

Black Holes: Thermodynamics and Statistical Mechanics

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Abstract

Black holes are both the simplest systems we can study and amongst the most complex. It is the only known system where we must study General Relativity, Quantum Field Theory, Thermodynamics, Statistical Mechanics, and Quantum Information Theory. This paper reviews and offers criticism of black hole thermodynamics and the statistical definition of horizon entropy. The review is structured with respect to the black hole mechanics, Hawking radiation, Euclidean partition functions, statistical thermodynamic ensembles, microscopic state counting, generalized entropies, Page curves and extended phase spaces with dynamical horizons. Established equations for the different black hole solutions (Schwarzschild, charged, rotating, and anti de Sitter) will be compared, and for each solution, four theoretical charts will be constructed based on the equations. The constructed charts give additional support for the area-entropy law and the surface gravity-temperature relationship. For an asymptotically flat Schwarzschild black hole, we find that the temperature is inversely proportional to mass, entropy is proportional to mass squared, and the heat capacity is negative. Thus, depending on charge, rotation, confinement, and the selected ensemble, canonical stability may or may not exist. In the supersymmetric and extremal regimes, we find that state counting reproduces the Bekenstein-Hawking entropy, though we do not find a single state for all realistic black holes. While many recent works have elucidated the problem of unitarity in black hole thermodynamics, recent extensions of the black hole thermodynamic law to horizons outside of equilibrium suggest that we may not be forced to restrict our reasoning to stationary systems. Overall, we can better confirm black hole thermodynamic universality over other specific microscopic explanations.

The horizon is both a thermodynamic limit and reveals the quantum statistical structure of spacetime.

Keywords: black-hole thermodynamics; Bekenstein-Hawking entropy; Hawking radiation; statistical mechanics; quantum gravity; information paradox; entanglement entropy; black-hole chemistry

1. Introduction

A classical black hole can be described as a region of spacetime that can be thought of as a boundary. Once a signal crosses this boundary, it can no longer be directed to the outside universe. In stationary general relativity, one can define an exterior state that is due to a limited number of quantities that are conserved. These quantities are mass (M), angular momentum (J), and electric charge (Q). From this description, black holes superficially seem to be the antipodes of other thermodynamic systems. Black holes do not have a surface in the classical sense, do not have a molecular structure, and classically only absorb matter and never radiate anything (and even appear to maintain this property in quantum theory). However, the simplicity is really complex. If a region of spacetime that contains a boundary classically defined as “invisible to the outside” absorbs one of the boxes of matter that contains entropy, the outside observer also loses this entropy. If the black hole’s horizon is considered to have no entropy, then the second law of thermodynamics is violated. To account for this, Bekenstein argued that the black hole’s horizon area is in fact the entropy, thus making the area increase a thermodynamic fact and not a glum fact of geometry. (Bekenstein 1973)

Hawking formalized the analogy by illustrating that quantum fields generate thermal-like flux in black-hole spacetime at temperatures related to the surface gravity (Hawking, 1975). Following this logic, the black-hole horizon area A and surface gravity κ are equivalent to the black-hole entropy S and temperature T , respectively. S and T are defined so that they imply the validity of the first law of thermodynamics, the second law of thermodynamics in a generalized form, a gravitational partition function, and a series of the stability of equilibria. The purely thermal final state represents the paradox of the black-hole-semistatics, as it appears to destroy all correlations and is in direct contradiction to the unitarity

of quantum mechanics. Thus, black-hole thermodynamics combines a successfully integrated theory of semiclassicalism with an infinite array of unresolved statistical-mechanical theories.

The horizon represents a merger of geometry, causality, quantum mechanics, and information. As a result, black-hole entropy S increases with area instead of with volume, as is the case with ordinary thermodynamic systems, indicating that the independent degrees of freedom in a gravitational system may be determined by the system's boundary. The temperature and the near-horizon geometry may not be determined by a material equation of state. We may even postulate that the horizon's heat capacity is negative, and this may be a reflection of the nature of gravitation. Black-hole thermodynamics also reflects the limitations and nature of statistical systems and the thermodynamic limit.

