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# ANALYTICAL STUDY OF NON-NEWTONIAN FLUID AND NAVIER-STOKES EQUATIONS

**G.SIDDARTHAN**

*Department of Mathematics,* Sathya College of Arts and Science, Kizhvisharam, Ranipet - 632509, *Tamilnadu, India.*

***Email**: itzgsiddarth@gmail.com*

## Abstract

This article gives a brief discussion of study on Non-Newtonian fluid and Navier-Stokes equation. A non-Newtonian fluid is a fluid that does not follow Newton's law of viscosity, that is, it has variable viscosity dependent on stress. Navier-Stokes equation in fluid mechanics is a partial differential equation that describes the flow of incompressible fluids. The aim of the present work is to study the Non-Newtonian fluid and Navier-Stokes equation. In this article, the non-Newtonian fluid can be characterized by a viscosity that varies with motion. For non-Newtonian fluids, the viscosity varies with the shear rate are studied. The behaviour of a fluid flow can be explained by the Navier Stokes equation.

***Key words:*** Mathematical methods, Non-Newtonian Fluids, Navier-Stokes equation

## 1. INTRODUCTION

A non-Newtonian fluid is a fluid that does not follow Newton's law of viscosity, that is, it has variable viscosity dependent on stress. In non-Newtonian fluids, viscosity can change when under force to either more liquid or more solid. Ketchup, for example, becomes runnier when shaken and is thus a non-Newtonian fluid. Many salt solutions and molten polymers are non-Newtonian fluids, as are many commonly found substances such as custard, toothpaste, starch suspensions, corn starch, paint, blood, melted butter, and shampoo [1].

Flow of non-Newtonian (non-linear) fluids occurs not only in nature, for example, mud slides and avalanches, but also in many industrial processes involving chemicals (polymers), biological materials (blood), food (honey, ketchup, yogurt), pharmaceutical and personal care items (shampoo, creams), etc. In general, these fluids exhibit certain distinct features such as shear-rate dependency of the viscosity (related to shear-thinning or shear-thickening aspects of the fluid), normal stress effects (related to die-swell and rod-climbing), creep or relaxation (viscoelasticity), yield stress effects (viscoplasticity), history effects (time dependent response), etc [2].

There are many phenomena in physics which are described by Navier–Stokes equations [3]. The mathematical modeling of the weather, ocean currents, water flow in pipes, channels and air flow around wings are described by Navier–Stokes equations. Many problems have been formulated in nonlinear partial differential equations, which face some difficulties in the way of analytical solutions [3]. Scientists turn to the numerical solutions according to the difficulty of the nonlinear terms in a described system of fluid flow [4]. Some scientists [5-6] turn to describe the physical problems in terms of nonlinear partial differential equations for special cases of fluid and flow properties.

In this paper, present work is to study the Non-Newtonian fluid and Navier-Stokes equation. In this paper, the convergence of solutions for incompressible viscous non-Newtonian fluids is investigated. We obtain the conclusion that the solutions of non-Newtonian fluids converge to the solutions of Navier–Stokes equations.

## 2. MATHEMATICAL FORMULATION

We consider the flow of an incompressible Newtonian fluid through a rectangular miniature cylinder with the z-axis being the pivotal way. The differential conditions overseeing the flow incorporate the coherence condition and the Navier–Stirs conditions as follows

$$\frac{\partial U_j}{\partial x_j} = 0 \qquad (1)$$

$$p\left(\frac{\partial y_j}{\partial x_j} + y_i \frac{\partial y_j}{\partial x_i}\right) = -\frac{\partial p}{\partial x_j} + \mu \frac{\partial^2 y}{\partial x_i \partial x_j} + p g_{i,}[i = 1,2,3; j = 1,2,3] \qquad (2)$$

Where p and $yi$ are respectively the fluid pressure and velocity vector, $gj$ is the gravitational acceleration, ρ and μ are respectively the fluid density and viscosity and $xi$ denotes coordinates. As the flow is axially symmetric, the velocity components in the x and y directions vanish, namely

