Analyzing Mathematical Models in the Transition to Network-Based Revenue Management: A Comprehensive Review

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Abstract - Airlines have expanded hub-and-spoke networks, leading to a notable rise in the importance of network-based revenue management systems. In their pursuit to maximize overall revenue, an airline's network RM-based system differentiates between local passengers using single resources and connecting or flow passengers utilizing multiple resources. Despite being theoretically ideal, these systems can be difficult to implement directly in real-world scenarios. The implementation of such systems often relies on assumptions that may lead to suboptimal decisions if incorrect. This article aims to outline the primary challenges practitioners may encounter during the transition to a network-based RM system. It begins with a brief overview of typical network-based RM models and then delves into three areas where this system could potentially lower an airline's revenue: forecasting optimization, and distribution. The goal is to elucidate each of these areas and their respective difficulties related to implementing a network RM system in detail so that fellow researchers can consider them within the field of network RM while providing practitioners insights for addressing these issues during the implementation process within their organization

Key Words: Airline hub-and-spoke networks, Revenue management, network-based RM models, forecasting, network optimization, distribution.

1.INTRODUCTION

1.1 Overview of Revenue Management (RM) Systems

Revenue Management systems often seen in the airline industry, offer a method for selling inventory (seats) to customers. Airlines typically do not charge the same price for every seat on a flight; instead, they base prices on the customer's willingness to pay (Lapp & Weatherford, 2013). The RM system aims to predict this willingness and offers price options accordingly. As an example, it is typical for lower fares to become unavailable as the departure date of the flight draws near (Oancea, 2015). Airlines set the price for a customer's preferred travel route using a two-step process. Initially, the RM system forecasts demand and solves an optimal seat allocation issue (Capocchi, 2018). It is important to highlight that RM systems usually function within a service class framework, where they determine the allocation of seats to various service classes or buckets rather than predicting the entire elasticity curve of demand (Mukhopadhyay et al., n.d). The airline offers five seats in M-class, five in H-class, and three in Q-class for purchase. The projected revenue is calculated using the average fare of each class (US$500 for M-class) multiplied by the number of allocated seats (Lapp & Weatherford, 2013. These available seats are then made accessible through the airline's distribution system.

The development of revenue management is partly linked to the advancements in information technology infrastructure accessible to carriers during implementation (Capocchi, 2018)). In the past, providing both availability and pricing simultaneously was challenging, leading to a two-step process. First, the airline's reservation system allocates seats and their respective class for sale. Then, the class is matched with a price that may or may not correspond exactly with its value. For instance, if an airline offers five seats in M-class for sale, they would then set an M-class fare within the $400-$600 range. The actual fare sale follows this sequence: (i) identify the lowest service class available; (ii) determine the associated fare for this service class; and finally (iii) complete the transaction (Oancea, 2015). This two-step method developed as a way to reduce the amount of messaging needed between airlines' inventory and distribution systems.In this next section, we provide a concise overview of this distribution strategy.

1.2 Overview of Airline Distribution System

An airline's main business is to sell flight tickets. However, the advancement of IT has greatly influenced how airlines market and ultimately sell individual tickets (Koo et al., 2011). In a perfect scenario, a potential customer wanting to travel from Phoenix (PHX) to Detroit (DTW) would inquire with all airlines operating on this route for the price of the fare, whether it's for a nonstop flight or one with connections. When airline distribution systems were first developed, the current form of the Internet did not exist; customers purchased tickets through traditional travel agents (TAs) and these were booked via a global distribution system (GDS)(Wang, 2010). The introduction of online (Internet-based) distribution was an addition to existing infrastructure rather than replacing the established methods (Alamdari & Mason, 2006).

Realizing that IT systems were not as advanced as they are now and considering the significant costs of inventory and pricing messaging exchange, we examine how airlines currently distribute and sell tickets (Karthik & Mitra, 2016). We assess three separate processes: (i) checking for availability, (ii) setting a ticket price, and (iii) finalizing the sale. These processes are carried out sequentially to complete the ticket sale (Koo et al., 2011). We'll go back to our previous scenario of a client wanting to travel from PHX to DTW. In order to make the booking, she contacts a TA, which could be a physical agency or an online platform, commonly known as OTAs - online travel agents.

The TA should begin by inquiring about the availability of the preferred flight. TAs typically have access to GDSs that compile real-time flight availability information from various airlines, such as Sabre, Travel-Port and Amadeus (Wang, 2010). Subsequently, the GDS can be used to check for available flights from PHX to DTW. It is important for the TA to specify whether it's a nonstop flight or a connection (PHX–DFW–DTW), depending on the airline preference. Following this request, the GDS will provide information about which fare classes are available for purchase. An example of this response is illustrated, where different letter/number combinations indicate specific fare classes (e.g., A9 represents class A with 9 available seats). Based on the provided information, it is evident that the availability of flight options and fare classes is determined through the use of Global Distribution Systems (Lapp & Weatherford, 2013). These systems allow travel agents to access real-time information on flight availability and fare classes from multiple airlines, enabling them to provide customers with up-to-date options (Archdale, 1993)

In the next step, the TA needs to inquire about the price of a specific fare class. For instance, they may find that service class K is available for the PHX DTW nonstop flight, To obtain the price, the TA must request from the GDS system for the published price for K-class. The GDS interprets these filed fares and matches them with their corresponding classes before providing a price for the itinerary as depicted..

In the last stage, the TA will then proceed to reserve the specific travel plan, such as a flight from PHX to DTW in service class K. With multiple transactions taking place, it is possible that other TAs may have made additional bookings between checking for availability and pricing and making the actual sale (Capocchi, 2018). In these situations, when requesting the final purchase command, "the reservation system may indicate that a K-class fare for PHX–DTW is no longer available. At this point, the TA would need to restart the process by asking for availability and pricing in a higher fare class or exploring alternative routes or airlines. These steps illustrate the process by which travel agents utilize Global Distribution Systems to access real-time flight availability and fare class information from multiple airlines (Capocchi, 2018).

1.3 Sample Revenue Management approach

An RM system is tasked with deciding how many seats to allocate for each service class (Gallego & Topaloğlu, 2019). In the previous section's example, a combination of forecasting and optimization was used to determine that one seat should be available for purchase in the K service category. We will now present an illustration showing how an inventory management system would determine the appropriate number of seats to allocate for each service class.

