics of many continuous-time and discrete-time systems is described by high order differential and difference equations respectively. The presence of small parameters such as time constants, masses, moments of inertia, inductances and capacitances is the source for increased order of the system. The solution of these high order stiff systems poses a problem and requires special numerical methods. Singular Perturbation Methodology (SPM) alleviates these problems by suppressing the small parameters. More specifically, SPM removes system's stiffness, reduces the order of the system, satisfies the given boundary conditions and gives an approximate solution closer to the exact solution.

A system in which the suppression of a small parameter results in degeneration of the dimension of the system is called a singularly perturbed system. Such a system possesses widely separated groups of eigenvalues exhibiting slow and fast phenomena or time-scale phenomena. Singular perturbed and time-scale systems are synonymous [5]. SPM in continuous control systems has matured enough [1-4]. SPM in discrete control systems is being developed [2, 5-13] and its applications are not thoroughly explored. To fill that gap, here a fifth order discrete power system model [14] with three time scales is considered. It is modeled as a two parameter singularly perturbed system. Then an IVP is studied using the SPM extended up to second-order approximation.

2. Discrete Two-Parameter Singular Perturbation Method (DTPSPM)

The two-parameter and multi-parameter problems in discrete systems are studied extensively [6-8, 10-12]. Here we overview and extend the DTPSPM up to second-order approximation in state variable form from control view. Consider a three-time-scale linear time-invariant stable system described by

$$\begin{bmatrix} x_0\,(k+1) \\ x_j\,(k+1) \end{bmatrix} = \mathrm{A} \begin{bmatrix} x_0\,(k) \\ \mu_j x_j\,(k) \end{bmatrix} + \mathrm{B}\ \mathrm{u}(k) \qquad (1)$$

where j=1,2;

$$\mathrm{A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{22} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}, \mathrm{B} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

and

state vector $x_{j-1}\,(k) \in \Re^{n_j}$, j=1, 2, 3.

A_{ij} and B_i are matrices of approximate dimensionality μ_1, μ_2. are the small positive scalar parameters related, such that ($\mu_j/\mu_{j-1}) \to +0$, as $\mu_j \to +0, j = 2$. The control vector u(k) $\in \Re^r$ is independent of the small parameters. Redefining the new parameters as $\varepsilon_1 = \mu_1$ and $\varepsilon_2 = \frac{\mu_2}{\mu_1}$, Eq. (1) may be written as

$$\begin{bmatrix} x_0\,(k+1) \\ x_1\,(k+1) \\ x_2\,(k+1) \end{bmatrix} = \mathrm{A} \begin{bmatrix} x_0\,(k) \\ {}_1x_1\,(\ ) \\ \varepsilon_1\varepsilon_2 x_2\,(k) \end{bmatrix} + \mathrm{B}\ \mathrm{u}(k) \qquad (2a)$$

With initial conditions

$x_j(k=0) = x_j(0)$, j = 0,1,2 (2b)

(2) gives the general form of singularly perturbed two-parameter discrete control system

When ε_2 is made equal to zero in (2), $x_2(0)$ is lost and when ε_1 and ε_2 are neglected $x_1(0)$ and $x_2(0)$ are sacrificed. In other words, when the small parameters ε_1 and ε_2 are suppressed in (2a), the resulting degenerate subsystem

$$\begin{bmatrix} x_0^{00}(k+1) \\ x_1^{00}(k+1) \\ x_2^{00}(k+1) \end{bmatrix} = \mathrm{A} \begin{bmatrix} x_0^{00}(k) \\ 0 \\ 0 \end{bmatrix} + \mathrm{B}\ \mathrm{u}(k) \qquad (3a)$$

is of reduced order n_0 and cannot satisfy all the given initial conditions (2b). Hence the discrete system (2) is in the singularly perturbed form. That is

$$x_0^{00}(0) = x_0(0);\ \ x_j^{00}(0) \neq x_j(0),\ j=1,2. \qquad (3b)$$

The (n_1+n_2) initial condition lost in the process of degeneration are recovered by the following SPM.

