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An Inventory Model for Deteriorating Items with Price Dependent Demand Under Fuzzy and Neutrosophic Fuzzy Environment

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ABSTRACT:
This paper represents an inventory model of deteriorating items under fuzzy and neutrosophic environment without allowing shortages in inventory. Demand is price dependent and is compared between crisp, fuzzy and neutrosophic fuzzy demand. Realistically it is observed that although the cycle time of any supply chain system is uncertain but uncertainty is mainly dependent on the selling price. So, here we have compared the model using crisp, fuzzy and neutrosophic fuzzy demand. The result is illustrated with the help of numerical example under fuzzy and neutrosophic fuzzy demand.

KEYWORDS:
Inventory, Fuzzy and Neutrosophic Fuzzy Price dependent Demand.

INTRODUCTION:

Inventory plays an inventory role in any supply chain. The main objective of maintaining an inventory is to provide a cushion between supply and demand for smooth and efficient running of the supply chain's operation. In any production inventory system, we face uncertainty associated with the different parameters such as demand, raw supply material various relevant costs, rate of deterioration and etc. to solve this kind of problems we use fuzzy and neutrosophic set theory. In 1970, Bellman and Zadeh [1] introduced fuzzy set theory for solving decision making problems. Then Wang, Tang and Zhao [2] developed model on fuzzy economic order quantity inventory model without backordering. Sen and Chakrabarti [3] developed An Industrial Production Inventory model with deterioration under neutrosophic fuzzy optimization. Sen and Chakrabarti [4] developed An EOQ model for healthcare industries with exponential demand pattern and time dependent elayed deterioration under fuzzy and neutrosophic environment. Saha [5] developed Fuzzy Inventory Model for Deteriorating Items in a Supply Chain System with Price Dependent Demand and Without Backorder. Chakrabarti and Chaudhuri [6] developed An EOQ model deterioratng items with a inear trend in demand and shortages in all cycles. Saha and Chakrabarti [7] modeled A Supply chain inventory model for deterioration items with price dependent emand under fuzzy environment. Kundu and Chakrabarti [8] brought in a model on Impact of carbon emission policies on manufacturing, remanufacturing and collection of used item decision with price dependent return rate. Goyal [9] developed an EPQ model with stock dependent demand and time varying deterioration with shortages under inflationary environment. Saha and Chakrabarti [10] framed Fuzzy EOQ model for time-dependent deteriorating items and time-dependent demand with shortages. Manna [11] proposed An EOQ Model for deteriorating item with non-linear demand under inflation, time discounting and a trade credit policy. Mullai et al [12] suggested A single valued Neutrsophic inventory Model with Neutrosophic Random Variable. Mullai and Sangeetha [13] also proposed Neutrosophic Model and Price breaks. Shee and Chakrabarti [14] put forward Fuzzy inventory Model for deteriorating items in a supply chain system with time dependent demand rate. Soni and Shah [15] modeled Optimal ordering policy for stock dependent demand under

progressive payment. Mondal, Bhunia and Maiti [16] schemed an inventory system of ameliorating items for price dependent demand rate. Jaggi and Tiwari designed [17] Effect of deterioration on two warehouse inventory model with imperfect quality. Saha and Chakrabarti [18] also schemed A Fuzzy inventory model for deteriorating items with linear price dependent demand in a supply chain.
Here we discuss the basics of Triangular Fuzzy System [19] and Neutrosophic Set and System [20] to enhance the development of our model.

BASIC PRELIMINERIES:

Fuzzy Set

A Fuzzy Set A is defined by a membership function μ_A(x) which maps each and every element of X to [0, 1]. i.e. μ_A(x) → [0,1], where X is the underlying set. In simple, a fuzzy set is a set whose boundary is not clear. On the other hand, a fuzzy set is a set whose element are characterized by a membership function as above.
A triangular fuzzy number is a fuzzy set. It is denoted by A = <a, b, c> and is defined by the following membership function:

$$\mu_A(x) = \begin{cases} 0, & a \le x, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{c-x}{c-b}, & b \le x \le c, \\ 0, & x \ge c, \end{cases}$$

