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Application of Ordinary Differential Equation

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Abstract –This paper deals with various applications of Ordinary Differential Equation(ODE) in engineering. It is useful in many engineering problems. It is used to calculate movement or flow of electricity, motion of objects like pendulum. Also it is useful to explain thermodynamics concepts. It is useful in various physical phenomena's. Thus, It has wide range of applications.

Key Words: Ordinary Differential Equation (ODE), Electrical Circuits, Simple Harmonic Motion, Newton's Law of Cooling, Rate of Decay

1. INTRODUCTION:

Differential Equation: An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called differential equation.

Ordinary Differential Equation: A differential equation involving derivatives with respect to single independent variable is called an ordinary differential equation.

Ordinary differential equation has wide applications in various areas of engineering.

2. APPLICATIONS:

1) **Electric Circuit Analysis:** By using First order ordinary differential equation in L-R & C-R circuits we can find the current (i) & voltage (v) in the circuit when inductance L or capacitance C & Resistance R is given.

Example:

A circuit has in series an electromotive force given by $E = 100 \sin 40t$ V, a resister of 10Ω and an inductor of 0.5 H. If the initial current is 0, find the current at t > 0.

Solution:

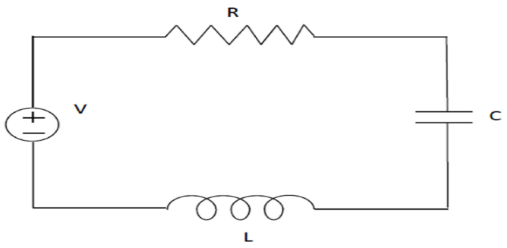

Fig. 1

By Kirchhoff's Law,

$$L\frac{di}{dt} + Ri = E$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$

$$\frac{di}{dt} + 20i = 200\ sin40t$$

Here I.F. = $e^{\int 20\,dt}$

$= e^{20t}$

Solution is –

$$ie^{20t} = \int e^{20t}\,200\ sin40t\ dt + c$$

$$= 200\int e^{20t}\ sin40t\ dt + c$$

$$ie^{20t} = 2e^{20t}(sin40t - 2cos40t) + c$$

$$i = 2(sin40t - 2cos40t) + ce^{-20t}$$

As initial current $i(0) = 0,\ \ c = 4$

So, $i = 2(sin40t - 2cos40t) + 4e^{-20t}$

2) **Simple Harmonic Motion:**

Simple Harmonic Motion is a type of oscillatory motion where restoring force is proportional to displacement when an object moves to and fro along the fixed line then the motion is said to be Simple Harmonic Motion.

Simple Harmonic Motion serves as mathematical model for various motions such as oscillation of spring, pendulum & molecular vibrations.

Example:

A molecule begins from rest, a distance 10 cm from settled point O. It moves along a horizontal straight line towards O under the influence of an attractive force at

O. This force at any time varies as the distance of the particle from O. If the acceleration of the particle is 16 cm/s^2 directed towards O when the particle is 1 cm from O, describe the motion.

Solution—

Assume that the particle starts from point A at t = 0.Take a fixed point O as the origin & choose OA as the positive direction. Let P be the situation of the molecule whenever. Since the magnitude of the force of attraction towards point O is proportional to the distance from point O.

We have from Newton's Law,

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad \ldots\ldots\ldots\ldots\ldots..1.$$

As acceleration is 16 cm/s^2 directed towards O when the particle is 1 cm from O, we have, x = 1 and