2.1 Singular perturbation method for IVP

(i) *Outer solution*

As a first step towards the proposed perturbation method for (2), double asymptotic power series expansions are assumed for outer series solution

$x_v(k)=\sum_{i,j=0}^{q} x_v^{ij}(k)\varepsilon_1^i\varepsilon_2^j$, v=0, 1, 2. (4) where q is the order of the desired approximation.

By substituting (4) in (2a) and equating coefficients of like powers of $\varepsilon_1^i\varepsilon_2^j$, we obtain a set of equations. For the zero-order approximation ($\varepsilon_1^0\varepsilon_2^0$) the resulting equation is the same as that given by (3). For the first-order approximation,

$$\varepsilon_1^1:\ \begin{bmatrix} x_0^{10}(k+1) \\ x_1^{10}(k+1) \\ x_2^{10}(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^{10}(k) \\ x_1^{00}(k) \\ 0 \end{bmatrix} \tag{5a}$$

$$\varepsilon_2^1:\ \begin{bmatrix} x_0^{01}(k+1) \\ x_1^{01}(k+1) \\ x_2^{01}(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^{01}(k) \\ 0 \\ 0 \end{bmatrix} \tag{5b}$$

For the second-order approximation

$$\varepsilon_1^1\varepsilon_2^1:\ \begin{bmatrix} x_0^{11}(k+1) \\ x_1^{11}(k+1) \\ x_2^{11}(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^{11}(\ ) \\ x_1^{01}(k) \\ x_2^{00}(k) \end{bmatrix} \tag{5c}$$

$$\varepsilon_1^2:\ \begin{bmatrix} x_0^{20}(k+1) \\ x_1^{20}(k+1) \\ x_2^{20}(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^{20}(k) \\ x_1^{10}(k) \\ 0 \end{bmatrix} \tag{5d}$$

$$\varepsilon_2^2:\ \begin{bmatrix} x_0^{02}(k+1) \\ x_1^{02}(k+1) \\ x_2^{02}(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^{02}(k) \\ 0 \\ 0 \end{bmatrix} \tag{5e}$$

Similar equations can be easily formed for higher-order approximations.

(ii) *Boundary layer correction solutions*

In order to recover the initial conditions lost in the process of degeneration and to provide the necessary initial data for solving outer equation (5), the following transformations are used.

$$x_{0C1}(\mathrm{k})=\frac{x_0(k)}{\varepsilon_1^{k+1}}; \tag{6a}$$

$$x_{0C2}(k)=\frac{x_0(k)}{\varepsilon_1^{k+1}\varepsilon_2^{k+1}}; \tag{6b}$$

$$x_{1C1}(\mathrm{k})=\frac{x_1(k)}{\varepsilon_1^{k}}; \tag{6c}$$

$$x_{1C2}(k)=\frac{}{\varepsilon_1^k\varepsilon_2^{k+1}}\ ; \qquad (6d)$$

$$x_{2C1}(k)=\frac{x_2(k)}{\varepsilon_1^k}; \qquad (6e)$$

$$x_{2C2}(k)=\frac{x_2(k)}{\varepsilon_1^k\varepsilon_2^k} \qquad (6f)$$

where (x_{01}, x_{11}, x_{21}) refers to the BLC 1 and (x_{02}, x_{12}, x_{22}) refers to the BLC 2.
Using (6) in (2) the subsystem for the BLC 1 is given by

$$\begin{bmatrix}\varepsilon_1 x_{0c1}(k+1)\\ x_{1c1}(k+1)\\ x_{2c1}(k+1)\end{bmatrix}=A\begin{bmatrix}x_{0c1}(k)\\ x_{1c1}(k)\\ \varepsilon_2 x_{2c1}(k)\end{bmatrix} \qquad (7a)$$

and the subsystem for the BLC 2 is given by

$$\begin{bmatrix}\varepsilon_1\varepsilon_2 x_{0c2}(k+1)\\ \varepsilon_2 x_{1c2}(k+1)\\ x_{2c2}(k+1)\end{bmatrix}=A\begin{bmatrix}x_{0c2}(k)\\ x_{1c2}(k)\\ x_{2c2}(k)\end{bmatrix} \qquad (7b)$$

