Apply Adomian Decomposition Method (ADM) and Haar Wavelet Method (HWM) for Applications of linear Differential Equations

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Abstract

In this work, the numerical solution of the equations arising in simple harmonic motion and free oscillation is found using the Adomian decomposition method (ADM). For comparison analysis, the Haar wavelet method (HWM) is used. Some numerical examples have been performed to illustrate the accuracy of the present work.

Keywords: Oscillatory motion, Adomian decomposition method (ADM), Haar wavelet method (HWM), Simple Pendulum

1. INTRODUCTION

The Adomain Decomposition Method (ADM) is a powerful tool for solving linear or nonlinear functional equation . Adomain Decomposition Method is extended to the calculations of the non-differential function. The main advantage of this method is that it can be applied directly to all types of differential and integral equations, linear or non-linear, homogeneous or inhomogeneous, with constant or variable coefficients. Another important advantage is that, the method is capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution [2]. Decomposition method has developed for solving frontier problems of physics in [1]. Analytic solution of nonlinear boundary-value problems in several dimensions with the help of decomposition method has presented in [2]. Convergence analysis of decomposition methods has been presented in [3]. Adomian decomposition method has presented for solving fourth order integro-differential equations in [4]. Adomian decomposition method for solving second order ordinary differeial equations has presented in [5]. Adomian decomposition method and Haar wavelet methods have been presented for solving some oscillatory problems arising in science and engineering in [6]. Adomian decomposition method has been presented for solving nonlinear systems in [7]. Haar wavelet methods have been developed for solving differential and integral equations in [8-12].

2. THE ADOMAIN DECOMPOSITION METHOD (ADM)

Consider differential equation

$$Lu + Ru + Nu = (x) \qquad (1)$$

Where N represents non-linear factor, L represents the highest order derivative which is supposed to be invertible and R represents a linear differential factor, whose orderis less than L.From equation (1), we get

$$Lu = (x) - Ru - Nu \qquad (2)$$

As L is invertible, therefore L^{-1} exists. Multiply Equation (3.2) with L^{-1}, we obtain

$$L^{-1}Lu = L^{-1}(x) - L^{-1}Ru - L^{-1}Nu \qquad (3)$$

After simplification, from (3), we get

$$u = C + Dx + L^{-1}(x) - L^{-1}Ru - L^{-1}Nu \qquad (4)$$

where C and D are constants of integration and can be obtained from the initial or boundary conditions. Adomian method approximate the solution of Equation (1) inthe form of infinite series.

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \qquad (5)$$

and decomposing the non-linear operator N as

$$N(u) = \sum_{n=0}^{\infty} A_n \qquad (6)$$

where A_n represents the Adomian polynomials as discussed in **[**2,3**]** and are givenby

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i u_i\right]_{\lambda=0} \quad , \qquad n = 0,1,2,3, \ldots$$

Putting (3) and (6) into (4) we obtain

$$\sum_{n=0}^{\infty} u_n = C + Dx + L^{-1}G(x) - L^{-1}R\left(\sum_{n=0}^{\infty} u_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right)$$

The recursive relationship is found to be

$u = G(x)$

$u_{n+1} = -L^{-1} - L^{-1}A_n$

Using the above recursive relationship, we can make solution of u as

$$u = \lim_{n\to\infty} \phi_n(u) \qquad (7)$$

Where

$$\phi_n(u) = \sum_{i=0}^{n} u_i \qquad (8)$$

3. HAAR WAVELETS

The amplitudes of the switched rectangular waveforms that make up the orthogonal family of Haar functions can vary from one function to the next. A wavelet family, also known as a basis, is made up of a series of rescaled square-shaped functions called Haar wavelets. The Haar wavelet function $h_i(x)$ is defined in the interval $[\alpha\ \gamma]$ as:

$$h_i(x) = \begin{cases} 1, & \alpha \le x < \beta \\ -1, & \beta \le x < \gamma \\ 0, & otherwise \end{cases}$$

