

# Analysis of Laminated Composite Plate with Sinusoidal Load Using Meshless Method

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**Abstract** - Meshless methods are very accurate in solving engineering problems so here we solve the problem of laminated composite plate with sinusoidal load using meshless method and same problem is also solved by using Abaqus software after that we compare the results. Value of deflection of laminated composite plate under sinusoidal load found from meshless method is very closest to the value found from Abaqus software. Here radial basis function finite difference method is used which is very important and accurate meshless method.

**Key Words:** Meshless method, laminated plate, Radial basis function, Abaqus software.

## 1. INTRODUCTION

In the design process of an advanced engineering system, engineers must undertake the courses of modeling, simulation, analysis and visualization. Differential equations and boundary conditions are abstract, and often highly approximate, characterizations of physical process in engineering. However, exact solutions to these differential equations are often possible only for problems defined in simple geometrical domains and mostly constrained to linear problems [1]. To solve differential equations governing the engineering processes occurring mostly in practice, many types of numerical methods have been proposed and developed such as the finite difference method (FDM), the finite element method (FEM) and the boundary element method (BEM), etc. FEM possesses many attractive features and has become one of the most important advances in the field of numerical methods [2].

An important of FEM is that it divides a continuum into a finite number of elements to model the problem. The individual elements are connected together by a topological map called mesh [1]. The common characteristic of the meshes is that each of them has several Connecting nodes and there is some information concerning the relation of nodes.

The continuity of field variables within the domain spreads through the adjacent meshes and related nodes. The governing differential equations, whether ordinary differential equations (ODEs) or partial differential equations (PDEs), can be transformed into weak-form formulations on the discretized sub-domains by means of certain principles, such as variational method, minimum potential energy principle or principle of virtual work [10]. Using the properly predefined meshes and the field discretization method, a set of algebraic equations are generated. After assembling the equations of all the meshes and imposing of proper boundary conditions, the system equations governing the problem domain can be formed and thereafter solved.

The FEM has been thoroughly developed and is widely used in engineering field due to its versatility for complex geometry and flexibility for different types of problems. Most practical engineering problems related to solids and structures are currently solved using well developed FEM commercial packages. Despite of the robustness in numerical analysis, there are still some limitations or inconveniences in the FEM [3]. For example the data preparation in the course of mesh generation and model conversion from physical model to finite element data is an extremely burdensome and time-consuming task [4]. Another factor may be that the secondary variables such as strains and stresses by the FEM are much less accurate than the primary variables such as displacements, temperature, etc. At the same time, the problems of computational mechanics grow ever more challenging. For instance, in the simulation of manufacturing processes, such as extrusion and modeling, it is necessary to deal with extremely large deformations of the mesh; while in computations of castings the propagation of interfaces between solids and liquids is crucial [6].

In simulations of failure processes, it is required to model the propagation of cracks with arbitrary and complex paths. In the development of advanced materials, methods which can track the growth of phase boundaries and extensive micro-cracking are required. However, these problems are not well suited to conventional computational methods such as the finite element method [9]. To overcome these problems, meshfree or meshless methods have been developed and achieved remarkable progress in recent years.

Meshfree methods use a set of nodes scattered within the problem domain as well as sets of nodes scattered on the boundaries of the domain to represent the problem domain and its boundaries. For most meshfree methods, these sets of scattered nodes do not form a mesh, which means no prior information on the relationship between the nodes is required for at least the interpolation or approximation of the unknown functions of field variables [8]. So far, many meshfree methods have found important applications and shown great potential to become powerful numerical tools. The use of local numerical schemes, such as finite differences produces good conditioned matrices [9]. By combining finite differences and radial basis functions it is possible to obtain a versatile meshless method, the radial basis function-finite difference technique (RBF-FD) [11]. To solve large engineering problems when compared to traditional global RBF collocation method, since the conditioning of the problem is greatly improved. Due to its excellent results, many authors are increasingly using local versions of the RBF method in diverse areas of physics and engineering [12–17]. Still, the most favorable grid distribution, stencil size and shape parameter in RBF-FD method remains an open problem. A recent comparison between meshless weak and strong formulations for boundary value problems shows how sensitive the method is to the shape parameter, especially when Neumann boundary conditions are used.

Some limitations in mesh generation, remeshing and making the approximation scheme in customary mesh-based methods such as FEM and FVM tend the general interests to use the meshless methods that remove the limitations of classical mesh-based methods. In meshless methods, only a cloud of points without any Information about nodal connections is used to discretize the domain.