Double asymptotic power series expansions are implemented for BLC

$$x_{vcs}(k) = \sum_{i,j\geq 0}^{q}[x_{vcs}^{ij}(k)]\,\varepsilon_1^i\varepsilon_2,\quad v = \ ,1,2.\ s = 1,2. \qquad (8)$$

(a) *BLC 1*

Substituting (8) in (7a) and collecting coefficients gives for the zero-order approximation

$$\varepsilon_1^0\varepsilon_2^0:\ \begin{bmatrix}0\\ x_{1C1}^{00}(k+1)\\ x_{2C1}^{00}(k+1)\end{bmatrix}=A\begin{bmatrix}x_{0C1}^{00}(k)\\ x_{1C1}^{00}(k)\\ 0\end{bmatrix} \qquad (9)$$

For the first-order approximation

$$\varepsilon_1^1:\ \begin{bmatrix}x_{0C1}^{00}(k+1)\\ x_{1C1}^{10}(k+1)\\ x_{2C1}^{10}(k+1)\end{bmatrix}=A\begin{bmatrix}x_{0C1}^{10}(k)\\ x_{1C1}^{10}(k)\\ 0\end{bmatrix} \qquad (10a)$$

$$\varepsilon_2^1:\ \begin{bmatrix}0\\ x_{1C1}^{01}(k+1)\\ x_{2C1}^{01}(k+1)\end{bmatrix}=A\begin{bmatrix}x_{0C1}^{01}(k)\\ x_{1C1}^{01}(k)\\ 0\end{bmatrix} \qquad (10b)$$

For the second-order approximation

$$\varepsilon_1^1\varepsilon_2^1:\ \begin{bmatrix}x_{0C1}^{01}(k+1)\\ x_{1C1}^{11}(k+1)\\ x_{2C1}^{11}(k+1)\end{bmatrix}=A\begin{bmatrix}x_{0C1}^{11}(k)\\ x_{1C1}^{11}(k)\\ x_{2C1}^{10}(k)\end{bmatrix} \qquad (11a)$$

$$\varepsilon_1^2:\; x^{20}(k+1) = A\; x^{20}(k) \qquad (11b)$$
$(k+$

$$\varepsilon_2^2:\; \begin{bmatrix} 0 \\ x_{1\,C1}^{02}(k+1) \\ x_{2C1}^{02}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C1}^{02}(k) \\ x_{1C1}^{02}(k) \\ x_{2C1}^{01}(k) \end{bmatrix} \qquad (11c)$$

(b) *BLC 2*

Inserting (8) in (7b) and collecting coefficients, we get for the zero-order approximation

$$\varepsilon_1^0\varepsilon_2^0:\; \begin{bmatrix} 0 \\ 0 \\ x_{2C2}^{00}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C2}^{00}(k) \\ x_{1C2}^{00}(k) \\ x_{2C2}^{00}(k) \end{bmatrix} \qquad (12)$$

For the first-order approximation

$$\varepsilon_1^1:\; \begin{bmatrix} 0 \\ 0 \\ x_{2C2}^{10}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C2}^{10}(k) \\ x_{1C2}^{10}(k) \\ x_{2C2}^{10}(k) \end{bmatrix} \qquad (13a)$$

$$\varepsilon_2^1:\; \begin{bmatrix} 0 \\ x_{1\,C2}^{00}(k+1) \\ x_{2C2}^{01}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C2}^{01}(k) \\ x_{1C2}^{01}(k) \\ x_{2C2}^{01}(k) \end{bmatrix} \qquad (13b)$$