Suppose an airline typically begins managing a flight 150 days before it departs. For instance, let's consider the case of flight number 1725 from PHX to DTW. When dealing with a single flight using a 100-seat aircraft, the RM system must determine the distribution of seats among different service classes, also known as class-protection levels. Based on historical demand data (Lapp & Weatherford, 2013), for example, the system may decide to allocate 50 seats in the M-class, 25 in the H-class, and another 25 in the Q-class. This allocation aims to align fare offerings with customers' willingness-to-pay tendencies demonstrated by an elasticity curve.

In addition to determining the number of seats available for sale in each class, airlines also employ other methods to limit access to inventory (Birbil et al., 2014). For instance, an airline may impose a rule on service class Q that demands this fare be sold 30 days before departure. Once this deadline has passed, customers can no longer buy this fare, even if the RM system indicates some inventory is still available. Other methods for restricting access include limitations based on days-to-departure, specific days of the week, minimum stay requirements and blackout dates (Capocchi, 2018).

It is important to highlight that RM systems typically do not take into account other fencing methods. In other words, the RM system's objective is to calculate the seat allocation within a specific class category without taking into consideration pricing restrictions affecting ticket sales (Huang & Liang, 2011). This further emphasizes the distinct decisions made regarding setting seat availability based on service classes and determining the actual price for the next ticket sale (Capocchi, 2018).

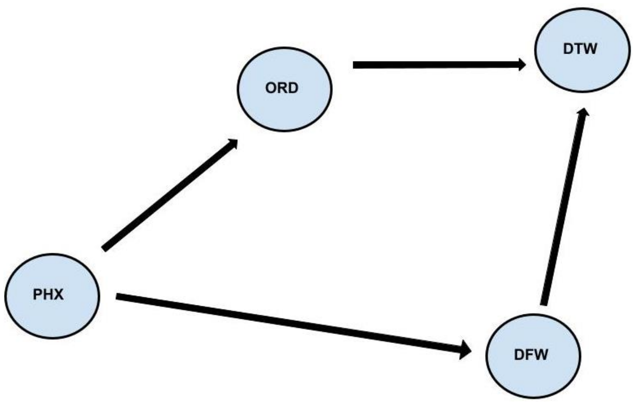
2. REVENUE MANAGEMENT STRATEGIES

Airlines typically employ one of two business approaches: hub-and-spoke or point-to-point. In a point-to-point model, airlines focus on transporting passengers directly from a specific origin to a particular destination (Lederer & Nambimadom, 2014). On the other hand, in a hub-and-spoke system, airlines use feeder flights to transport passengers to a central hub and then provide connecting service to various destinations through specific paths known as origin–destination pairs (Birbil et al., 2014). Major legacy carriers often adopt the hub-and-spoke model, while many low-cost carriers favour the point-to-point strategy. Making inventory decisions that align with the chosen business strategy is crucial for maximizing revenue (Bieger & Wittmer, 2011). One common revenue management strategy is to allocate a certain number of seats in each fare class based on historical demand data and customers' willingness-to-pay tendencies (Lapp & Weatherford, 2013).

2.1 Leg-based revenue management

Carriers opting for a point-to-point approach typically utilize a leg-focused RM strategy. In this scenario, the carrier makes all RM decisions at the level of individual legs, forecasting demand and service class at that level (Gallego & Topaloğlu, 2019). Seat allocations are then determined based on an optimization model at the leg-class level, often using an Expected Marginal Seat Revenue (EMSR)method (Babić et al., 2020). We will now expand our original example to illustrate the determination of service class allocations. Using this demand forecast, we can then determine the anticipated value for each seat on the specific flight. If the plane had a total of 10 seats, then according to the algorithm, seven seats would be allocated to M-class and three seats to H-class for the optimal solution. Therefore, this flight's availability would include seven seats in M-class and three seats in H-class.

Figure 1: PHX–DTW travel network example



2.2 Network-based Revenue management

Carriers that use a hub-and-spoke model typically employ a network-informed RM system. This type of system conducts both forecasting and optimization at the level of origin-destination (OD) classes (Oancea, 2015). In the given scenario, rather than having a direct service from PHX to DTW, an airline with a hub in Chicago requires passengers to first fly from PHX to ORD and then proceed on from ORD to DTW. This type of carrier utilizes a network-aware system and must make more intricate forecasts by predicting passenger routes through the ORD hub (Hsiao & Hansen, 2011). Furthermore, if this same carrier introduces the option for passengers to travel from PHX to Dallas before continuing on to DTW, it will need to forecast both the PHX–ORD–DTW and PHX–DFW–DTW routes. The upcoming section 'Forecasting issues' will outline some challenges associated with network-based forecasting. Additionally, apart from making forecasts for OD and service class levels, the carrier is also required to conduct optimization at the network level. For network-based revenue management, carriers need to forecast demand and optimize seat allocations at the level of origin-destination classes (Huang & Liang, 2011). Similarly to the previous example based on legs, the carrier needs to determine which inventory classes should be offered for the PHX–DTW itinerary. Carriers utilize network optimization techniques in order to make such decisions. A brief overview of some commonly used network-based optimization models found in practice will be provided in the next section.

3. NETWORK REVENUE MANAGEMENT MODELS

Airline business models have developed over time to depict the structure of their networks, such as the hub-and-spoke model. That is, instead of determining the number of seats to sell for each leg in the network, these models are designed to make optimal network decisions(Birbil et al., 2014). In this section, we present the five common network RM approaches and show how each one of these attempts to solve the problem of maximizing network revenue, including the (i) deterministic linear program (LP), (ii) stochastic LP, (iii) dynamic program (DP), (iv) probabilistic bid price (ProBP) and (v) displacement-adjusted virtual nesting (DAVN) approach(Lapp & Weatherford, 2013).

Before presenting each of the network RM models, we explain the idea of a bid price or the leg-based indifference point for passenger transport. This term is commonly used in most network RM models to denote the next acceptable fare for a specific flight segment. Essentially, the airline sets a (bid) price that represents the minimum amount required from customers to secure accommodation on a flight (Hsiao & Hansen, 2011). The notion of bid prices is expanded to cover multi-leg itineraries, requiring customers to compensate at least an aggregate amount equal to all bid prices across each flight segment they request (Karthik & Mitra, 2016).