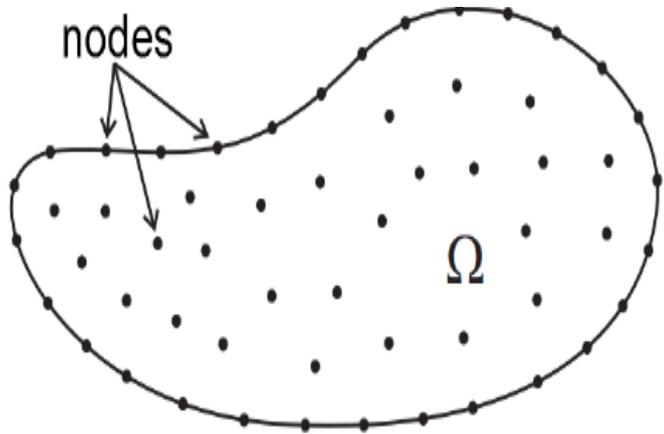


Fig-1: Distribution of nodes on domain and boundary [1].

One of the most favorite meshless methods is the RBF method which is constructed by radial kernels. They are positive definite, rotationally and translationally invariant. These features make its application straightway specially for approximating the solution of problems with high dimensions. The RBFs contain two useful specifications: a set of scattered centers with possibility of selecting their positions and existence of a free positive parameter known as the shape parameter [2].

## 2. RBF-FD METHOD

Radial basis function finite difference method represents a local meshless approach which has been shown to work well for large scale problems. Radial basis functions have been popular choices for meshless methods. Therefore one frequently refers to this approach as the radial basis-finite difference method (RBF-FD). Application of RBFs to compute derivatives on unstructured grids was first introduced by Kansa and then formally proposed as RBF-FD approach in [2]. Since then, this method has been continuously improved and applied to numerical modeling for various processes including convection–diffusion [7], and heat flow [14]. Use of multiquadric function theoretically provides a good convergent meshless method.

In modern conventions, the shape parameter is inversely proportional to the average distance between the node points [5]. Use of very small shape parameters may result in an approximation that is similar to overfitting. Therefore, it is recommended to keep the shape parameter on the optimal side. In practice, the use of small parameters in RBF-FD results in ill-conditioning, so that a stable algorithm is required for precise evaluation. The existence of the free shape parameter can be considered as an advantage of the RBFs. Without any additional computational cost or any change in other parameters of the method, varying the value of the shape parameter can cause more accurate results.

## 3. PROBLEM OF COMPOSITE PLATE

Firstly a plate of CFRP (carbon fibre reinforced polymer) having four layers and stacking sequence of 0°/90°/0°/90° is used in the static analysis. Meshless method is used in order to find transverse deflection and rotational deflection of the laminated composite plate. Stress and deformation relation for each layer (k) is given as follows.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{32} \end{Bmatrix}^{(k)}$$

$$Q_{11}^{(k)} = \frac{E_1^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}} \quad Q_{22}^{(k)} = \frac{E_2^k}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}$$

$$Q_{12}^{(k)} = \nu_{12}^{(k)} Q_{22}^{(k)} \quad Q_{33}^{(k)} = G_{12}^{(k)} \quad Q_{55}^{(k)} = G_{13}^{(k)}$$

## 4. MATERIAL PROPERTY

A square composite plate made of carbon fibre reinforced material (CFRP) have side a = 1 and thickness ratio a/h = 1 is considered. The plate is composed of four equally thick layers with layup (0°/90°/90°/0°). Since the plate has symmetric lay-up, the formulation is simplified in order to consider transverse deflection and rotational deflection. Following are the material properties if the plate E<sub>1</sub> = 200GPa, E<sub>2</sub> = 178GPa, E<sub>3</sub> = 178 GPa, G<sub>12</sub> = G<sub>13</sub> = 89GPa, G<sub>23</sub> = 35.6GPa, ν<sub>12</sub> = 0.25 as succinct as possible.

