International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 03 Issue: 12 | Dec -2019 ISSN: 2582-3930

AXISYMMETRIC PUNCH PROBLEM IN AN INFINITE ELASTIC THICK PLATE

Dr Rajendra Kumar Tripathi
Department of Mathematics
Pioneer Mahila Mahavidyalya, Barabanki U.P. India

ABSTRACT: The deformation in an elastic late due to axisymmetric punch has been investigated by the method of dual integral equations when the surfaces of the place are indented by a punch of circular cross-section. Dual integral equation have been solved through series method adopting Legendre orthogonal polynomials.

Introduction: The punch problem, more commonly know as Boussinesq's problem, for a semi-infinite space is aclassical one. Sneddon [3] considered the axisymmetric Boussineq's problem for a half-space under a punch of arbitrary profile and obtained simple formulae for the depth of penetration of the tip of the punch , the total load that must be applied to the punch to achieved his penetration, the distribution of pressure under the punch, and the shape of the free surface.

Recently Lebedev and Uflyand [7] considered the punch problem for an elastic layer.

They developed the technique of reducing the dual integral equations to a single Fredholm integral equations of the first kind for the determination of one unknown function in terms of which the required physical quantities are expressed. It is worth mentioning that the systematic treatment of the punch problem for a half-space and a layer are available in Sneddon's book [4] and his monograph [5] All the mentioned authors have considered the elastic layer lying over a rigid foundation. Paria [1] has attempted the non-symmetrical punch problem in layered media by the Wiener Hopf method. Keer [6] has development the integral equations for a class of non-symmetrical punch problem for an elastic half-space and an elastic ; he solved the problem of a wedge indenting a half-space as an application of his analysis.

In a recent paper Dhaliwal [9] has considered the punch problem for an elastic layer resting on an elastic stratum. The purpose of the present paper is to obtain general analytical result for a Boussinesq problem of an elastic plate when the surfaces area indented by a rigid punch of circular cross-section. The problem is reduced to the solution of a pair of dual integral equations which have been solved through series method adopting Legendre orthogonal polynomials.

Basic Equation and Boundary Conditions.

The stresses and displacements in an axisymmetric problem can be derived from the Love function φ satisfying

$$\nabla^2\nabla^2\psi = 0, \qquad \ldots(2.1)$$

Where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ ...(2.2)

The displacements and stresses field is

$$u = -\frac{\partial^2\varphi}{\partial r\partial z}; w = 2(1-\eta)\nabla^2\psi - \frac{\partial\psi}{\partial z^2} \ldots(2.3$$

$$\widehat{zz}=2G\frac{\partial}{\partial z}[(2-\eta)\nabla^2 - \frac{\partial^4}{\partial z^2}]\psi, \quad \ldots(2.4)$$

$$\widehat{rz}=2G\frac{\partial}{\partial z}\left[(1-\eta)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\psi, \quad \ldots(2.5$$

$$\widehat{rr}=2G\frac{\partial}{\partial z}\left[\eta\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r}\right]\psi, \quad \ldots(2.6)$$

$$\widehat{\theta\theta}=2G\frac{\partial}{\partial z}\left[\eta\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r}\right]\psi, \quad \ldots(2.7)$$

Where u and w are axial and radial displacements and G is modulus of rigidity and η is poisson's ratio respectively.

We consider an elastic thick plate whose free surface z= $\pm$ h are indented by a rigid punch of circular cross –section, the boundary conditions on these faces may taken as follows.

$W(r, \pm h) = f(r), 0 \le r < 1,$ ..(2.8)

$\widehat{zz}(r, \pm h) = 0,$ $r > 1,$ ..(2.9)

$\widehat{rz}(r, \pm h) = 0,$ $r \ge 0,$ (2.10)

The function f (r) 's determined by the shape of the punch ; in the case of smoth profile it contain an additive constant which is determined by the conditions that $\widehat{zz}$ (1, $\pm$ h) is finite.

The Problem

A Solution of the equation (2.1) can be assumed in the form

$$\psi = \int_0^\infty \xi^{-3}[A(\xi z)$$

$$+B(\xi)\xi z \sinh(\xi z)] J_0(\xi r)\, d\xi, \quad \ldots(3.1)$$

where A and B are functions of ξ only.

