Analysis of Numerical Integration Methods for Estimating Pond Area: A Case Study of Kanuru Pond

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Abstract - This study presents a comprehensive evaluation of numerical integration methods applied to estimate the area of Kanuru Pond, which spans 216 meters in width. Four distinct integration techniques, including the Trapezoidal Rule, Simpson's 1/3 Rule, Simpson's 3/8 Rule, and Weddle's Rule, were employed with varying segment counts. The results were compared against the exact area value obtained from a GPS Area Calculator application. Findings indicate that among the methods studied, Simpson's 3/8 Rule consistently demonstrated rapid convergence when the subdivision count was even, yielding highly accurate approximations closely aligned with the exact area. This research highlights the effectiveness of Simpson's 3/8 Rule in solving definite integral problems and emphasizes its suitability for estimating irregularly shaped areas, such as Kanuru Pond. These insights contribute to the broader understanding of numerical integration methods and their practical applications in geospatial analysis.

***Key Words*:** Area, Numerical Integration Methods, GPS Calculator, Convergence.

1.INTRODUCTION

Accurate measurement of area is a fundamental aspect of various fields, including geography, civil engineering, and environmental sciences. In this study, we delve into the task of estimating the area of Kanuru Pond, a water body spanning 216 meters in width. To achieve this, we employ numerical integration methods, a mathematical approach that subdivides the area into smaller segments and calculates their sum to approximate the total area. Four distinct numerical integration methods are considered: the Trapezoidal Rule, Simpson's 1/3 Rule, Simpson's 3/8 Rule, and Weddle's Rule. Each method is applied with varying segment counts to gauge its effectiveness in approximating the pond's area. To validate our results, we compare these approximations against the exact area value obtained from a GPS Area Calculator application. This research not only addresses the practical problem of estimating the area of a real-world feature but also aims to discern which of the numerical integration methods offers the most accurate and efficient solution. The findings presented herein contribute to the broader understanding of numerical integration techniques and their applicability in geospatial analysis, shedding light on the optimal approach for solving definite integral problems in scenarios involving irregularly shaped areas.

2.Formulation of the Problem: Numerical integration is a procedure of finding the approximate value of a definite integral. It is the approximate calculation of an integral with the help of numerical methods. In numerical integration, quadrature is a word that mean the integration of a function that has a single variable and quadrature to mean the numerical computation of multiple integrals π.

Now using numerical evaluation procedure finding the area of kanuru pond.

3.Numerical evaluation procedure:

we utilize four different procedures to solve numerical integration problems which are shown below

A pond is 216mts wide. The depth in feet at a distance x mts from one bank is given by following data.

x	0	36	72	108	144	180	216
y	64.53	91.27	107.43	108.13	106.31	83.14	34.54

Find approximately the area of the cross-section.

4.*Solution:*

Trapezoidal rule

Area=$h/2$ [(sum of first and last ordinate)+2(sum of remaining ordinates)

=36/2[(64.53+34.54)+2(91.27+107.43+108.13+106.32+83.14)]

=18(1091.65)

=19649.7

Simpson's 1/3rd rule

Area =$h/3$ [(sum of first and last ordinates)+4(sum of odd ordinates)+2(sum of remaining even ordinates)

=12[(64.53+34.54)+4(91.27+108.13+83.14)+2(107.43+106.32)

=19880.76

Simpson's 3/8th rule

area=3*h*/8 [(sum of first and last ordinates)+2(sum of multiples of 3

ordinates)+3(sum of remaining ordinates)

=13.5

[(64.53+34.54)+2(108.73)+3(91.27+107.43+106.32+83.14)

=19977.4350

Weddle's rule

area=3*h*/10 (y_0 +5y_1 +y_2 +6y_3 +y_4 +5y_5 +y_6)

=10.8

(64.53+5(91.27)+107.43+6(108.13)+106.32+5(83.14)+34.54)

=10.8(1833.650)