For the second-order approximation

$$\varepsilon_1^1\varepsilon_2^1:\; \begin{bmatrix} x_{0C2}^{00}(k+1) \\ x_{1C2}^{10}(k+1) \\ x_{2C2}^{11}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C2}^{11}(k) \\ x_{1C2}^{11}(k) \\ x_{2C2}^{11}(k) \end{bmatrix} \qquad (14a)$$

$$\varepsilon_1^2:\; \begin{bmatrix} 0 \\ 0 \\ x_{2C2}^{20}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C2}^{20}(k) \\ x_{1C2}^{20}(k) \\ x_{2C2}^{20}(k) \end{bmatrix} \qquad (14b)$$

$$\varepsilon_2^2:\; \begin{bmatrix} 0 \\ x_{1\,C2}^{01}(k+1) \\ x_{2C2}^{02}(k+1) \end{bmatrix} = A \begin{bmatrix} x_{0C2}^{02}(k) \\ x_{1C2}^{02}(k) \\ x_{2C2}^{02}(k) \end{bmatrix} \qquad (14c)$$

For higher-order approximations, similar equations may be obtained.

(iii) *Total series solution (TSS)*

The total series solution is given by the outer solution (4) and the two BLC solutions (8) as

$$x_0^q(k)\; \sum_{i,j\geq 0}^{q} [x_0^{ij}(k) + \varepsilon_1^{k+1} x_{0C1}^{ij}(k)\; + \varepsilon_1^{k+1}\varepsilon_2^{k+1} x_{0C2}^{ij}(k)]\varepsilon_1^i\varepsilon_2^j \qquad (15a)$$

$$x_1^q(k)\; \sum_{i,j\geq 0}^{q} [x_1^{ij}(k) + \varepsilon_1^{k} x_{1\,C1}^{ij}(k)\; + \varepsilon_1^{k}\varepsilon_2^{k+1} x_{1C2}^{ij}(k)]\varepsilon_1^i\varepsilon_2^j \qquad (15b)$$

$$x_2^q(k)\; \sum_{i,j\geq 0}^{q} [x_2^{ij}(k) + \varepsilon_1^{k} x_{2C1}^{ij}(k)\; + \varepsilon_1^{k}\varepsilon_2^{k} x_{2C2}^{ij}(k)]\varepsilon_1^i\varepsilon_2^j \qquad (15c)$$

where q is the order of the desired approximation.

(iv) Initial conditions

The determination of the necessary initial conditions for the solution of the outer equations (3) and (5) and the BLC equations (9) - (14) is a vital step in singular perturbation analysis. These are determined based on the fact that the total series solution (15) consisting of outer and BLC solutions should satisfy the given initial conditions. Then the following relations are obtained. For zero-order approximation

$$\varepsilon_1^0\varepsilon_2^0:\ x_0^{00}(0) = x_0(0) \tag{16a}$$

$$\varepsilon_1^0\varepsilon_2^0:\ x_{1C1}^{00}(0) = x_1(0) - x_1^{00}(0) \tag{16b}$$

$$\varepsilon_1^0\varepsilon_2^0:\ x_{2C2}^{00}(0) = x_2(0) - x_2^{00}(0) - x_{2C1}^{00}(0) \tag{16c}$$

For first- order approximation

$$\varepsilon_1^1:\ x_0^{10}(0) = -x_{0C1}^{00}(0) \tag{17a}$$

$$\varepsilon_2^1:\ x_0^{01}(0) = 0 \tag{17b}$$

$$\varepsilon_1^1:\ x_{1C1}^{10}(0) = -x_1^{10}(0) \tag{17c}$$

$$\varepsilon_2^1:\ x_{1\,C1}^{01}(0) = -x_1^{01}(0) - x_{1C2}^{00}(0) \tag{17d}$$