Defuzzification of Triangular fuzzy number

Defuzzification, i.e. Signed distance for A = <a, b, c>, a triangular fuzzy number, the signed distance of A measured from O_1 is given by

$$d(A, O_1) = \tfrac{1}{4}(a + 2b + c)$$

Neutrosophic Fuzzy Set

A neutrosophic set is characterized independently by a truth-membership function $\alpha(x)$, an indeterminacy-membership function $\beta(x)$, and a falsity-membership function $\gamma(x)$ and each of the function is defined from X → [0,1]

Single valued Trapezoidal Neutrosophic Number

A single valued trapezoidal neutrosophic number $\tilde{A} = \langle (a, b, c, d){:}\ \rho_A, \sigma_A, \tau_A \rangle$ is a special neutrosophic set on the real number set R, whose truth membership, indeterminacy membership, and a falsity –membership is given as follows:

$$\mu_{\bar{A}}(x) = \begin{cases} (x-a)\rho_{\bar{A}}/(b-a) & (a \le x < b) \\ \rho_{\bar{A}} & (b \le x \le c) \\ (d-x)\rho_{\bar{A}}/(d-c) & (c < x \le d) \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_{\bar{A}}(x) = \begin{cases} (b - x + \sigma_{\bar{A}}(x-a)))/(b-a) & (a \le x < b) \\ \sigma_{\bar{A}} & (b \le x \le c) \\ (x - c + \sigma_{\bar{A}}(d-x))/(d-c) & (c < x \le d) \\ 1 & \text{otherwise} \end{cases}$$

$$\varphi_{\bar{A}}(x) = \begin{cases} (b - x + \tau_{\bar{A}}(x-a)))/(b-a) & (a \le x < b) \\ \tau_{\bar{A}} & (b \le x \le c) \\ (x - c + \tau_{\bar{A}}(d-x))/(d-c) & (c < x \le d) \\ 1 & \text{otherwise} \end{cases}$$

Operations on Single valued Trapezoidal Neutrosophic Number

Let $\tilde{A} = \langle(a_1, b_1, c_1, d_1): \rho_{\bar{A}}, \sigma_{\bar{A}}, \tau_{\bar{A}}\rangle$ and $\tilde{B} = \langle(a_2, b_2, c_2, d_2): \rho_{\bar{B}}, \sigma_{\bar{B}}, \tau_{\bar{B}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $\mu \ne 0$, then

1. $\tilde{A} + \tilde{B} = \langle(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle$
2. $\tilde{A} - \tilde{B} = \langle(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle$
3. $\tilde{A}\,\tilde{B} = \begin{cases} \langle(a_1a_2, b_1b_2, c_1c_2, d_1d_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle & (d_1 > 0, d_2 > 0) \\ \langle(a_1d_2, b_1c_2, c_1b_2, d_1a_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle & (d_1 < 0, d_2 > 0) \\ \langle(d_1d_2, c_1c_2, b_1b_2, a_1a_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle & (d_1 < 0, d_2 < 0) \end{cases}$
4. $\tilde{A}/\tilde{B} =$
$$\begin{cases} \langle(a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle & (d_1 > 0, d_2 > 0) \\ \langle(d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle & (d_1 < 0, d_2 > 0) \\ \langle(d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2): \rho_{\bar{A}} \wedge \rho_{\bar{B}}, \sigma_{\bar{A}} \vee \sigma_{\bar{B}}, \tau_{\bar{A}} \vee \tau_{\bar{B}}\rangle & (d_1 < 0, d_2 < 0) \end{cases}$$
5. $\mu\tilde{A} = \begin{cases} \langle(\mu a_1, \mu b_1, \mu c_1, \mu d_1): \rho_{\bar{A}}, \sigma_{\bar{A}}, \tau_{\bar{A}}\rangle & (\mu > 0) \\ \langle(\mu d_1, \mu c_1, \mu b_1, \mu a_1): \rho_{\bar{A}}, \sigma_{\bar{A}}, \tau_{\bar{A}}\rangle & (\mu < 0) \end{cases}$
6. $\tilde{A}^{-1} = \langle(1/d_1, 1/c_1, 1/b_1, 1/a_1): \rho_{\bar{B}}, \sigma_{\bar{B}}, \tau_{\bar{B}}\rangle \quad (\tilde{A} \ne 0).$

Defuzzification of Neutrosophic Set.