The aim of the network RM models described in this section is to calculate the offer price for the upcoming seat on each flight segment within the network.

3.1 Deterministic Linear Program

The deterministic LP is potentially the most straightforward method for addressing a network RM issue. Its goal is to enhance income by choosing particular units of (passenger) demand for inclusion in the solution. The optimization also produces leg bid prices that can be utilized to calculate a total sum of bid prices, or hurdle rate, for a specific itinerary (Kunnumkal et al., 2012). Two pieces of information are needed to formulate this optimization problem: (i) a demand forecast and (ii) remaining capacity (seats) for each of the flight legs in the network. As the name of this network RM approach suggests, any randomness around the forecast of demand is not taken into account when solving the optimization problem (Oancea, 2015).

The deterministic LP formulation is expressed as The set of all flight legs(L), The set of all itineraries(I). The set of all fares(F) and parameters (δil) A parameter that is 1 if itinerary i uses flight leg l and is 0 otherwise, (λi) A parameter that indicates the fare of itinerary i,(di) A parameter that indicates the demand for itinerary i, (cl) A parameter that indicates the remaining capacity on leg l. Variables are (xi) A variable that is a positive integer if itinerary i is selected for inclusion in the solution. sub to constraints Demand Constraint (1) Capacity Constraint (2) (Lapp & Weatherford, 2013). The formulation maximizes revenue by selecting itineraries with the highest reward, considering demand and capacity limitations. Constraint represents the deterministic demand constraint where xi must be less than or equal to the demand forecast. Constraint depicts the capacity limit based on flight legs used in a particular itinerary; passenger demand counts towards flight capacity. Finally, Constraint mandates that the x variable must be an integer, limiting transportation to whole passengers only (Lapp & Weatherford, 2013). This version of the network RM model is straightforward, but it efficiently calculates bid prices for all flights in the network. These bid prices can then guide decisions on which fares to make available for purchase. Interestingly, these bid prices are derived as a result of solving the linear program relaxation (Oancea, 2015).

Solving the integer programming representation mentioned earlier as a linear program offers shadow prices for the capacity restriction. According to linear programming principles, the

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shadow price for this capacity limitation signifies an increase in the objective function for each additional unit of capacity (Bridgelall, 2022). To put it differently, the shadow price reflects the amount a traveler needs to pay in order to secure the next seat on a specific flight segment. In simple terms: if a passenger is prepared to offer this particular amount as compensation, then the airline ought to be willing to accommodate this passenger on that leg of travel (Oancea, 2015). To calculate the cost for the next seat on a complete set of flights (known as an itinerary), the bid prices can be combined to create a threshold value for that specific journey. It's important to highlight that including shadow prices may not always result in optimal compensation for the itinerary (Milne et al., 2018). However, this is currently a widely adopted method among airlines.

3.2 Stochastic Linear Program model

The deterministic LP incorporates the anticipated value of the demand prediction in creating the demand constraint. Since demand predictions are seldom completely accurate, the stochastic LP takes into account the discrepancy related to the demand forecast. The main aim of the stochastic LP is to optimize the expected revenue of the network while upholding identical constraints as those in place for a deterministic LP (Weatherford & Khokhlov, 2012).

Building upon the symbols introduced in the prior section, we will now present the expression of the stochastic linear programming model as proposed by Williamson in 1992. Additional parameter f(xi) A parameter that indicates the random demand distribution for itinerary i. The stochastic LP formulation remains largely similar, with the only notable difference being in the adjustment of the objective function. In contrast to the deterministic LP, in this case, ensuring adherence to the demand constraint is incorporated into the objective function. Consequently, only two constraints are required: Constraint for observing remaining seat capacity and for maintaining integrality on decision variables (Lapp & Weatherford, 2013). Similar to solving the deterministic LP, solving the stochastic LP also involves obtaining bid prices for each flight leg in the network from the dual variables of the capacity constraint. The stochastic LP, like its deterministic counterpart, is based on certain assumptions such as instantaneous demand arrival. In the following sections, we will explore how these assumptions impact and influence bid prices. (Kemmer et al., 2012).

3.3 Dynamic program model

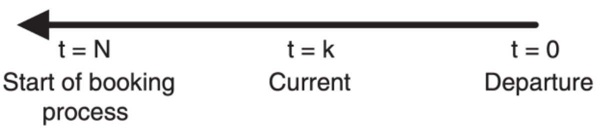
The DP approach contrasts with the use of linear programming approaches. In this methodology, Decisions are made at specific time intervals throughout the flight's booking horizon in this approach. We now examine how the DP methodology can be applied to establish a bid price control mechanism for an airline network based on work done by, Consider the airline network with m legs and n OD itineraries with p fare classes for each itinerary (Weatherford & Khokhlov, 2012).. The bid price is determined through a dynamic programming approach, where decisions are made at discrete time intervals throughout the booking horizon of a flight (Topaloğlu, 2009).

Similar to deterministic and stochastic linear programming, the objective is to determine a range of bid prices for each: (i) time period, (ii) itinerary, (iii) remaining capacity and (iv) remaining demand. However, the dDP approach creates a bid price structure that is more intricate than the linear programming approach. The LP generates a single value for each leg in the network, regardless of remaining capacity, remaining demand, or time period (Weatherford & Khokhlov, 2012).

The dynamic programming problem can be segmented into stages (time periods), where a decision needs to be made at each stage that impacts the objective function. In the context of airline networks, the decision involves selecting between various itineraries/fare classes available, with the goal being to maximize total revenue (Oancea, 2015). Time is measured in reverse (time t represents a point t periods from the end of the horizon) and denoted by 'k' as illustrated in Figure 2. At each time period, we analyze and graph all possible decision outcomes. Each stage is associated with a number of states, representing the various possible conditions of the system at that stage. The state of the airline network is described by a vector x indicating remaining leg capacities and matrix D denoting remaining demands for itinerary i and fare class g.

The policy decision at each stage transforms the current state into a state associated with the beginning of the next stage. If itinerary i at fare class g is sold (accepted), the network's state changes from x to x−Ai and from dig to dig−1. The solution procedure aims to find an optimal policy for all stages, prescribing decisions for each possible state. An optimal path involves deciding which itineraries to accept in order to maximize revenue while considering demand and capacity constraints within the airline network problem (Lapp & Weatherford, 2013).