Where $\alpha = \frac{k}{m}, \beta = \frac{k+0.5}{m}, \gamma = \frac{k+1}{m}, m = 2^j and\ j = 0,1,2,3,4,\ldots\ldots, J. J$ denotes the level of resolution. The integer $k = 0,1,2,3,\ldots\ldots, m-1$ is the translation parameter. The index i is calculated as $i = m + k + 1$.The minimal value $i = 2$ and the maximal value of $i = 2^{j+1}$.The collection points are calculated as

$$x_i = \frac{(l-0.5)}{2M}, l = 1,2,3,4,\ldots\ldots\ldots,2M$$

The operational matrix P, which is $2M \times 2M$, is calculated as below

$$P_{1,i}(x) = \int_0^{x_1} h_i\,(x)dx$$

$$P_{n+1,i}(x) = \int_0^x P_{n,i}\,(x)dx \qquad n = 1,2,3,\ldots$$

4. NUMERICAL EXAMPLES

Example1. Consider the Pendulum equation (un-damped motion)

$$\frac{d^2x}{dt^2} + \mu^2 x = 0 \qquad (9)$$

The exact solution is $x(t) = cos\mu t$. Let $\mu = 1$ and initial condition $x(0) = 1,\ x'(0) = 0$. Apply Adomian decomposition method we obtain

$$L^{-1}(x) = -L^{-1}(x)$$

$$x(t) = x(0) + x'(0) - \int_0^t \int_0^t x(t)dtdt$$

$$x(t) = 1 - \int_0^t \int_0^t x(t)dtdt \qquad (10)$$

$$x(t) = \sum_{n=0}^{\infty} x_n$$

From (10) we have

$$\sum_{n=0}^{\infty} x_n = 1 - \int_0^t \int_0^t \sum_{n=0}^{\infty} x_n(t)dtdt \qquad (11)$$

$$x_0 + x_1 + x_2 \ldots = 1 - \int_0^t \int_0^t (x_0 + x_1 + x_2)dtdt \qquad (12)$$

From (12) we have

$$x_0 = 1$$

$$x_1 = -\int_0^t \int_0^t x_0 dtdt = -\frac{t^2}{2}$$

$$x_2 = -\int_0^t \int_0^t x_1 dtdt = \frac{t^4}{24}$$

$$x_3 = -\int_0^t \int_0^t x_2 dtdt = -\frac{t^6}{720}$$

The solution is

$$x = x_0 + x_1 + x_2 + x_3 + \cdots$$

$$x(t) = 1 - \frac{t^2}{2} + \frac{t^4}{24} - \frac{t^6}{720} + \cdots = cost$$

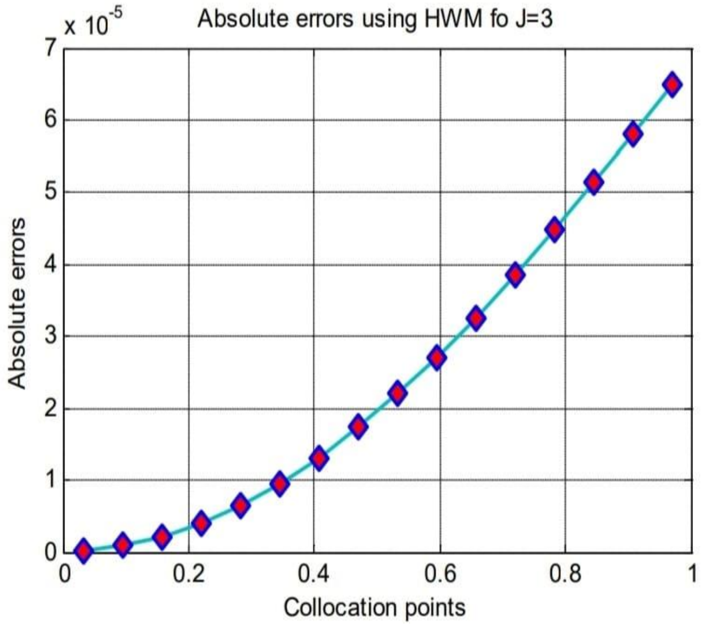

Figure 1 :Shows absolute error obtained in case of HWM foe example1

Figure 1 shows the absolute error of numerical solutions using Haar wavelet method (HWM) for J=3. Figure 2 shows the comparison of numerical results obtained with ADM (using four terms approximation) and HWM of Example 1.