A single valued trapezoidal neutrosophic numbers of the form $\tilde{A} = \langle(a, b, c, d); \rho, \sigma, \tau\rangle$ can be defuzzified by finding its respective score value $K(\tilde{A})$

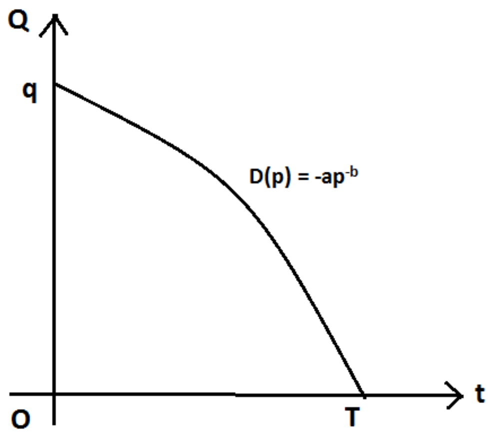
$$\frac{1}{K(\tilde{A})} = \frac{1}{16}[a + b + c + d][2 + \mu_{\tilde{A}} - \pi_{\tilde{A}} - \emptyset_{\tilde{A}}]$$

ASSUMPTIONS AND NOTATIONS:

i. Demand is dependent on selling price i.e. $D = ap^{-b}$, where a, b and p is the selling price. p is first considered as crisp then fuzzy and then neutrosophic fuzzy.
ii. θ is rate of deterioration and is constant.
iii. Replenishment is instantaneous and lead time is zero.
iv. The cycle time is uncertain.
v. Shortages are not allowed.
vi. q is the initial stock level at the beginning of every inventory.
vii. The total deterioration items is D.
viii. T is the length of a cycle.
ix. I(t) is the inventory level at any time t.
x. h is the holding cost per unit time.
xi. A is the setup cost per cycle.
xii. C is the deterioration cost per unit.
xiii. TAC is the total inventory cost.
xiv. TAC^{+} is the total fuzzy inventory cost.
xv. TAC^{++} is the total neutrosophic fuzzy inventory cost.

MATHEMATICAL MODEL:

The inventory level is zero at time t = 0. It accumulates in the time period [0, T] due to production at the constant rate. After that inventory level decreases due to demand and deterioration and reaches to zero at t = T.



The change in the inventory level can be described by the following differential equation:

Case 1 (Crisp Model)

$I'(t) + \theta I(t) = -ap^{-b} \quad , 0 \le t \le T$

With boundary condition I(0) = q and I(T) = 0

The solution of the above differential equation is given by

$I(t) = qe^{-\theta t} + (ap^{-b}/\theta)(e^{\theta T} - 1)$

With I(T) = 0, we have

$q = (-ae^{-b}\ /\theta)(e^{\theta t} - 1)$

Hence, the solution is

$I(t) = ap^{-b}[(T-t) + \frac{\theta}{2}(T-t)^2]$ [neglecting $O(\theta)$]

The inverse in a cycle is given by:

$I_T = \int_0^T I(T)\, dt$

$= ap^{-b}\left[\frac{T^2}{2} - \frac{\theta T^3}{6}\right]$

Total deterioration is a cycle is given by

$D = q - \text{total demand}$

$= q - \int_0^T ae^{-b}\, dt$

$= ½\, ae^{-b}\theta T^2$

Average cost of the system is given by

$TAC = \frac{1}{T}[A + C.D + hI_T]$

$$TAC = \frac{1}{T}\left[A + ½\, C. ae^{-b}\theta T2 + h\left[ap - b\left[\frac{T^2}{2} - \frac{\theta T^3}{6}\right]\right]\right]$$

Case 2 (Fuzzy Model, when selling price p is Fuzzy)