Figure 2: Flow of time in standard DP decision-making.



The DP method allows for finding all the best routes for the airline network (various combinations of selling different itineraries/fare classes), leading to maximum potential revenue (Kemmer et al., 2012). Bid prices are then computed only for states that are part of one of these optimal routes. This differs significantly from the LP solution and is far more intricate. The LP solution may involve two instances of itinerary a and three instances of itinerary b, whereas the DP needs to list out all possible sequences (Bilegan et al., 2014). For instance, among many possible sequences we have: (i) AABB, (ii) ABABB, (iii) BBABA and (iv)BBBAA. The deterministic DP method involves the next stage's state being fully determined by the current stage's state and policy decision. In contrast, in the probabilistic scenario, the model assumes a probability distribution for the next state.

3.4 Probabilistic bid price model

In contrast to conventional network-based approaches, there are heuristic RM methods designed to incorporate "network-

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awareness" without explicitly modeling actual revenue flow between connecting itineraries in a network. Solution strategies that utilize this approach are commonly known as "intermediate OD" solutions (Wright et al., 2010). Contrary to the conventional network RM models that were previously discussed, intermediate OD solutions do not need a complete forecast of demand at the itinerary level (Birbil et al., 2014). For instance, instead of predicting the service class demand from DTW to Los Angeles through a hub in PHX, intermediate OD solutions focus on forecasting demand at the individual flight leg level: DTW–PHX and PHX–LAX. Additionally, this approach aims to incorporate network awareness into these forecasts by also considering flight leg-level demand. One example of an intermediate OD approach is the ProBP method.

Not all airlines have the capability to conduct a comprehensive "origin/destination" forecast, but they still function within a hub-and-spoke system where passengers mainly make connections at major hubs (Oancea, 2015). In situations where forecasting is conducted for specific flight legs, airlines adjust the forecast to accommodate some of the traffic flow across those legs. It's worth noting that all models discussed so far have focused on optimizing revenue management based on given forecasts. The ProBP approach operates at both the forecast and optimization levels of a network revenue management system (Lapp & Weatherford, 2013). To elaborate on this method, let's start with an illustration. The DTW–LAX OD pair using the DTW–PHX leg for forecasting the quantity of passengers based on their service class. Another forecast is conducted specifically for estimating demand on the PHX–LAX leg. The ProBP approach involves adjusting past bookings that contribute to the forecast.

If we make a booking from Detroit (DTX) to Los Angeles (LAX) for $400. That is, the passenger paid a total of $400 to fly the full itinerary. Now In the realm of leg-based bookings, we are unable to allocate a $400 reservation to each individual leg such as DTWPHX and PHXLAX for forecasting purposes. If this were done, the forecaster would start predicting $400 bookings separately for each leg, suggesting a total revenue potential of $800, which one would be inaccurate. Over the long run, this approach would imply a continuous push toward higher service classes (Oancea, 2015) The crucial element for the ProBP lies in an appropriate allocation process. While neither leg individually yielded $400, both contributed to the total value of $400. For instance, a RM system might determine that the DTWPHX leg produced $250 in value, while the PHXLAX leg generated $150 worth of value (Lapp & Weatherford, 2013). The forecaster will identify the true impact of each component on the network

Figure 3: DTW–LAX flight connection through the PHX hub.

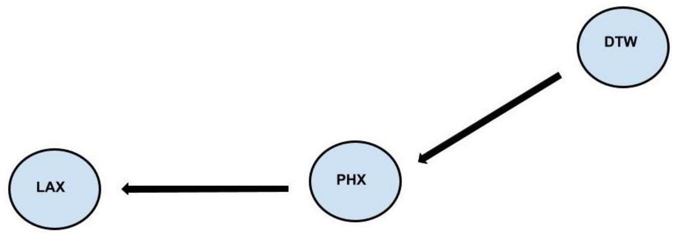
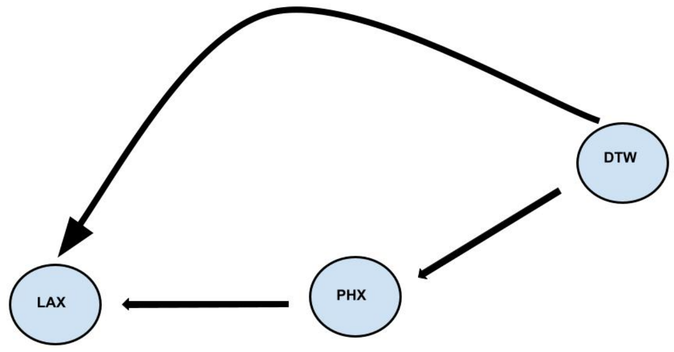


Figure 4: Sample prorated method for the previous DTW – LAX schedule.



Carriers experienced with the conventional EMSR method for forecasting and optimization on a leg-by-leg basis can employ a similar approach of expected revenue value to set bid prices for each leg. In the ProBP approach (Kemmer et al., 2012), the RM system utilizes a probabilistic proration function to distribute historical OD fares across individual legs. Subsequently, carriers will apply their forecasts to calculate demand expectations and error values. Combined, the leg-based fare times the probability will generate a leg-based EMSR value (Oancea, 2015). The EMSR approach traditionally involves using the EMSR values to calculate the actual availability of seats for each service class. In the ProBP method, the EMSR value is determined based on the fare of the itinerary. Once the ProBPs are determined, one would simply add the bid prices across all the legs that are used to create a future itinerary (Lapp & Weatherford, 2013). For example, suppose that we have two flights, one from DTW to PHX and a subsequent one from PHX to San Francisco (SFO). In this instance, we will utilize the EMSR values from each individual flight segment to calculate the total OD bid price for the next available seat on this route.The advantage of using the ProBP approach lies in its simplicity, as it leverages familiar existing concepts for many yield managers. Therefore, this heuristic network RM control method has a straightforward implementation process.