$y_1 = y_x = 0$ and $y_2 = y_y = 0$. Thus the continuity equation (1) becomes

$$\frac{\partial y_3}{\partial x_3} = \frac{\partial y_z}{\partial z} = 0$$

Which gives rise to $y_3$= v = v (x, y, t)

As the flow is horizontal, $g_3 = g_z = 0$, and hence eq. (2) becomes

$$p\left(\frac{\partial v}{\partial t}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial z}$$

We consider the liquid stream driven by the weight field with a pressure angle q(t) which can be communicated by a Fourier arrangement, to be specific

$$\frac{\partial p}{\partial z} = q(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega t) + b_n sin(n\omega t)] \qquad (3)$$

As the issue is pivotally symmetric, we just need to think about a quadrant of the cross-segment in the calculation.

By applying the Navier slip conditions in the primary quadrant of the rectangular cross segment, as in the paper by Wu *et al*., (2008) [7] for each time t, we have

$$\frac{\partial v}{\partial y}(x, 0) = 0; 0 \le x \le a$$
$$\frac{\partial v}{\partial x}(0, y) = 0; 0 \le y \le b$$

$$v(x,b) + l\frac{\partial v}{\partial y}(x,b) = 0; 0 \le x \le a$$

$$v(a,y) + l\frac{\partial v}{\partial x}(a,y) = 0; 0 \le y \le b \qquad (4)$$

## 3. NON-NEWTONIAN FLUID

A non-Newtonian fluid can be characterized by a viscosity that varies with motion. Most non-Newtonian fluids have a molecular chain structure [8]. In some of these fluids, the molecules tend to orient in planes of maximum tension resulting in a decrease in viscosity with an increasing velocity gradient; these fluids are called pseudo-plastics or 'shear-thinning' fluids. With increasing shear rate, the fluid is 'thinning'. In others, the viscosity will increase with an increasing velocity gradient. The fluid is called dilatant or 'shear-thickening.' Many biological fluids such as blood and polymer solutions are shear-thinning, while suspensions (paints) are shear-thickening. For non-Newtonian fluids, the viscosity varies with the shear rate. The relation between shear stress and shear rate for structure viscous fluids can be written as a power law (with $K$ as a constant factor):

$$\tau = K\frac{du^n}{dy} \quad ---------(5)$$

with

$n < 1$ for pseudoplastic fluids (viscosity decreases with shear rate, shear-thinning);

$n = 1$ for Newtonian fluids and $K = \mu$ (constant viscosity);

$n > 1$ for dilatant fluids (viscosity increases with shear rate, shear-thickening).

For the above-mentioned fluids, the shear stress will disappear when the shear rate approaches zero. Some other fluids will start to deform only when the shear stress exceeds a certain critical level. These fluids are called plastic fluids (e.g., toothpaste or even blood). One needs to overcome a given shear force before the fluid flows. This critical value of the corresponding shear stress is called the yield stress, $\tau_0$. For such fluids, the shear stress can be generally defined as

$$\tau^{1/m} = \tau_o^{1/m} + K \frac{du^{1/m}}{dt} \text{ ---------}(6)$$

With

$m$ = 1: the Bingham model;

$m$ = 2: the Casson model.

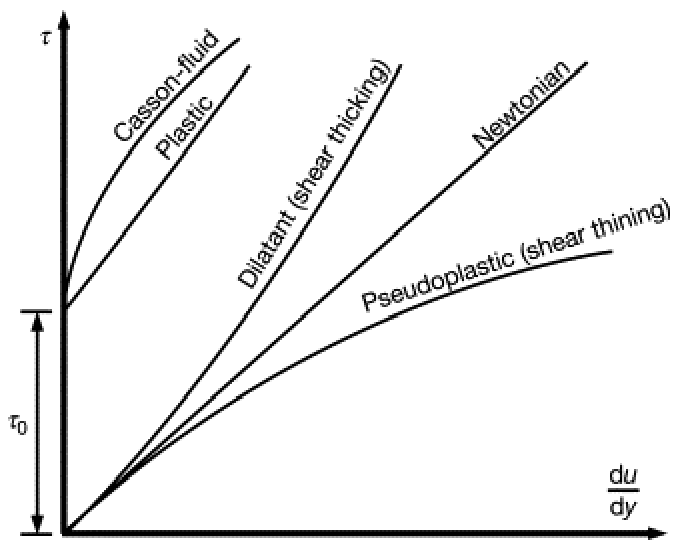


Fig**. 1.** gives the different kinds of fluids and their typical viscosity evolution as a function of shear rate.