$$\varepsilon_1^1:\ x_{2C2}^{10}(0) = -x_2^{10}(0) - x_{2C1}^{10}(0) \tag{17e}$$

$$\varepsilon_2^1:\ x_{2C2}^{01}(0) = -x_2^{01}(0) - x_{2C1}^{01}(0) \tag{17f}$$

For second - order approximation

$$\varepsilon_1^1\varepsilon_2^1:\ x_0^{11}(0) = -x_{0C1}^{01}(0) - x_{0C2}^{00}(0) \tag{18a}$$

$$\varepsilon_1^2:\ x_0^{20}(0) = -x_{0C1}^{10}(0) \tag{18b}$$

$$\varepsilon_2^2:\ x_0^{02}(0) = 0 \tag{18c}$$

$$\varepsilon_1^1\varepsilon_2^1:\ x_{1C1}^{11}(0) = -x_1^{11}(0) - x_{1C2}^{10}(0) \tag{18d}$$

$$\varepsilon_1^2:\ x_{1C1}^{20}(0) = -x_1^{20}(0) \tag{18e}$$

$$\varepsilon_2^2:\ x_{1\,C1}^{02}(0) = -x_1^{02}(0) - x_{1\,C2}^{01}(0) \tag{18f}$$

$$\varepsilon_1^1\varepsilon_2^1:\ x_{2\,C2}^{11}(0) = -x_{2\,C1}^{11}(0) - x_2^{11}(0) \tag{18g}$$

$$\varepsilon_1^2:\ x_{2\,C2}^{20}(0) = -x_2^{20}(0) - x_{2\,C1}^{20}(0) \tag{18h}$$

$$\varepsilon_2^2:\ x_{2\,C2}^{02}(0) = -x_2^{02}(0) - x_{2\,C1}^{02}(0) \tag{18i}$$

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(v) Algorithm

A close examination of the selected initial conditions suggests the algorithm. For a particular order of approximate solution, first the outer solution is to be found. Next, add the BLC corresponding to the least singular transformation. Continuing this process, finally add the BLC corresponding to the most singular transformation. Note that proper care should be taken in evaluating the zero-, first- , second- and higher- order approximate solutions by adding corrections in an appropriate manner depending on the value of k.

3. Application of DTPSPM

Consider a fifth order power system model sampled with 0.7s [14]. The resulting system is given by

$$\begin{bmatrix} x_0(k+1) \\ x_1(k \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.9147 & 0.0253 & 0.0125 & 0.0075 & 0.0051 \\ -0.0602 & 0.8893 & -0.0003 & 0.0456 & 0.0295 \\ -0.0195 & 0.7016 & 0.2465 & 0.0209 & 0.0192 \\ -1.4300 & -0.0219 & -0.0138 & 0.2399 & -0.0063 \\ -1.1124 & -0.0125 & -0.0089 & 0.3388 & 0.0259 \end{bmatrix} \begin{bmatrix} x_0(k) \\ \cdots \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0.0097 \\ . \\ 0.0198 \\ 1.5006 \\ 1.1545 \end{bmatrix} u(k) \qquad (19a)$$

The eigenspectrum of this system

(0.8928 + 0.0937i, 0.8928 - 0.0937i, 0.2506 + 0.0251i, 0.2506 - 0.0251i, 0.0296)

clearly indicates three-time-scale nature. Hence it is represented as a two-parameter system as shown below.

$$\begin{bmatrix} x_0(k+1) \\ \cdots \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.9147 & 0.0253 & 0.0625 & 0.0375 & 0.2550 \\ -0.0602 & 0.8893 & -0.0015 & 0.2280 & 1.4750 \\ -0.0195 & 0.7016 & 1.2325 & 0.1045 & 0.9600 \\ -1.4300 & -0.0219 & -0.0690 & 1.1995 & -0.3150 \\ 1.1124 & 0.0125 & -0.0445 & 1.6940 & 1.2950 \end{bmatrix} \begin{bmatrix} x_0(k) \\ x_1 \cdots \\ \varepsilon_1 x_2(k) \\ \varepsilon_1 x_3(k) \\ \varepsilon_1 \varepsilon_2 x_4(k) \end{bmatrix} + \begin{bmatrix} 0.0097 \\ . \\ 0.0198 \\ 1.5006 \\ 1.1545 \end{bmatrix} u(k)$$

where $\varepsilon_1 = 0.2$; $\varepsilon_2 = 0.1$. (19b)