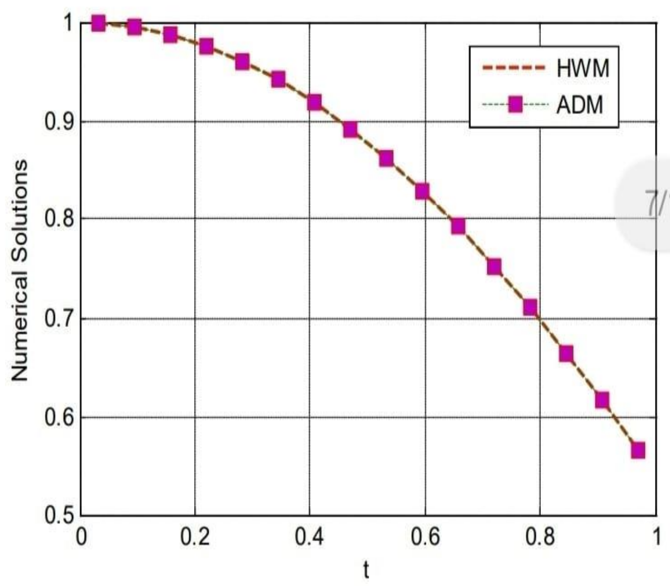

Figure 2: Shows comparison of numerical results of HWM and ADM of example 1

Example 2. Consider the Pendulum equation (Damped motion)

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \mu^2 x = 0 \tag{13}$$

The exact solution is $x(t) = (1+t)e^t$. Let $\mu = 1,\ k = 1$ and the initial conditions

$x(0) = 1,\ x'(0) = 0$. Apply Adomian decomposition method we have

$$L^{-1}(x) = -L^{-1}(x) - L^{-1}(x')$$

$$x(t) = 1 + 2t - 2\int_0^t x\,dt - \int_0^t\int_0^t x\,(t)dtdt \tag{14}$$

Now

$$x(t) = \sum_{n=0}^{\infty} x_n$$

From (14) we have

$$\sum_{n=0}^{\infty} x_n = 1 + 2t - 2\int_0^t \sum_{n=0}^{\infty} x_n\,(t)dt - \int_0^t\int_0^t \sum_{n=0}^{\infty} x_n\,(t)dtdt \tag{15}$$

$x_0 + x_1 + x_2 + \cdots = 1 + 2t - 2\int_0^t (x_0 + x_1 + x_2 + x_3 + \cdots)dt - \int_0^t\int_0^t x_0 + x_1 + x_2 + x_3 + \cdots$

$$x_0 = 1 + 2t$$

$$x_1 = -2\int_0^t x_0\,dt - \int_0^t\int_0^t x_0\,dtdt = -2t - \frac{5}{2}t^2 - \frac{t^3}{3}$$

$x_2 = -2\int_0^t x_1\,dt - \int_0^t\int_0^t x_1\,dtdt = 2t^2 + 2t^3 + \frac{3}{8}t^4 + \frac{1}{60}t^5$

The solution is

$$x(t) = x_0 + x_1 + x_2 + \cdots = 1 - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{8} + \frac{t^5}{30} - \frac{t^6}{144} + \cdots$$

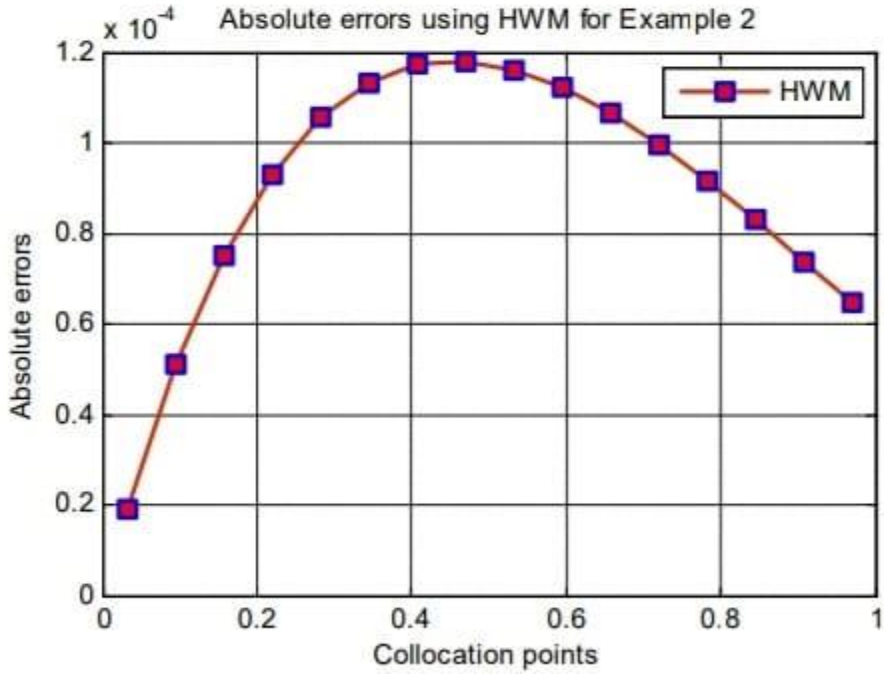

Figure 3: Shows error obtained in case of HWM for example 2

Figure 3 shows the absolute errors of numerical solutions using Haar wavelet method (HWM) for J=3. Figure 4 shows the comparison of numerical results obtained with ADM (using four terms approximation) and HWM of Example 2