## 4. NAVIER-STOKES EQUATIONS IN FLUID MECHANICS

Navier-Stokes equation in fluid mechanics is a partial differential equation that describes the flow of incompressible fluids. Mathematically, the behaviour of a fluid flow can be explained by the Navier Stokes equation. This equation is able to identify forces that occur in a fluid flow. The forces included in it are the force of objects and the force of the surface. Navier Stokes Equation is a differential form of momentum equation. Since the general equation used for the fluid problem is part of the momentum conservation law that is the second law of the newton, where force is the rate of momentum change [9].

$$\vec{F} = \frac{d}{dt}\left(m.\vec{V}\right) \quad -----------(7)$$

where F is the force vector, m is mass, and V is the velocity vector.

The Navier–Stokes equations for the motion of an incompressible, constant density and viscous fluid are [10]

$$\frac{\partial q}{\partial t} + (q.\nabla)q = \frac{1}{\rho}\nabla P + v\nabla^2 q \ ---------(8)$$

$$div\ q = 0 \ ---------(9)$$

where q (x, t) denotes the velocity vector, P (x, t) the pressure, and the constants ρ and v are the density and kinematic viscosity, respectively. This system is considered in three (or sometimes two) spatial dimensions with a specified initial velocity field

$$q(x,0) = q_o(x) \ ---------- (10)$$

and physically appropriate boundary conditions: for example, zero velocity on a rigid boundary, or periodicity conditions for flow on a torus. This nonlinear system of partial differential equations (PDEs) has proved to be remarkably challenging, and in three dimensions the fundamental issues of existence and uniqueness of physically reasonable solutions are still open problems.

It is often useful to consider the Navier–Stokes equations in nondimensional form by scaling the velocity and length by some intrinsic scale in the problem, for example, in Reynolds' experiment by the mean speed U and the diameter of the pipe d. This leads to the non- dimensional equations

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\nabla P + \frac{1}{R}\nabla^2 q \text{ ---------}(11)$$

$$div\, q = 0 \text{ ---------}(12)$$

where the Reynolds number R is

$$R = U\, {}^{d}/_{v} \quad \text{----------} (13)$$

In many situations, the size of R has a crucial influence on stability. Roughly speaking, when R is small the flow is very sluggish and likely to be stable. However, the effects of viscosity are actually very complicated and not only is viscosity able to smooth and stabilize fluid motions, sometimes it actually also destroys and destabilizes flows.

The Euler equations, which predate the Navier–Stokes equations by many decades, neglect the effects of viscosity and are obtained from (8) by setting the viscosity parameter $v$ to zero. Since this removes the highest-derivative term from the equations, the nature of the Euler equations is fundamentally different from that of the Navier–Stokes equations and the limit of vanishing viscosity (or infinite Reynolds number) is a very singular limit. Since all real fluids are at least very weakly viscous, it could be argued that only the the Navier–Stokes equations are physically relevant. However, many important physical phenomena, such as turbulence, involve flows at very high Reynolds numbers ($10^4$ or higher). Hence, an understanding of turbulence is likely to involve the asymptotics of the Navier–Stokes equations as $R \to \infty$. The first step towards the construction of such asymptotics is the study of inviscid fluids governed by the Euler equations:

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\nabla P \text{ ---------}(14)$$

$$div\, q = 0 \text{ ---------}(15)$$

Stability issues for the Euler equations are in many respects distinct from those of the Navier–Stokes equations and in this article we will briefly touch upon stability results for both systems.

## CONCLUSION

The present work gives brief study on the Non-Newtonian fluid and Navier-Stokes equation. In this paper, the non-Newtonian fluid can be characterized by a viscosity that varies with motion. For non-Newtonian fluids, the viscosity varies with the shear rate are studied. Mathematically, the behaviour of a fluid flow has been explained by the Navier Stokes equation.

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