The initial conditions are given as

$x_0(0) = 2$; $x_1(0) = -1$; $x_2(0) = 1$; $x_3(0) = 5$; $x_4(0) = -4$

This IVP is solved using the SPM given in Section 2. The solutions for zero, first, second-order approximations are obtained and compared with the exact solution as shown in the Table 1. From the table we observe that

(i) The degenerate solution, obtained by making ε_1 and ε_2 equal to zero in (19b), is unable to satisfy the initial conditions $x_2(0)$, $x_3(0)$ and $x_4(0)$.

(ii) The zero-order solution, obtained from (15), incorporates BLCs and hence it recovers the initial conditions $x_2(0)$, $x_3(0)$ and $x_4(0)$. Thereafter, i.e., $k \geq 1$, it remains equal to the degenerate solution.

(iii) The first-order solution improves the zero-order solution and is closer to the exact solution.

(iv) The second-order solution improves the first-order solution and is much closer to the exact solution.

x(k)	Degenerate Solution	Zero- order Solution	First- order Solution	Second-order Solution	Exact Solution
$x_0(0)$	2	2	2	2	2
$x_1(0)$	-1	-1	-1	-1	-1
$x_2(0)$	-0.7563	1	1	1	1
$x_3(0)$	-1.6293	5	5	5	5
$x_4(0)$	-1.2847	-4	-4	-4	-4
$x_0(1)$	1.8138	1.8138	1.8637	1.8603	1.8434
$x_1(1)$	-0.9481	-0.9481	-0.7204	-0.7199	-0.8384
$x_2(1)$	-0.7208	-0.7208	-0.3698	-0.3659	-0.4466
$x_3(1)$	-1.3373	-1.3373	0.2052	0.1995	-0.1266
$x_4(1)$	-1.0578	-1.0578	-2.1483	-2.0963	0.5137
$x_0(2)$	1.6448	1.6448	1.6772	1.6756	1.6708
$x_1(2)$	-0.8907	-0.8907	-0.7519	-0.7543	-0.7852
$x_2(2)$	-0.6808	-0.6808	-0.7276	-0.7201	-0.7070
$x_3(2)$	-1.0722	-1.0722	-1.4595	-1.4501	-1.1446
$x_4(2)$	-0.8213	-0.8513	-1.3563	-1.3499	-0.9110
$x_0(3)$	1.4917	1.4917	1.5082	1.5069	1.4960
$x_1(3)$	-0.8295	-0.8295	-0.7567	-0.7573	-0.8161
$x_2(3)$	-0.6372	-0.6372	-0.7307	-0.7513	-0.7763
$x_3(3)$	-0.8318	-0.8318	-1.1289	-1.1301	-1.1305
$x_4(3)$	-0.6640	-0.6640	-1.0692	-1.0786	-1.0994
$x_0(4)$	1.3531	1.3531	1.3559	1.3522	1.3336
$x_1(4)$	-0.7659	-0.7659	-0.7399	-0.7301	-0.8379
$x_2(4)$	-0.5913	-0.5913	-0.7150	-0.7936	-0.8188
$x_3(4)$	-0.6141	-0.6141	-0.8301	-0.8591	-0.8744
$x_4(4)$	-0.4945	-0.4945	-0.7899	-0.8592	-0.9340
$x_0(5)$	1.2280	1.2280	1.2192	1.2000	1.1870
$x_1(5)$	-0.7010	-0.7010	-0.5489	-0.6091	-0.8302
$x_2(5)$	-0.5440	-0.5440	-0.6844	-0.7316	-0.8316
$x_3(5)$	-0.4174	-0.4174	-0.5699	-0.5751	-0.5806
$x_4(5)$	-0.3412	-0.3412	-0.5474	-0.5999	-0.6309
$x_0(6)$	1.1152	1.1152	1.0971	1.0615	1.0565
$x_1(6)$	-0.6352	-0.6352	-0.6579	-0.6972	-0.7930
$x_2(6)$	-0.4960	-0.4960	-0.6421	-0.7652	-0.8150
$x_3(6)$	-0.2398	-0.2398	-0.3198	-0.3221	-0.3025
$x_4(6)$	-0.2028	-0.2028	-0.3294	-0.3461	-0.3613
$x_0(7)$	1.0137	1.0137	0.9885	0.9771	0.9417
$x_1(7)$	-0.5709	-0.5709	-0.6008	-0.6823	-0.7314
$x_2(7)$	-0.4480	-0.4480	-0.5908	-0.6604	-0.7713
$x_3(7)$	-0.0801	-0.0801	-0.1044	-0.1049	-0.0518
$x_4(7)$	-0.0782	-0.0782	-0.1349	-0.1348	-0.1154