a = $\frac{d^2x}{dt^2} = -16$

By 1, $\frac{k}{m} = 16$

$$\frac{d^2x}{dt^2} = -16x \qquad \ldots\ldots\ldots\ldots\ldots\ldots 2.$$

Since the particle starts from rest 10 cm from O, we have x = 10, v = 0 at t = 0

$$\frac{d^2x}{dt^2} = -16x \qquad at\ x = 10$$

$$\frac{dx}{dt} = 0 \qquad at\ t = 0 \ldots\ldots\ldots\ldots.3.$$

Let, $\frac{dx}{dt} = v$ so that,

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$$

By 2,

$$v\frac{dv}{dx} = -16x$$

$$v\,dv = -16x\,dx$$

$$\frac{v^2}{2} = -16\frac{x^2}{2} + c$$

$$v^2 = -16x^2 + c_1$$

$$v = 0\ \ when\ \ x = 0, \qquad c_1 = 1600$$

Thus, $v^2 = 16(100 - x^2)$

$$v = \frac{dx}{dt} = \pm 4\sqrt{100 - x^2}$$

Separating the variables,

$$sin^{-1}\left(\frac{x}{10}\right) = \pm 4t + c_2$$

As $x = 10, at\ t = 0$

We have, $c_2 = \frac{\pi}{2}$

Thus,

$$sin^{-1}\left(\frac{x}{10}\right) = \frac{\pi}{2} \pm 4t$$

$$X = 10\cos 4t$$

This shows that the particle start at x=10 when t= 0 the proceeds through O to the place x = -10 from where it returns to origin passes through and goes to x=10.The cycle then repeats over & over again. This behavior is similar to that of the bob of the pendulum swinging back & forth. It is an example of Simple Harmonic Motion.

3) **Newton's Law of Cooling:** Let the temperature of a body at any time t be T units & that of the surrounding be T_0 ($T_0 < T$). The Newton's law of cooling states that the rate of cooling is proportional to the difference in the temperature.

$$\frac{dT}{dt} \propto (T - T_0)$$

$$\frac{dT}{dt} = -k(T - T_0)$$

$$\frac{dT}{(T - T_0)} = -k\,dt$$

$$\log(T - T_0) = -kt + \log c$$

$$\log\left(\frac{T - T_0}{c}\right) = -kt$$

$$T - T_0 = ce^{-kt}$$

$$T = T_0 + ce^{-kt}$$

Example:

Water at temperature 100^0c cools in 10 minutes to 88^0c

in a room temperature 25^0c. Find the temperature of water after 20 minutes.

Solution—

Let T^0c be the temperature of water at time t .Then the rate of cooling is directly proportional to T-25.

$$\frac{dT}{dt} \propto T - 25$$

$$\frac{dT}{dt} = -k(T - 25)$$

which gives,

$$T - 25 = ce^{-kt}$$

At $t = 0, \quad T = 100$

$$c = 75$$

$$T - 25 = 75e^{-kt}$$

At $t = 10, \quad T = 88$

$$63 = 75e^{-10k}$$

$$e^{-10k} = \frac{21}{25}$$

At, $t = 20, \quad T - 25 = 75e^{-20k}$

$$T = 25 + 52.92$$

$$T = 77.92$$

is temperature of water after 20 minutes.

4) **Rate of Decay:** A radioactive substance such as radium disintegrates and its mass is converted to energy. It emits energy and losses mass. This process is described as radioactive decay. The rate of decay is directly proportional to the mass left at any instant. It gives

$$m = ce^{-kt}$$

This equation describes the process and gives the mass of substance at any time t.

Example:

The half life of radioactive substance is the time needed for one half of a given amount to disintegrate. Express this in term of time constant. If for the substance the half life is 1600 years, show that the amount left in 100 years, show that the amount left in100 years is$(\frac{1}{2})^{\frac{1}{16}}$ times the original mass.

Solution—

If m is the mass of the substance at any time t,

$$\frac{dm}{dt} = -km$$

$$m = ce^{-kt}$$

When $t = 0, m = m_0$

$$m_0 = c$$

$$m = m_0 e^{-kt}$$

When $t = 1600, m = \frac{m_0}{2}$

$$\frac{m_0}{2} = m_0 e^{-1600k}$$

$$\frac{1}{2} = e^{-1600k}$$

When t = 100,

$$m = m_0 e^{-100k}$$

$$= m_0 (e^{-1600k})^{\frac{1}{16}}$$

$$= m_0 (\frac{1}{2})^{\frac{1}{16}}$$

3. CONCLUSIONS

By using first order and second order ordinary differential equations we can solve many engineering problems. Thus, ODE has most important role in every engineering concept.

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