3.5 Displacement-adjusted virtual nesting model

To apply the ProBP method mentioned earlier, various proration schemes can be utilized. Common options include Y-fare or mileage proration, which allows the RM system to compute the revenue contribution of each leg in the system. However, neither of these proration methods captures the actual displacement of down-line traffic. In simpler terms, there is no link between Y-fare (or mileage) and how likely a leg is to have a higher (or lower) bid price (Oancea, 2015). Therefore, a more advanced approach involves using actual network displacement for proration. Williamson demonstrates this in his DAVN method that utilizes historical OD fare and actual displacement at booking time for proration purposes. With adjustments made to forecast full itinerary fares on a per-leg basis, optimization can then calculate leg availability (Wittman et al., 2018). As previously described, traditional approaches like EMSR provide leg-level availability through implementations such as EMSRa (partitioned versus nested class), or EMSRb (super buckets). This however only generates availability at the leg level; thus creating availability at the OD level requires carriers to adopt one of several outlined approaches next (Kemmer et al., 2012).

Using the "limiting service class" approach is one way to generate OD availability. For example, if the DTW-PHX leg is restricted up to service class Q and H-class is the lowest available class on the PHX-LAX leg, then the presented price for customers would be based on a service class H price point since it holds more value than Q-class on both legs. A difference involves using the EMSR method to generate bid prices for each leg in order to produce availability for specific origins and destinations. For instance, if the incremental seat value for the DTW-PHX leg is $120, and the EMSR value for the PHX-LAX leg is $150, a bid price of $270 could be formulated by the reservation system. This would then result in an appropriate pricing point being offered for the DTW-LAX itinerary. As demonstrated in this section, the DAVN approach shares a major advantage with the ProBP in terms of its straightforward forecasting method (Wittman et al., 2018). In DAVN, the leg forecaster seeks to find a balance point between local and flow passengers that generates displacement-adjusted value for the leg. Additionally, research has confirmed the effectiveness of both DAVN and ProBP through simulation within a competitive large airline network. The results indicated revenue increases of 1–2 percent compared to leg EMSR when network load factors were between 78–87 per cent (Lapp & Weatherford, 2013).

4. REVENUE FORECASTING ISSUES

4.1 Overview of airline forecasting

The RM system's input consists of a demand estimate for each upcoming flight to be sold. This estimate includes two key details: (i) customers' willingness-to-pay and (ii) time of arrival(Cleophas et al., 2019). In an ideal scenario, airlines would prefer precise forecasts in the form of an elasticity curve, within the 'Sample RM approach' section, for a specific future itinerary period. With such accurate information, airlines could efficiently organize customer willingness-to-pay and maximize consumer surplus. However, real-world implementation of this process is far more intricate than this idealized concept suggests(Vinod, 2021).

Before the advent of network RM models, predictions were made at the flight leg/class level. This involved creating a statistical forecast based on historical demand for a specific future flight and class(Poelt, 2011). For instance, for a particular flight (DTW–PHX on Friday, 20 December at 17:35) and class (e.g., M), forecasters would estimate the expected demand and its corresponding margin of error (variance). These forecasts typically relied on past demand data from previous departures of the same flight, in addition to considering factors like day-of-week(Huang & Liang, 2011).Typically, forecasting methods also accounted for trend and seasonality elements to adjust projections for cyclical variations. example unconstrained demand profile for flight 239 from DTW to PHX

We observe that the unrestricted demand signifies the anticipated remaining demand from the forecast time until the day of departure. This demand measure is frequently known as "demand-to-come" or pickup among different airlines. The prediction for this "demand-to-come" is typically made at various detailed levels as Individual Flight Leg, Directional Level (e.g., DTWPHX vs PHXDTW), Day-of-the-week (e.g., Monday vs Saturday), Time of day (e.g., Morning vs Afternoon), Class of Service (e.g., Y-class vs H-class)

To create a forecast like this, it's important to balance the level of detail with the margin of error. As the granularity increases, so does the potential for noise around the signal (i.e., error) (Oancea, 2015). For instance, a forecast that includes day-of-week, time-of-day and price point may only be based on 52 observations in a year (such as all morning flights on Fridays for service class H). When setting up an RM system, it's crucial to evaluate this trade-off between more detailed forecasts and the possibility of increased noise (Lapp & Weatherford, 2013).

The network RM models discussed previously (in the section titled 'Network RM models') all rely on a demand prediction as an input. However, in the case of network RM models, this forecast necessitates a higher level of detail - specifically at the full origin/destination level. This includes determining how many travellers use the DTW-PHX flight to travel from DTW through PHX to reach Burbank, California. We will refer to a forecast at this passenger origin/destination level as an OD forecast. Consequently, along with other levels of granularity such as day-of-week and time-of-day, we further divide the forecast based on another dimension. For instance, for a flight from DTW to PHX, we now predict the OD composition of these passengers. It's important to note that in this scenario, DTW represents one end point and PHX serves as the central location in a hub-and-spoke network; therefore all passenger itineraries originate at DTW.

To provide an example of this level of detail, let's revisit the journey taken by flight Traveling from Detroit to Phoenix on December 20, 2023. In a network RM system, it is essential for the forecaster not only to predict the number of passengers taking the DTW–PHX flight and then connecting to the PHX–SBA flight but also their likely service class (B, M or H). Considering that a typical aircraft accommodates around 100 passengers and that DTW–SBA accounts for approximately 2.17% of traffic from DTW–PHX, forecasting passenger numbers for specific routes like DTW–SBA can result in expectations exceeding two passengers with associated error values. Therefore, when implementing a network RM system, it is important to consider potential inaccuracies caused by increased forecast errors (Cleophas et al., 2019)

5. OPTIMIZATION ISSUES

In this part, we will discuss possible challenges from the perspective of optimization when incorporating a network RM system. Specifically, we will utilize our knowledge of the network RM strategies outlined in the 'Network RM models' section and our practical experience in deploying such systems for major international carriers. Similar to addressing forecasting issues, it is recommended that readers assess the implementation of a network RM system and devise carrier-specific resolutions for each mentioned challenge.