## 5. MODELLING

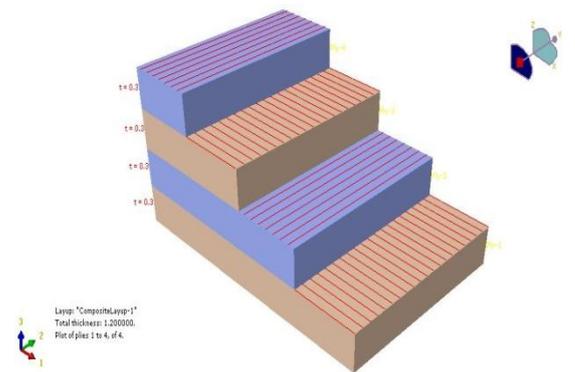


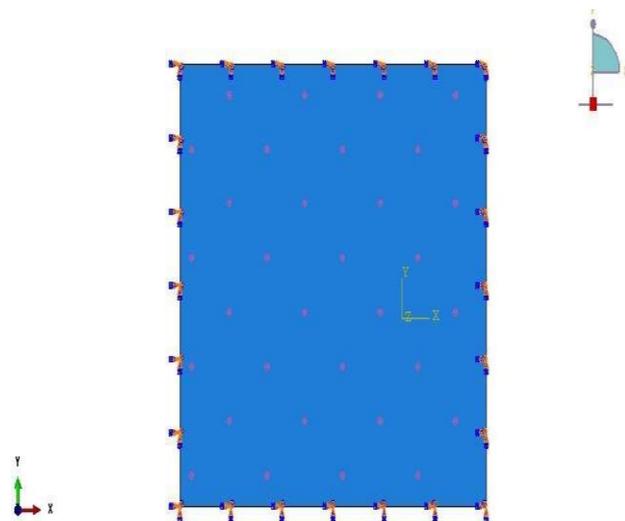
Fig-2: Ply stacking sequence of composite plate of four layer prepared on ABAQUS.

Model of four layer laminated composite plate having stacking sequence (0°/90°/0°/90°) has been modelled on FEM software ABAQUS.

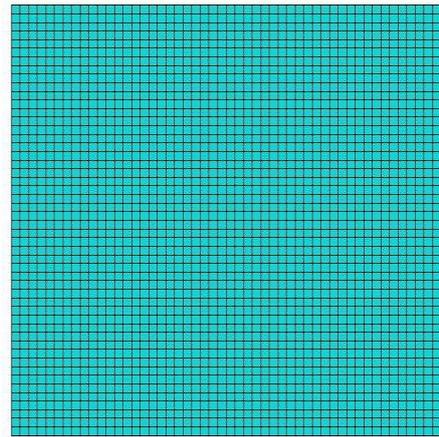
## 6. LOADING AND BOUNDARY CONDITIONS

Sinusoidal load of magnitude 1500 KN is applied on the plate of four layer having stacking sequence of 0°/90°/0°/90°.

Boundary conditions are all four edges of the plate are fixed thus movement of edges is restricted.



**Fig-3:** Shows boundary conditions of the plate. Fixed at all four edges.



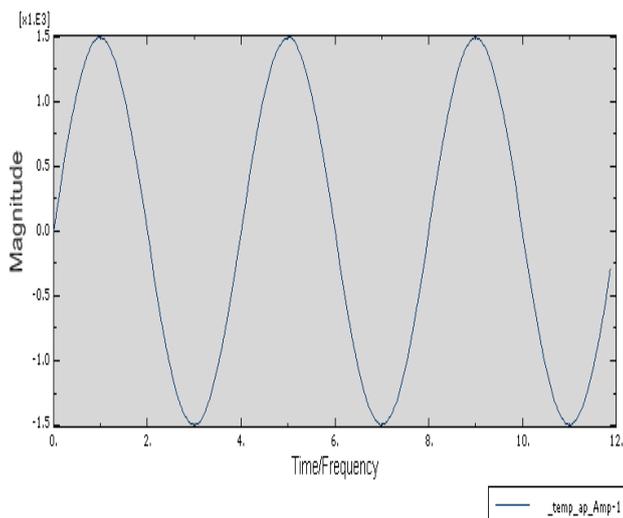
**Fig-5:** Meshing on the plate.

### 8. RESULT AND DISCUSSION

As the number of nodes increases, error in value of deflection decreases as the number of nodes increases and at 1600 nodes error reduces to a minimum value and after that at 2000 nodes value of deflection almost repeats itself. when sinusoidal load of magnitude 1500 KN is subjected to laminated composite plate (0°/90°/0°/90°) of four layer with boundary condition of all four edges are fixed. It is solved by the help of ABAQUS. Transverse deflection of the plate is given in mm while rotational Deflections of the plate are given in radians.