$x_0(8)$	0.9225	0.9225	0.8925	0.8791	0.8419
$x_1(8)$	-0.5071	-0.5071	-0.5356	-0.5704	-0.6511
$x_2(8)$	-0.4005	-0.4005	-0.5330	-0.6192	-0.7051
$x_3(8)$	0.0637	0.0637	0.0872	0.1099	0.1689
$x_4(8)$	0.0340	0.0340	0.0391	0.0500	0.1024
$x_0(9)$	0.8407	0.8407	0.8017	0.7811	0.7563
$x_1(9)$	-0.4449	-0.4449	-0.4645	-0.4901	-0.5571
$x_2(9)$	-0.3540	-0.3540	-0.4708	-0.5169	-0.6217
$x_3(9)$	0.1927	0.1927	0.2568	0.3001	0.3605
$x_4(9)$	0.1346	0.1346	0.1933	0.2365	0.2922
$x_0(10)$	0.7674	0.7674	0.7283	0.7099	0.6838
$x_1(10)$	-0.3847	-0.3847	-0.3909	-0.4096	-0.4541
$x_2(10)$	-0.3087	-0.3087	-0.4049	-0.4996	-0.5259
$x_3(10)$	0.2249	0.2249	0.3321	0.4309	0.5245
$x_4(10)$	0.2249	0.2249	0.3368	0.4176	0.4554

Table 1: Comparison of various series solutions with the exact solution

4. Conclusions

The dynamics of many continuous-time and discrete-time systems is described by high order differential equations. The solution of these high order stiff systems poses a problem and requires special numerical methods. SPM solves these problems by removing system's stiffness and reducing the order of the system. SPM in discrete control systems is being developed and its applications are not thoroughly explored. In order to fill this gap, here a fifth-order discrete power system model with three time scales is considered. It is modeled as a two parameter singularly perturbed system. Then an IVP is studied using the SPM for two parameters. These systems possess three widely different clusters of eigenvalues giving rise to one group of slow mode, and two groups of fast and faster modes. The suppression of the two small parameters affects the corresponding two sets of initial conditions, besides causing the degenerate subsystem to be of reduced order. A SPM has been presented where the approximation solution has been obtained in terms of the outer solution and the two BLC solutions corresponding to the two small parameters. The BLC solutions, obtained from the transformed systems, are meant mainly for the recovery of those initial conditions that are lost in the process of degeneration. SPM extended up to second-order approximation. In most of the papers on SPM the results are shown up to first-order approximation only. Here we have presented the results up to second-order approximation giving more justification to the proposed method. As the order of approximation increases, the mean square error between the exact and approximate solutions decreases.

It appears that the SPM is daunting due to the BLC equations. But it is not so as the corrections used in the SPM are to be calculated for only a limited number of values of k depending on the order of approximation. Also some functions of TSS may have trivial solutions as demanded by the selection of boundary conditions. Next we apply SPM to boundary value and optimal control problems which are of practically important and difficult compared to IVP.

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