5.1 Bid price generation

In the "Deterministic LP" section, RM optimization models generate bid prices for each leg of the network. The shadow price of flight capacity represents the value that an airline may accept for transportation on a single flight leg (Karthik & Mitra, 2016). For example, a passenger flying from PHX to ORD has a bid price of $210 determined by the airline's RM optimizer. If another passenger wants to fly from PHX to DCA via ORD, the airline needs to calculate a bid price for both legs. In this case, it would be $490 ($210 from PHX to ORD and $280 from ORD to DCA). With this method, the airline can establish a minimum acceptable ticket price for all the routes available in its network. However, adding bid prices together to create these fares may become less precise over time as more bookings are made before any further adjustments are made (Wittman et al., 2018). The shadow price, calculated as a by-product of the optimization, reflects the opportunity cost of a unit change to the capacity constraint. However, its effectiveness is limited within a certain range of changes in the right-hand side. If this range is exceeded, it necessitates additional pivot steps to establish a new optimal basis (Oancea, 2015). For example, an LP solution may indicate that the shadow price for a specific leg is $210 and is valid for -1 seat to +3 seats; beyond this range, the shadow price no longer applies.

While having only the bid price of the next seat available may be adequate for some airlines, it may not be enough for all. Take a major airline that handles 300,000 bookings per day or 3.5 bookings per second. Even larger airlines may process around 15–45 bookings per second across their extensive networks. Therefore, the likelihood of a shadow price remaining valid for an entire day is relatively low and an updated solution to this problem must be found quickly(Kemmer et al., 2012). For a large network, this expectation is rather unrealistic and addressing the validity of bid prices becomes essential. Furthermore, practitioners should consider various scenarios in which bid prices become outdated and assess the potential revenue implications of such situations(Gallego & Topaloğlu, 2019).

5.2 Bid price gradient

To address with the problem discussed in the preceding section, certain carriers create estimations of a bid price curve. This involves calculating a form of distribution around the bid price and utilizing it to progress the availability calculation without having to recalculate the actual bid price. Essentially, this assumes that changes in bid prices remain fairly consistent regardless of other network alterations (Birbil et al., 2014).

The network RM systems discussed in the section on "Network RM models" are designed to calculate the bid price based on the current system state, which includes factors like the number of seats booked, remaining capacity, and expected/achievable demand for a flight (Huang & Liang, 2011). When working with a network RM model, it's important to note that the bid price is specific to each individual seat and should ideally be recalculated every time a new booking occurs (Lapp & Weatherford, 2013).

To deal with the issue of accurate and up-to-date bid price values, some network RM strategies have incorporated methods to estimate the bid price curve. This involves predicting the progression of bid prices based on historical data as bookings are made. While there may be variations in how different network RM systems carry out these approximations, the underlying concept remains consistent(Chaneton & Vulcano, 2011). The current bid price is determined using a network optimization model, which is then applied to an approximation curve that offers bid prices for all remaining seats on a specific flight(Capocchi, 2018). These curve approximations should be developed at the flight leg level, taking into account all relevant characteristics essential for forecasting purposes such as market conditions, day-of-week trends, time-of-day patterns etc(Li et al., 2014).

5.3 Bid price approximation within a Global Distribution System

Regardless of the approach used to compute a series of bid prices for a specific route in the network, these actual amounts may not be directly usable within GDSs(Wittman et al., 2018). As previously mentioned, airlines use GDSs to publish their inventory (availability). These GDSs are contractually obligated to offer seats by fare class that match what the host airline has available. However, due to limitations in many GDS systems, representing bid price values becomes quite restrictive and necessitates certain approximations. In essence, while the solution to the network RM problem may generate a set of bid prices, a carrier may encounter challenges when trying to provide these exact bid prices via the GDS due to system constraints and hence must make certain approximations (Capocchi, 2018).

Approximating bid price curves is a commonly used method to ensure that the bid price value increases as the number of available seats decreases. However, current reservation systems face challenges in storing complete bid price curves for flights. For instance, even if all the bid prices for 50 remaining seats on a specific flight could be calculated, existing reservation systems lack the capability to store these values during availability calculations. A major airline operating 3000 daily flights bookable up to 365 days in advance would need to store an immense amount of bid prices - approximately 109,500,000 - assuming an average of 100 seats per flight. While this volume can easily be stored by modern computers, reservations systems still require adaptation to accommodate storage needs for bid price values. Therefore, Global Distribution Systems offer methods to approximate sets of bid prices instead (Lapp & Weatherford, 2013).

In one approach, the GDS allows the carrier to provide a linear approximation along with a notification parameter to determine when a new approximation should be fetched. The carrier provides an intercept and slope of the bid price curve (e.g., $200 and $10). For each incremental booking, the bid price increases by $10 from an initial value of $200. Additionally, the carrier may set the notification parameter to five bookings so that once the bid price reaches $250, a new intercept and slope are requested from the carrier. With this scheme, determining piecewise linear approximation of bid prices is necessary for optimizing upload schedules and minimizing revenue leakage. In a perfect scenario, a carrier would aim to share the complete pricing curve with each of its stakeholders. GDSs. This way, a linear approximation of a likely non-linear curve could be avoided (Huang & Liang, 2011).

6. DISTRIBUTION ISSUES

6.1 Pricing versus inventory

As outlined in the 'Distribution overview' section, airlines make separate inventory decisions from the prices set for specific inventory classes. This means that an airline sets a certain fare class value it is willing to sell. After a passenger selects an itinerary and respective fare classes, the airline (or GDS) will then provide a price for the specific itinerary linked to, but not necessarily determined by, the class of service offered to the customer (Capocchi, 2018).

Before we delve into the issue of separating pricing from availability, let's consider an illustration of how network availability is established. If a bid price for the LAX to PHX route at $320 and another bid price for the PHX–FLL route at $185. To generate OD availability, these bid prices are combined to form a hurdle rate of $505. An airline would then identify the lowest class fare that yields at least $505 in revenue. For instance, if the airline offers an H-class fare for $510 in this scenario, it exceeds the required hurdle point and thus service class H becomes available for purchase. Based on this network RM model calculation (Kemmer et al., 2012), each additional seat sold beyond the next one contributes to a calculated gradient - assuming a linear gradient here as an example with increments of $10 per seat. Consequently, subsequent hurdle points (for this specific itinerary) will be set at increasingly higher fares such as $515 and $525 until all seats on the respective aircraft are occupied. Note that once it reaches $515, exceeding no longer applies to the H-class fare; therefore only one seat in service class H becomes available while achieving further availability necessitates higher fare classes (Lapp & Weatherford, 2013)

At this stage, the RM system has translated a specified minimum rate into a specific fare category. The pricing system then determines the exact price for this travel route. For instance, the airline might have introduced a new cost of $520 for an H-class ticket from LAX to FLL. It's important to note that depending on how the RM system generates availability, the calculated hurdle point may be compared with either a historical value (e.g., $510) or with the currently listed fare of $520. This disparity between availability and pricing arises because an airline cannot set a price for every individual hurdle point value that can be computed.