$I'(t) + \theta I(t) = -a\tilde{p}^{-b} \quad, 0 \le t \le T$

With boundary condition $I(0) = q$ and $I(T) = 0$

The solution of the above differential equation is given by

$I(t) = qe^{-\theta t} + a\tilde{p}^{-b}(e^{\theta T} - 1)/\theta$

With $I(T) = 0$, we have

$q = (-ae^{-b}/\theta)(e^{\theta t} - 1)$

Hence, the solution is

$I(t) = a\tilde{p}^{-b}[(T - t) + \frac{\theta}{2}(T - t)^2]$ [neglecting $O(\theta)$]

The inventory in a cycle is given by:

$I_T = \int_0^T I(T)\, dt$

$= a\tilde{p}^{-b}\left[\frac{T^2}{2} + \frac{\theta T^3}{6}\right]$

Total deterioration is a cycle is given by

$D = q - \text{total demand}$

$= q - \int_0^T ae^{-b}\, dt$

$= ½\, ae^{-b}\theta T^2$

Average cost of the system is given by

$TAC^{+} = \frac{1}{T}[A + C.D + hI_T]$

$$TAC^{+} = \frac{1}{T}\left[A + ½\, C. ae^{-b}\theta T^2 + h\left[a\tilde{p}^{-b}\left[\frac{T^2}{2} + \frac{\theta T^3}{6}\right]\right]\right]$$

Case 3 (Fuzzy Model, when selling price P is Neutrosophic Fuzzy)

$I'(t) + \theta I(t) = -a\tilde{P}^{-b} \quad, 0 \le t \le T$

With boundary condition $I(0) = q$ and $I(T) = 0$

The solution of the above differential equation is given by

$I(t) = qe^{-\theta t} + a\tilde{P}^{-b}(e^{\theta T} - 1)/\theta$

With I(T) = 0, we have

$q = (-ae^{-b}/\theta)(e^{\theta t} - 1)$

Hence, the solution is

$I(t) = a\widetilde{P}^{-b}[(T-t) + \frac{\theta}{2}(T-t)^2]$ [neglecting O(θ)]

The inventory in a cycle is given by:

$$I_T = \int_0^T I(T)\,dt = a\widetilde{P}^{-b}\left[\frac{T^2}{2} + \frac{\theta T^3}{6}\right]$$

Total deterioration is a cycle is given by

$$D = q - \text{total demand} = q - \int_0^T ae^{-b}\,dt = \tfrac{1}{2}\, ae^{-b}\theta T^2$$

Average cost of the system is given by

$$TAC^{++} = \frac{1}{T}[A + C.D + hI_T]$$

$$TAC^{++} = \frac{1}{T}\left[A + \tfrac{1}{2}\,C.ae^{-b}\theta T^2 + h\left[a\widetilde{P}^{-b}\left[\frac{T^2}{2} + \frac{\theta T^3}{6}\right]\right]\right]$$

Problem and Solution Procedure:

The problem is to minimize TAC, TAC^+, TAC^{++}

Here optimization is done using Lingo Software.

Illustrative example

For Crisp Model: A = 200, a = 100, p = 50, b = 0.05, h = 20, θ = 0.05, c = 18

For Fuzzy Model: A = 200, a = 100, p = <48, 50, 52>, b = 0.05, h = 20, θ = 0.05, c = 18

For Neutrosophic Fuzzy Model: A = 200, a = 100, P = < (48, 50, 52: ϵ), (47, 51, 52: δ), (48, 51, 53: γ) >, b = 0.05, h = 20, θ = 0.05, c = 18

Comparison of Models:

TAC = 823 at T = 0.479

TAC^+ = 822 at T = 0.481

TAC^{++} = 815 at T = 0.476

Conclusions:

We have developed an inventory model for deteriorating items with price dependent demand under fuzzy and neutrosophic environment. This model is without shortages. Here the comparison of the model has been depicted with its crisp, fuzzy and neutrosophic demands and hence its cost. For the fuzzy model, we have used triangular fuzzy number and trapezoidal single valued neutrosophic number as its defuzzification method and we obtain the minimum cost for each case. We also observe that the result is quite impressive with neutrosophic fuzzy parameters and the cost is much less compare to its crisp and fuzzy model. In the present situation, the fuzziness occurs in the different parameters which affects to the whole inventory management.

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