**Binary Relations with Different Properties of Structural Patterns in Network Information Flow**

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**Abstract**

The research paper intends to introduce innovative problems of structural model of network information flow which is inspired by patterns of Binary relations through the logical operators in the network flow, supporting parallel and distributed system. In this paper an effort is made to find out the properties of binary relations through the matrix connectivity of point to point network by converting the network into its equivalent connectivity of matrix between nodes, so that communication algorithm to connectivity can be developed for its relations. The logical operations will be introduced between the vectors of nodes or edges of the connectivity matrix. These operations will be used to explore the properties of binary relations with different patterns of the architecture for network information flow. The logical operations will be performed to between nodes & edges, in which number of vertexes connected to one another. The objective is to find out found vertex - vertex relation between edges through the matrix relation and to prove the tautology for the symmetric property is true, through logical operation.

***Keywords: Network Connectivity, Matrix Relation, Structural Model, Interconnection Network.***

**Introduction**

Structural networks display both patterns of network and matrix relations between nodes as vertices to edges in their structure. We introduce the information sources in network flow to determine the structural models connectivity descriptions with relational property between nodes. In the research paper effort has been made to point out the different properties of binary relations using logical operations between nodes for the connectivity of network patterns [1][3]. The logical operations will be used to find a degree of each node between two edge and number of vertexes connected to one another in figure 2, we found vertex - vertex relation between edges through the matrix relation with different patterns and proved the tautology for the symmetric property is true, through logical operators in network flow. Network flows, which can be embodied as a set of nodes representing structural models and a set of links representing connections, shows the many different structural properties of network patterns, representing the topological structure with logical properties through the matrix relation[4]. The arrangement and connectivity between nodes in a network model is referred to binary relations as Symmetric, Anti symmetric, Transitive, Reflexive, etc.

In this paper an attempt to represent properties of various relationship with multiple nodes has been made, which is explored through connectivity & complexity with logical operations between nodes and algorithm[8], for linear network flow problems [6][7][9]. The purpose of this paper is to clarify the binary relation with different properties and patterns for the multiple nodes with matrix connectivity. In this paper it is discussed in some points a simple model of network patterns with different example of network structure for relationships and we used different matrix relations for binary relations and showing the matrix relation table with logical operation illustrating various figure and table to each patterns of network [5].

**Structural Relations with different Patterns**

The Properties of structural Relationships through the mathematical model, a binary relation represents two [sets](https://en.wikipedia.org/wiki/Set_(mathematics)) X and Y is a set of [proper pairs](https://en.wikipedia.org/wiki/Ordered_pair) (a & b) consisting of elements a of X and elements b of Y. Assuming the information of binary relation, an element *a* is related to X and an element *b is related to Y* if and only if the pair (*a*, *b*) belongs to the set. Binary relations are used in many patterns of mathematical models in [graph theory](https://en.wikipedia.org/wiki/Graph_theory), in arithmetic, in [linear data](https://en.wikipedia.org/wiki/Linear_algebra) structure and many more operations.

**Structural relationship models in network flow:**

In graph theory a network flow is directed-graph where each edge has a capacity and each edge receives a flow connected by vertex. A network patterns can be used in logical operation to represent the matrix, they determines vertex to vertex connectivity between nodes of the interconnection network. A network is a graph G = (v, e) where v is vertices and that connected between edges, we may assume that if (u, v) ∈ e then (v, u) is also a member of edge and then represents set e (v, u) = 0

***v1  v5***

***v4 v2 v3***

Figure 1. Structural relationship models in network flow.

Here is not a Reflective, this graph is shows all relation is Symmetric in figure 1.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | ***v1*** | ***v2*** | ***v3*** | ***v4*** | ***v5*** | ***link*** |
| ***v1*** | 0 | 1 | 0 | 0 | 0 | 1 |
| ***v2*** | 1 | 0 | 1 | 1 | 1 | 4 |
| ***v3*** | 0 | 1 | 0 | 0 | 0 | 1 |
| ***v4*** | 0 | 1 | 0 | 0 | 0 | 1 |
| ***v5*** | 0 | 1 | 0 | 0 | 0 | 1 |

1. *v1Rv2* ∧ *v2Rv1→v1Rv2*
2. *v2Rv3 & v3Rv2→v2Rv3*
3. *v4Rv2 & v2Rv4→v4Rv2*
4. *v2Rv5 & v5Rv2→v2Rv5*
5. *v1Rv3 & v3Rv1→v1Rv3*
6. *v4Rv5 & v5Rv4→v4Rv5*

Table 1. Matrix Representation of figure 1.