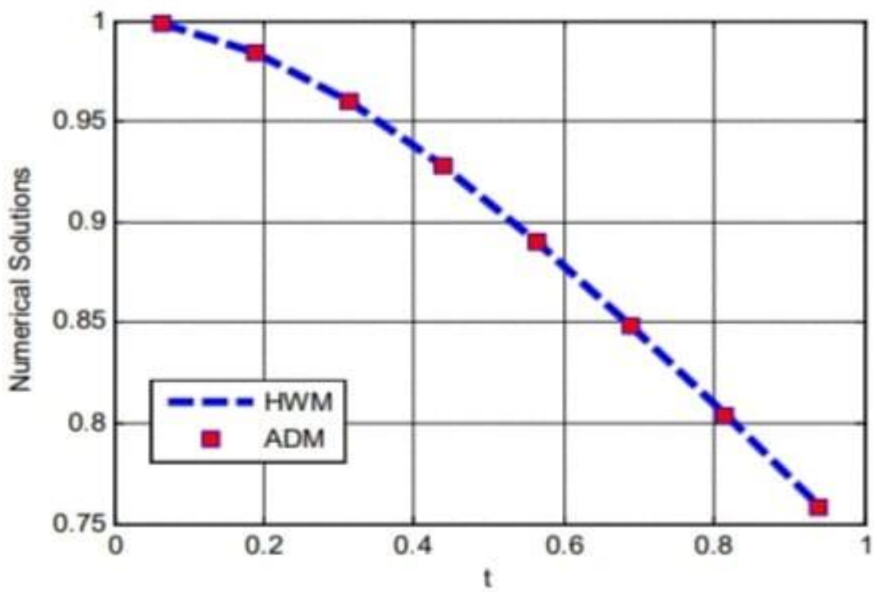

Figure 4: Comparison of numerical results of HWM and ADM of example 2

Example 3: Consider the Un-damped pendulum equation

$$\frac{d^2x}{dt^2} + \mu^2 x - \frac{1}{6}\mu^2 x^3 = 0 \quad (16)$$

Let $\mu = 1$, and the initial conditions $x(0) = 1,\ x'(0) = 0$. Apply Adomian decomposition method we have

$$x(t) = 1 - \int_0^t \int_0^t x(t)dtdt + \frac{1}{6}\int_0^t \int_0^t x^3(t)dtdt \qquad (17)$$

Putting

$$x(t) = \sum_{n=0}^{\infty} x_n$$

From equation

$$\sum_{n=0}^{\infty} x_n = 1 - \int_0^t \int_0^t \sum_{n=0}^{\infty} x_n(t)dtdt + \frac{1}{6}\int_0^t \int_0^t \left[\sum_{n=0}^{\infty} x_n\right]^3 dtdt \qquad (18)$$

From equation (18) we have

$$x_0 = 1$$

$$x_1 = -\int_0^t \int_0^t x_0\, dtdt + \frac{1}{6}\int_0^t \int_0^t {x_0}^3(t)dtdt = 1 - \frac{5}{12}t^2$$

$$x_2 = -\int_0^t \int_0^t x_1\, dtdt + \frac{1}{6}\int_0^t \int_0^t {x_1}^3 + 3{x_0}^2 x_1 + 3{x_1}^2 x_0 dtdt$$

The solution is

$$x_n(t) = x_0 + x_1 + x_2 + \cdots$$

$$x(t) = 1 - \frac{5}{12}t^2 - \frac{5}{288}t^4 - \frac{5}{1728}t^6 - \frac{125}{580608}t^8 + \cdots$$

4. CONCLUSION

In this paper only linear equation are considered, but this method are also applicable for non-linear system. From above numerical data, it is concluded that Adomian decomposition method (ADM) and Haar wavelet method (HWM) are powerful mathematical technique for solving application of linear differential equations such as simple pendulum oscillation problems arising in many applications of science and engineering. For more accuracy the number terms may be increased.

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