**Table(1).** Deflections of four layer laminated composite plate under sinusoidal load at different number of nodes.

S.No.	Deflection	400 nodes	800 nodes	1200 nodes	1600 nodes	2000 nodes	ABAQUS result
1	w(mm)	6.375	6.361	6.358	6.335	6.335	5.998
2	$\phi_x$	0.301	0.312	0.32	0.324	0.325	0.34
3	$\phi_y$	0.299	0.301	0.306	0.31	0.31	0.335

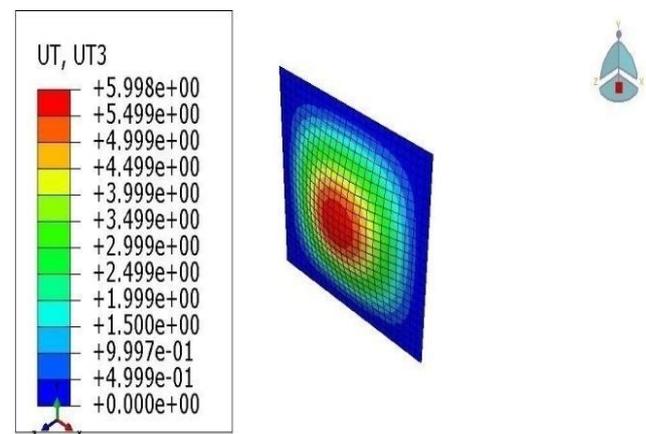


**Fig-4:** Sinusoidal loading on the plate.

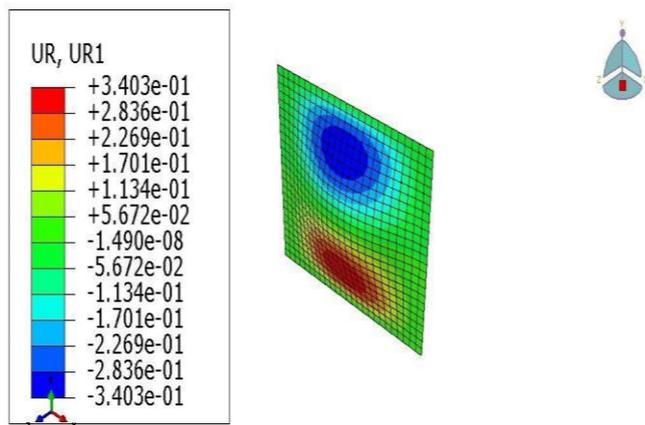
In this way edges of the plate can not move in any direction when the load is applied. Load is applied at the center of the plate. In this work, applied load is not constant. It varies between its maximum positive value and minimum negative value. Results obtained from MATLAB code is compared with the results of ABAQUS.

### 7. MESHING

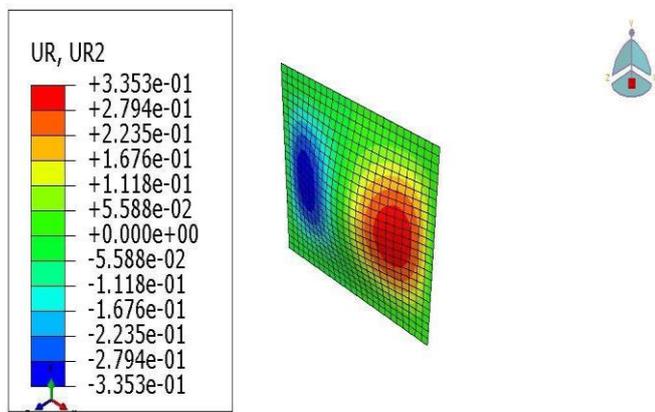
Meshing of laminated composite plate has been done on ABAQUS. Type of Element used in Meshing is S4R: A4 node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane strains. In this number of elements are 2500 and number of nodes are 2000. Meshed part shown in figure given below.



**Fig-6:** Transverse deflection w of composite plate of four layer with sinusoidal load.



**Fig-7:** Shows rotational deflection  $\phi_x$  of composite plate of four layer with sinusoidal load.



**Fig-8:** Rotational deflection  $\phi_y$  of composite plate of four layer with sinusoidal load.

Above figures show the variation in the value of deflection in different portions of the plate. Results obtained with RBF-FD method for the same case of plate has been compared with the results of ABAQUS. RBF-FD method gives good accuracy and faster convergence as compared to meshbased method.

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