Here Table 1 shows the logical relation between vertex to vertex in figure 1 (v2, v5 shows maximum connectivity between nodes). In figure 1 we have shows this graph which are converted in vertex – vertex relations for the symmetric position by matrix tables shown in table 1(a) & (b). Now we are using logical operation "AND" with implication between vectors of matrix show in table 1(a) & (b). We have indicated and proof that the tautology for the symmetric property [2].

**i.e.**

|  |  |  |  |
| --- | --- | --- | --- |
| **S. No.** | P | Q | P → Q |
| **1** | T | T | T |
| **2** | T | F | F |
| **3** | F | T | T |
| **4** | F | F | T |

***(a)Symmetric Relation in matrix from the logic of binary relation:***

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| V1 | V2 | V3 | (V1∧V2) | (V2∧V3) | (V1∧V3) | (V1∧V2) ∧ (V2∧V3) | (V1∧V2) ∧ (V2∧V3)→ (V2∧V3) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | F | T | F | T |
| T | F | F | F | F | F | F | T |
| F | T | T | F | T | F | F | T |
| F | T | F | F | F | F | F | T |
| F | F | T | F | F | F | F | T |
| F | F | F | F | F | F | F | T |

***(b)Symmetric Relation in matrix from the logic of binary relation:***

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| V4 | V2 | V5 | (V4∧V2) | (V2∧V5) | (V4∧V5) | (V4∧V2) ∧ (V2∧V5) | (V4∧V2) ∧ (V2∧V5)→ (V4∧V4) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | F | T | F | T |
| T | F | F | F | F | F | F | T |
| F | T | T | F | T | F | F | T |
| F | T | F | F | F | F | F | T |
| F | F | T | F | F | F | F | T |
| F | F | F | F | F | F | F | T |

Table 2. (a) & (b) binary relation shows tautology of figure 1.

**Properties of Binary Relations with Different Patterns**

Now we are representing the structural relation between nodes of any graph by converting matrices for each relation. We are assuming that the relation between node of vertex - vertex that may be possible and we will take one for self node/self vertex in our connectivity matrix. We have represented and indicated other different patterns for each binary relation with matrix in Network Information Flow.

**V1 V2 V3**

Figure 2. Graph Structure with 3 node.

**Matrix representation of Symmetric:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ***v1*** | ***v2*** | ***v3*** | ***degree*** |
| **v1** | 0 | 1 | 0 | **1** |
| **v2** | 1 | 0 | 1 | **2** |
| **v3** | 0 | 1 | 0 | **1** |

Table 3. Matrix relation of figure 2

***Symmetric Relation in matrix from the logic of binary relation:***

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| V1 | V2 | V3 | (V1∧V2) | (V2∧V3) | (V1∧V3) | (V1∧V2) ∧ (V2∧V3) | (V1∧V2) ∧ (V2∧V3)→ (V2∧V3) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | F | T | F | T |
| T | F | F | F | F | F | F | T |
| F | T | T | F | T | F | F | T |
| F | T | F | F | F | F | F | T |
| F | F | T | F | F | F | F | T |
| F | F | F | F | F | F | F | T |

Table 4. binary relation shows tautology of figure 2.

Here the table 1(a) found us degree of each node between two edge and number of vertexes connected to one another in figure 2, the table 2(b) found vertex - vertex relation between edge of figure 1 and proof the tautology for the symmetric property is true, through logical operation.

**Properties of Relation**

1. **Symmetric (a):** The Relation R on {4,2,3} given by {(4,4),(2,4),(3,4)} is Symmetric (all path are two way.

**v 1 v2**

Figure 3. Graph Structure with 2 node.