From a perspective of reconciliation, the filed fare is crucial for creating a historical record of bookings that can be used for future forecasting. For example, if a customer buys an H-class fare for $520, it's important to reconcile this with the actual amount paid in order to ensure that the demand profile aligns with its true value. While most systems may only capture booking information based on fare classes, incorporating the actual paid fare into class valuation is essential for future network revenue calculations, especially within OD systems where different products within a fare class may have varying prices (Oancea, 2015).

7. COMPARATIVE ANALYSIS ON MATHEMATICAL MODELS AND CONSIDERATIONS.

7.1 Analyst interaction with Revenue Management systems

One of the primary drawbacks of pure network RM systems is the absence of evident external factors that can be applied to the system. Airlines typically utilize yield managers who, quite justifiably, make decisions to influence the RM system in order for it to respond to events that may not be within its direct awareness (Capocchi, 2018). For instance, if a major sporting event is taking place in Atlanta, flight analysts managing traffic at ATL might need to modify the RM system's settings to capitalize on potential additional revenue from travelers coming specifically for the sports event. An analyst can influence the RM system at different points in the business process, like pre-forecast or post-forecast steps. Similarly, they can also affect pre-optimization or post-optimization steps. Each case can be broken down further to determine whether the analyst should influence the system at a leg level or full OD itinerary level (Kemmer et al., 2012). Thus far, all analyst impacts have been centered on adjusting optimization outputs further down the line. In contrast, in some carriers' business processes, analysts are required to operate at the forecast level. Instead of modifying bid prices or OD fare values, analysts adjust the forecasts that serve as input into the optimization system. With this approach, analysts can avoid downstream effects on other parts of the network (i.e., they bear responsibility only for expected traffic on their managed flights) (Oancea, 2015). However, they must effectively manage their forecast expectations, particularly at the individual OD level (Lapp & Weatherford, 2013). If managing individual ODs is too detailed, forecast influences can be aggregated to the flight-leg level where analysts simply tweak aggregate demand expectations.

There are numerous methods through which an analyst can engage with a RM system. This engagement is commonly influenced by the current business procedures(Kemmer et al., 2012). When introducing a network RM system, it's essential to create a business process that facilitates the analyst's comprehension of such a system, allowing for effective decision-making as needed (Wright et al., 2010).

Table 1. Analyst Intervention

Analyst Intervention Point	Network Level Impacted	Description
Pre-forecast	Flight-leg or OD	Analysts can adjust demand forecasts, either at the individual OD level or aggregated to the flight-leg level. This approach allows them to manage expectations for specific routes or origin-destination pairs.
Post-forecast	Flight-leg or OD	Analysts can modify forecasts after the initial generation by the system. This provides flexibility to adapt to unforeseen circumstances.
Pre-optimization	Leg	Analysts can directly influence bid prices for specific legs within the network. This can be beneficial for situations like major sporting events in a particular city, where analysts might increase bid prices to capitalize on potential revenue opportunities. However, overriding a bid price on a single leg might not effectively control specific ODs that utilize that leg.
Post-optimization	Leg or OD	Analysts can adjust OD fare values or override system-generated values for specific fares. For example, they might restrict access to a particular fare class on a route if necessary.

Table 2. A Review of Mathematical Models Used in Airline Network Revenue Management

Model	Focus	Key Points	Strengths	Weaknesses	Applications	Sources
Deterministic Linear Program (DLP)	Optimizing revenue under fixed demand	- Well-suited for predictable demand. - Limited in capturing demand uncertainty and dynamic pricing. - Useful for baseline solutions or network design.	- Easy to implement and solve. - Provides a clear understanding of optimal solutions for fixed demand scenarios.	- Does not reflect real-world demand variability. - Limited in dynamic pricing strategies.	- Initial network design and planning. - Benchmarking revenue under fixed demand assumptions.	(Ahn et al., 2020), (Gaul & Winkler, 2019),(An et al., 2021), (Klein et al., 2020), (Duduke & Venkataraman, 2021), (Subulan et al., 2016), (Szymański et al., 2021)
Stochastic Linear Program (SLP)	Optimizing revenue with demand uncertainty	- Accounts for demand variability in capacity allocation and pricing. - Computationally expensive for complex networks. - Relies on accurate demand forecasting. - Useful for analyzing the impact of demand uncertainty on revenue.	- Captures demand variability for more realistic decision-making. - Enables analysis of risk and potential revenue under different demand scenarios.	- Can be computationally expensive for large networks. - Reliant on accurate demand forecasting models.	- Analyzing the impact of demand uncertainty on revenue. - Evaluating alternative pricing strategies under uncertain demand.	(Shiina et al., 2023), (Imai et al., 2021), (Terciyanlı & Avsar, 2019), (Boer et al., 2002), (Bertsimas & Popescu, 2003)
Dynamic Program (DP)	Optimizing revenue with sequential decisions	- Well-suited for dynamic pricing and time-dependent factors. - Offers flexibility for complex networks. - Can suffer from "curse of dimensionality." - Requires efficient algorithms for real-world applications. - Useful for optimizing pricing in dynamic airline revenue management.	- Enables dynamic pricing strategies based on real-time information. - Can handle complex network structures with time-dependent factors.	- Computationally challenging for large networks with many decision stages. - Requires efficient algorithms for real-world applications.	- Optimizing pricing decisions in dynamic airline revenue management environments. - Analyzing the impact of time-dependent factors (e.g., cancellations, promotions).	(Huang & Liang, 2011), (Miyazawa et al., 2013), (Lagos et al., 2020), (Weatherford & Khokhlov, 2012), (Kunnumkal & Topaloğlu, 2010), (Zhang, 2011)