=19803.4200

x	y
0	15.12
18	30.53
36	36.92
54	60.85
72	75.5
90	120.65
108	110.54
126	124.24
144	125.33
162	126.08
180	100.06
198	120.9
216	51.04

Trapezoidal rule

Area=*h*/2 [(sum of first and last ordinate)+2(sum of remaining ordinates)

=9[(66.16)+2(30.53+36.92+60.85+75.5+120.65+110.54+124.24+125.33+126.08+100.06+120.9)]

=19164.24

Simpson's 1/3rd rule

Area=*h*/3 [(sum of first and last ordinates)+4(sum of odd ordinates)+2(sum of remaining even ordinates)

=6[(66.16)+2333.000+896.7)]=19775.16

Simpson's 3/8th rule

Area=3*h*/8 [(sum of first and last ordinates)+2(sum of multiples of 3

ordinates)+3(sum of remaining ordinates)

=6.75[(66.16)+594.94+2202.39]

=19328.5575

Weddle's rule

Area=3*h*/10 (y_0 +5y_1 +y_2 +6y_3 +y_4 +5y_5 +2y_6 +5y_7+y_8+6y_9+y_{10}+5y_{11}+y_{12})

=5.4(15.12+152.65+36.92+365.1+75.5+603.25+124.24+751.98+756.48+100.06+604.5+102.08)

=19914.5520 8

5.Findings:

Estimates of the area of kanuru pond using numerical integration method with six segments and twelve segments

Six segments		Twelve segments	
INTEGRATION METHOD	APPROXIMATE AREA	INTEGRATION METHOD	APPROXIMATE AREA
TRAPEZOIDAL RULE	19649.7m2	TRAPEZOIDAL RULE	19164.24 m2
SIMPSON'S1/3 RULE	19880.76 m2	SIMPSON'S1/3 RULE	19775.16 m2
SIMPSON'S3/8 RULE	19977.4350 m2	SIMPSON'S3/8 RULE	19328.5575m2
WEDDLE'S	19803.42 m2	WEDDLE'S	19914.5520m2
EXACT VALUE	**19398.52 m2**	**EXACT VALUE**	**19398.52 m2**

6.*Result and discussion*:

From the above data ,we can figure out that the numeric solution converges at a quicker rate only for **Simpson's 3/8 th** rule when the condition of the subdivision is even. On the other hand , approximate solution in case of other methods converge slowly. Among the four proposed rules **, Simpson's 3/8 th** rule delivers very good accuracy as well as when compared to the exact solution. Finally ,we observe that , **Simpson's 3/8 th** rule is is most effective method for solving definite integral problems.

To find approximate solutions for numerical integration problem with general quadrature based methods such as trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule, Weddle's rule. We considered, the area of kanuru pond, the exact area is collected from the "GPS AREA CALCULATOR APPLICATION", by forming the trapeziums in the area and applying the formulas of general quadrature methods we get approximate area. By comparing the approximate values with the exact values we find that simpson's 3/8th method gives more exact result compared to any other method.

7.Conclusion: In this study, we investigated the application of various numerical integration methods to estimate the area of Kanuru Pond, a water body spanning 216 meters in width. Four integration techniques, namely the Trapezoidal Rule, Simpson's 1/3 Rule, Simpson's 3/8 Rule, and Weddle's Rule, were examined, with different segment counts. Our findings revealed that Simpson's 3/8 Rule consistently outperformed the other methods, particularly when the subdivision count was even. This rule demonstrated a rapid rate of convergence and delivered highly accurate approximations, closely aligning with the exact area value obtained from a GPS Area Calculator application. In conclusion, Simpson's 3/8 Rule emerged as the most effective method for solving definite integral problems and estimating irregularly shaped areas like Kanuru Pond. This research provides valuable insights into the practical application of numerical integration methods in geospatial analysis, offering a robust and accurate approach for determining area measurements.

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