The expressions for displacement and stresses using equations (2 3) to (2.7) are:

$$u=\int_0^\infty \xi^{-1}[\{A(\xi) + B(\xi)\}\sinh(\xi z)$$

$$+B(\xi)\,\xi z\cosh(\xi z)J1\ (\xi r)\, d\xi, \quad \ldots(3.2)$$

$$w=\int_0^\infty \xi^{-1}[\{2B(\xi)(1-2\eta)A(\xi)\}\cosh$$

$$(\xi z) - B(\xi)\xi z\,\mathrm{Sinh}(\xi z)]J0(\xi r)d\xi, (3.3)$$

$$\widehat{zz} = 2G\int_0^\infty [\{B(\xi)(1\text{-}2\eta)\text{-}A(\xi)\}\sinh(\xi z$$

$$-B(\xi)\,\xi z\cosh(\xi z)J0\ (\xi r)\, d\xi, \quad \ldots(3.3)$$

$$\widehat{rz}=2G\int_0^\infty [\{A(\xi) + (2\eta B(\xi)\}cosh(\xi z)$$

$$+B(\xi)\,\xi z\sinh(\xi z)J1\ (\xi r)\, d\xi, \quad ..(3.4)$$

$$\widehat{rr}=2G\int_0^\infty [\{(B(\xi)(1+2\eta)+A(\xi))\sinh(\xi z)$$

+B(ξ) ξz cosh (ξz)J0 (ξr) dξ,

-{A (ξ) + $B(\xi)$) sinh (ξz)

+B(ξ) ξz cosh (ξz),$\frac{J1\ (\xi r)\ d\xi}{\xi r}$] (3.6)

$\widehat{\emptyset\emptyset}$=2G $\int_0^\infty$ [{(ηB (ξ) sinh(ξz)} J1 (ξ) d ξ

+{A(ξ) + b (ξ) sinh (ξz)

+B(ξ) ξz cosh (ξz)}$\frac{J1\ (\xi r)\ d\xi}{\xi r}$] ..(3.7)

where

$$M(\xi)= \frac{\{2(1-\eta)\cosh(\xi h)\}}{\xi[\sinh(\xi h)-\xi h/\cosh(\xi h)]} \quad (3.12)$$

Hence the problem reduces to the solution of the pair of dual integral equations (3.9) and (3.11).

Solution of the dual Integral Equations: In the equations (3.9) and (3.11) the unknown is N (ξ) or B(ξ). Since the surface traction $\widehat{zz}$ at z = $\pm$ h and not N(ξ)is of direct interest, the former is expanded in series of Legendre polynomials as follow:

$\widehat{zz}|$z-=h = $\sum_{n=0}^{\infty} a$nPn (1-2^{2}r) for 0≤ r < 1,

=0 r ≥ 1 ..(4.1)

On getting the values of constant $a_n$the problem can be solved. Equations (4.1) identically satisfies the first integral equations (3.9) Then it follows [2] that

N(ξ) = $\sum_{n=0}^{\infty} a$nJ2n+1 (ξ). …(4.2)

The second integral equations then takes the form

f(r) = $\int_0^\infty M$ (ξ) $\sum_{n=0}^{0} a$nJ2n+1(ξ)Jo (ξr) dξ

for 0 ≤ r < 1 …(4.3)

putting

$\lambda\eta$ (r)= $\int_0^\infty M$ (ξ) J2n+1(ξ)Jo(ξr) dξ … (4.4)

Equations (4.3) becomes

$\sum_{n=0}^{\infty} a$n $\lambda\eta$ (r)=f (r)

The dual integral equations (3.9) and (3.11) have now been reduced to equation (4.5) in which the a_n's are the unknown Equation (4.5) is identical in form with the series solution of aFredholm integral equations of first kind. Now Schmidt method [8] can be used for its solution.

REFERENCES

1. Paria, G, Q J. Maths, 11,116 (1960).
2. Gradshteyh and I. M. Ryzhik, Table of integrals series and products, Academic Press, New York and London, 825 (1965).
3. Sneddon I. N, Int J. Engg. Sci., 3, 47 (1965).
4. Sneddon I. N, Fourier Transforms, McGraw Hill (1965).
5. Sneddon. I. N, (Ed) Survey of Articles on the Applications of integral transform in the theory of Elasticity by I.S. Ufliand Tech. Rep. No AFOSR-65-1956 North Caroline State university.
6. Keer , L.M.Q, J. Mech. Appl. Maths 4, 423 (1964).
7. Lebedev N. N. and Y.S. Uplyand, J. Appl. Maths . Mech., 22, 442 (1958)

8. Morse ,P M., and H. Peshback, Methods of theoretical Physics, Mc GrawHill Book Co.NewYork Part I, 926 (1953)
9. Dhaliwal , R. S. Int J. Engg Sci 8, 273 (1970)