*v1Rv2 & v2Rv1 => v1Rv2 then Symmetric*

* R is Symmetric if for x, y ∊A, if (x, y) ∊R, than (x, y) ∊ R

1. ***Graph Representation of symmetric relation :***

**Rule:** if R is a symmetric relation, all links are bi-directional.

Example: **x** **y**   **e**

**z f**

**Fig. 4 Graph Representation of symmetric relation**

1. ***Matrix Representation of Symmetric Relations:*** *R1=xRy & yRx→xRy.*

|  |  |  |  |
| --- | --- | --- | --- |
|  | ***x*** | ***y*** | ***z*** |
| ***x*** | 0 | 1 | 0 |
| ***y*** | 1 | 0 | 1 |
| ***z*** | 0 | 1 | 0 |

Table 5. Binary relation of figure 4.

1. ***Symmetric and anti-Symmetric Relations:***

Compare the relations:

1. *A= {1,2,3)}, R1={(1,1), (1,2), (2,3), (3,3)}*
2. *A= {1,2,3)}, R2={(1,1), (1,2), (2,3), (3,3)}*
3. *A= {4,2,3)}, R3={(1,1), (1,2), (2,1), (2,3), (3,3)}*

R is neither symmetric nor anti-symmetric if it not symmetric and not symmetric.

* Symmetric: x R y ∊ y R x for all a and y
* Anti-Symmetric: x R y and y R x ∊ x=y neither: for some x and y: x R y, and y R x for others x R y is true, y R x is not true.

1. **Reflexive:** The Relation R on {1, 2} given R={(1,1), (2,2), (3,3)} is reflexive ( all Loops are present)

Figure 5. Example of Reflexive Relation.

**Matrix Representation of reflexive Relation:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ***v1*** | ***v2*** | ***v3*** | ***degree*** |
| ***v1*** | 1 | 1 | 1 | 3 |
| ***v2*** | 0 | 1 | 1 | 2 |
| ***v3*** | 0 | 0 | 1 | 1 |

Table 6. Binary relation for reflexive of figure 5.

*R = {(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)}*

*Reflexive: A = {1,2,3,1}, R1 = {(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)}*

*irreflexive: A = {1,2,3,1}, R2 = {(1,2),(2,3),(3,1),(1,1)}*

* *R4 is a Reflexive Relation.*
* *R2 is a irreflexive Relation.*

**3. Transitive:** The Relation R on {1,2,3} given by R={(1,4),(1,2),(2,1),(2,2),(2,3),(1,3)} is transitive.

*Example.*

v1 v2 v1 v2

v4 v3 v4 v3

Figure 6. Example of Transitive Relation.

1. *v1Rv2* , *v2Rv3 & v3Rv4 than v1Rv4*
2. *v1Rv2* ∧*v2Rv1, v2Rv3*∧*v3Rv2, v3Rv4*∧v4Rv3*→ v4Rv1*

**Matrix representation of Transitive Relation:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ***v1*** | ***v2*** | ***v3*** | ***v4*** |  |  | ***v1*** | ***v2*** | ***v3*** | ***v4*** |
| ***v1*** | 0 | 1 | 0 | 0 |  | ***v1*** | 0 | 1 | 1 | 1 |
| ***v2*** | 0 | 0 | 1 | 0 |  | ***v2*** | 0 | 0 | 1 | 1 |
| ***v3*** | 0 | 0 | 0 | 1 |  | ***v3*** | 0 | 0 | 0 | 1 |
| ***v4*** | 0 | 0 | 0 | 0 |  | ***v4*** | 0 | 0 | 0 | 0 |

Table 7. Binary relation for transitive of figure 6.

*Rule: if there is v1 link from v1 to v2 and v1 link from v2 to v3, than there must be v1 link from v1to v4.*

*A = {1,2,3,4}*

*R = {(1,2),(1,3),(1,4),(2,3),(2,3),(3,4)}*

*=> R is a transitive Relation.*

***Transitive Relations:***

Let R be a binary relation on set A.

R is transitive if for all x, y, z ∊ A, if (x, y) ∊ R and (y, z) ∊ R, than (x, z) ∊ R

*i.e...* (x R y ∊ y R z) ∊ x R z is true

**4. Anti-Symmetric:** A Relation R on a Set A is called Anti-Symmetric if (x,y) ∊ R and (y,x) ∊ R implies x=y.

*ex. R = {(1,1),(1,2),(3,2),(3,3)} is Anti-Symmetric*

v1 v2 v1 v2

v3

Figure. 7 Direct graphs.