Probabilistic Bid Price Model	Setting prices based on probabilistic bidding	- Accounts for competition among airlines. - Allows dynamic price adjustments based on competition and demand. - Requires advanced modeling and competitor data. - Computationally challenging for complex networks. - Useful for competitive airline networks with dynamic pricing.	- Considers competition in pricing decisions, leading to potentially higher revenue. - Enables dynamic pricing adjustments based on competitor bids and passenger demand.	- Requires advanced modeling techniques and data on competitor behavior. - Can be computationally expensive for complex networks with many competitors.	- Airline networks with significant competition and dynamic pricing strategies. - Analyzing the impact of competition on pricing and revenue.	(Talluri & Ryzin, 1998), (Belobaba & Jain, 2013), (Kumar et al., 2021), (Akan & Ata, 2009), (Gallego & Topaloğlu, 2019)
Displacement-Adjusted Virtual Nesting Model (DAVN)	Addressing demand displacement	- Improves on traditional models by considering demand displacement. - Provides a more accurate picture of revenue potential. - Computationally complex for large networks. - Requires careful model parameter calibration. - Useful for airlines concerned about demand displacement and maximizing network revenue.	- Provides a more accurate representation of revenue potential by accounting for demand displacement. - Useful for airlines with concerns about low-fare passengers displacing higher-fare options.	-Computationally complex for large networks with many booking classes. - Requires careful calibration of model parameters based on historical data.	- Airlines with concerns about demand displacement and maximizing network revenue. - Analyzing the impact of demand displacement on revenue strategies.	(Ryzin & Vulcano, 2008), (Belobaba & Jain, 2013), (Qiu, 2020), (Fry & Belobaba, 2016), (Lapp & Weatherford, 2013), (Ryzin & Vulcano, 2008)

8. CONCLUSIONS

The network-based revenue management (RM) necessitates the adoption of sophisticated mathematical models to optimize revenue generation across intricate airline networks. This review has undertaken a comprehensive examination of five key models that address various aspects of this challenge: Deterministic Linear Program (DLP), Stochastic Linear Program (SLP), Dynamic Program (DP), Probabilistic Bid Price Model, and Displacement-Adjusted Virtual Nesting Model (DAVN).

DLP offers a transparent and computationally efficient approach for initial network design under predictable demand. For instance, airlines launching new routes with limited historical data might utilize DLP to determine initial seat allocations and pricing based on market research and industry averages. However, limitations exist in its ability to capture real-world demand uncertainties and incorporate dynamic pricing strategies Miyazawa et al., 2013); (Klein et al., 2020);(Duduke & Venkataraman, 2021). SLP incorporates demand uncertainty into the model, providing a more realistic representation of the airline revenue management environment. This allows airlines to analyze risk and potential revenue under diverse demand scenarios. Imagine an airline planning for a major holiday weekend with historically high demand fluctuations. SLP can be used to model different demand possibilities (higher than usual, lower than usual) and estimate the potential revenue impact on each scenario. However, SLP models can be computationally expensive for very large and intricate networks, and their effectiveness is heavily reliant on the accuracy of demand forecasting (Imai et al., 2021); (Terciyanlı & Avsar, 2019). DP excels in handling dynamic pricing strategies and time-dependent factors, making it ideal for optimizing pricing decisions in a continuously evolving environment. Airlines can leverage DP to adjust prices closer to the departure date based on remaining inventory and real-time booking trends. For example, if a flight has many empty seats a few days before departure, DP can suggest lowering fares to attract last-minute bookings and maximize revenue. While it provides flexibility for complex networks, the "curse of dimensionality" can pose challenges for very large networks, necessitating the implementation of efficient algorithmic solutions (Huang & Liang, 2011);(Miyazawa et al., 2013); (Lagos et al., 2020); (Weatherford & Khokhlov, 2012).

Probabilistic Bid Price Models differentiate themselves from traditional models by considering the competitive landscape. This enables dynamic price adjustments based on competitor bids and passenger demand, potentially leading to higher revenue generation. Imagine a airline competing with a low-cost carrier on a popular route. A probabilistic bid price model can analyze competitor pricing strategies and adjust fares accordingly to capture demand at optimal price points. However, these models require advanced modeling techniques and access to competitor data, which can be challenging to obtain. Additionally, computational complexity can become an issue for networks with numerous competitors (Belobaba & Jain, 2013); (Kumar et al., 2021);(Gallego & Topaloğlu, 2019). DAVN addresses the crucial issue of demand displacement, a phenomenon where booking low-fare passengers may displace potential bookings at higher fares. This model provides a more accurate picture of revenue potential by accounting for this effect. For example, an airline might offer a limited number of discounted seats on a particular flight leg. DAVN can help determine the optimal number of discounted fares to maximize revenue while considering the potential displacement of full-fare passengers on connecting flights. However, DAVN models can be computationally demanding for large networks with many booking classes and require careful calibration of parameters based on historical data (Belobaba & Jain, 2013), (Qiu, 2020), (Fry & Belobaba, 2016).

The choice of the most suitable model depends on the specific needs and characteristics of the airline network. Airlines with predictable demand and a focus on initial network design might find DLP a valuable tool, For those facing significant demand uncertainty, SLP offers valuable insights. In dynamic environments with time-sensitive pricing, DP emerges as a powerful tool. Airlines operating in competitive markets can benefit from the strategic pricing capabilities of Probabilistic Bid Price Models. Finally, DAVN proves valuable for airlines concerned about demand displacement and maximizing network revenue.

8.1 Future Research Directions:

While these models offer significant capabilities, the growing complexity of airline network revenue management necessitates further exploration in several key areas:

Model Integration and Hybridization: Research could investigate how to integrate these models or develop hybrid approaches that combine the strengths of different models. This could lead to more comprehensive solutions that address a wider range of airline network revenue management challenges.

Machine Learning and Artificial Intelligence: Advancements in machine learning (ML) and artificial intelligence (AI) hold immense potential for airline revenue management. Research could explore incorporating ML and AI techniques to enhance demand forecasting accuracy and develop dynamic pricing strategies that adapt to real-time market conditions and customer behavior. This could lead to more optimized pricing decisions and increased revenue generation.

Real-Time Optimization and Algorithmic Efficiency: As network complexity and data volume increase, the need for real-time optimization and efficient algorithms becomes even more critical. Research could focus on developing efficient algorithms that enable airlines to make optimal pricing and capacity allocation decisions in real-time, maximizing revenue potential while considering computational constraints.

Incorporating External Factors: Future research could explore ways to incorporate external factors that can significantly impact airline revenue, such as weather events, economic fluctuations, and political instability. By integrating these factors into network revenue management models, airlines can develop more robust and adaptable strategies that account for the ever-changing business landscape.

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