**Matrix representation of Anti-Symmetric:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | ***v1*** | ***v2*** | ***v3*** |
| **1** | 0 | 1 | 0 |
| **2** | 0 | 0 | 1 |
| **3** | 0 | 0 | 0 |

Table 8. Binary relation of figure 7.

***Reflective:*** *v1Rv1, v1Rv2, v1Rv3, & v2Rv3.*

v1 v2

v5

v4 v3

Figure. 8 Example of reflective.

**Matrix representation:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | ***v1*** | ***v2*** | ***v3*** | ***v4*** | ***v5*** |
| ***v1*** | 1 | 1 | 0 | 1 | 1 |
| ***v2*** | 1 | 0 | 1 | 0 | 1 |
| ***v3*** | 0 | 1 | 0 | 1 | 1 |
| ***v4*** | 1 | 0 | 1 | 1 | 1 |
| ***v5*** | 1 | 1 | 1 | 1 | 0 |

Table 9. Binary relation of figure 8.

1. *v1 R v2 ^ v2 R v1*
2. *v1 R v3 ^ v3 R v1 Symmetric*
3. *v1 R v4 ^ v4 R v1*
4. *v1 R v5 ^ v5 R v1*

**Connection Patterns:**

Different network flows can be cited for reference to find out binary relations with different properties of patterns in network information flow. Binary relation between nodes can be established through vertex and edges, and also connectivity can assured through matrix relation. The structure given as under process the connectivity between nodes with following patterns of the interconnection network.

Figure. 9 Different patterns of connected graph.

***Matrix Representation:***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ***r0*** | ***r1*** | ***r2*** | ***r3*** |
| ***1*** | 1 | 0 | 1 | 0 |
| ***2*** | 1 | 1 | 0 | 0 |
| ***3*** | 0 | 1 | 0 | 0 |
| ***4*** | 0 | 0 | 1 | 1 |

***r0 2 r4***

***4 3***

***r2 4 r3***

***(a)***

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | ***r0*** | ***r1*** | ***r2*** | ***r3*** | ***r4*** | ***r5*** |
| ***1*** | 1 | 0 | 0 | 1 | 0 | 0 |
| ***2*** | 1 | 1 | 0 | 0 | 0 | 0 |
| ***3*** | 0 | 1 | 1 | 0 | 0 | 0 |
| ***4*** | 0 | 1 | 0 | 0 | 1 | 0 |
| ***5*** | 0 | 0 | 1 | 0 | 0 | 1 |
| ***6*** | 0 | 0 | 0 | 1 | 1 | 0 |
| ***7*** | 0 | 0 | 0 | 0 | 1 | 1 |

***r0* 2 *r4* 3 *r2***

**4 45**

**6 7**

***r3 r4 r5***

***(b)***

***r0***  ***r1***

1 ***r4*** 2

***r2*** 3 4 ***r3***

***(c)***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | ***r1*** | ***r2*** | ***r3*** |  | ***r4*** |
| ***1*** | 0 | 0 | 0 | 0 | 1 |
| ***2*** | 1 | 1 | 0 | 0 | 1 |
| ***3*** | 0 | 1 | 1 | 0 | 1 |
| ***4*** | 0 | 0 | 0 | 1 | 1 |

***r1 r2***

1 ***r3***  ***r4*** 4

2 3 5

***r2 r5***

***(d)***

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | ***r0*** | ***r1*** | ***r2*** | ***r3*** | ***r4*** | ***r5*** |
| ***1*** | 1 | 0 | 0 | 1 | 0 | 0 |
| ***2*** | 0 | 0 | 1 | 1 | 0 | 0 |
| ***3*** | 0 | 0 | 0 | 1 | 1 | 0 |
| ***4*** | 0 | 1 | 0 | 0 | 1 | 0 |
| ***5*** | 0 | 0 | 0 | 0 | 1 | 1 |

Table 10. Binary relation of figure 9 (a, b, c, d).

**Conclusion and Future work**

In this paper it is assumed that different properties of binary relation can be drawn easily through representation of logical operator with matrix relation between nodes. It determines topological and logical patterns both properties of various relations using logical operation to each structure between vertices & edges. In this paper an attempt has to be made to establish the binary relation with connectivity of nodes for the development of algorithms. The generalization that has to be drawn is that the use of logical operations shows that the binary relations may be possible and some more points can be discussed for validity in our future studies.

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