The main problem here is the distinction between macroscopic universality and microscopic ambiguity. Laws related to stationary black-hole mechanics, Hawking temperature, and the leading entropy formula hold strongly. The identity of states for the formula $S = k_B \ln \Omega$, on the other hand, remains framework-dependent. For certain black holes in string theory, supersymmetry allows computing the degeneracy of states. Various approaches to entropy and its associated boundary data also cross the boundary of holism to explain the connection between bulk spacetime and lower dimensional quantum theory. While these explanations can be somewhat flexible, they cannot be easily substituted for one another, and one must be careful to state their limitations.

The second issue deals with equilibrium. An isolated Schwarzschild black hole loses energy and increases temperature (which is counter to classical meaning). This means negative heat capacity and results in an unstable equilibrium. A stable equilibrium is possible in a finite cavity, AdS spacetime, or in some regions of charged and rotating parameter space. The extension of black hole thermodynamics interprets the cosmological constant (Λ) as a form of pressure and black hole mass as enthalpy, and reveals a phase transition of the liquid-gas critical point type (Kubizňák et al. 2017). These findings suggest that stability is not solely determined by the horizon geometry, but rather by the entire thermodynamic context.

This review aims to reconcile four disciplines that are usually treated independently, i.e., classical horizon mechanics, semiclassical radiation and Euclidean statistical mechanics, microscopic and entanglement-based entropy and extended or non-equilibrium thermodynamics. It addresses black holes of the Schwarzschild, Reissner–Nordström, Kerr/Kerr–Newman, and AdS varieties. Analytic relations and dimensionless graphs are deployed in preference to observational data. The documents cited in Section 2 are introduced to illuminate the historical and conceptual dimensions. The theoretical background and the key equations are provided in Section 3. The objectives and the methodology are outlined in Sections 4 and 5, respectively. The limitations of the methodologies and the comparative results are presented in Section 6. This review delineates the established consensus and the more speculative domains in which a deeper theory of quantum gravity will be required.

2. Literature Review

2.1 Foundational Transition from Mechanics to Thermodynamics

Bardeen, Carter, and Hawking (1973) described four laws regarding black holes in equilibrium. The first says that under certain conditions, the surface gravity of a black hole's horizon is constant. The first law deals with mass, area, angular momentum, and charge differentials. The second law is related to Hawking's area theorem, and the third law states that it is impossible to reduce the surface gravity of a black hole to zero through finite means. At this point of the theory, the relationship to thermodynamics was structural: κ was like temperature, A was like entropy, but a black hole had no physical temperature.

Bekenstein (1973) surmised that black hole area is a form of entropy, stating that if this was not true, the area would be in violation of the second law and all matter entropy would disappear. Hawking's calculation related to black hole entropy and provided the constant to the this equation, $T_H = \frac{\hbar \kappa}{2\pi c k_B}$, producing a result of $S_{BH} = \frac{k_B c^3 A}{4G\hbar}$. The works of Bekenstein and Hawking transformed an analogy into a system of black hole thermodynamics. The generalized second law states that the entropy of a black hole and the entropy of the universe combined can not decrease.

Gibbons and Hawking (1977) developed this black hole thermodynamics further into equilibrium statistical mechanics via the old Euclidean gravitational action. Analytic continuation into imaginary time (in the region of a nonextremal black hole) leads to a polar geometry. A conical singularity can be removed by imposing a finite (Euclidean) time period, which yields the inverse of temperature. The action, in this case, is a partition function of the semiclassical theory of

black holes and is related to the logarithm of a partition function. This approach yields a value of entropy equal to one quarter of the area of a black hole and allows for the comparison among different phases of black hole thermodynamics.

2.2 Geometrical and Noether-Charge Entropy

The quarter-area result is a case of Einstein gravity but is part of a broader class of geometrical constructions. In Wald (1993), it was shown that in generally covariant theories, the entropy of stationary black holes can be expressed as a Noether charge on a cross-section of the horizon. Higher-order curvature corrections do not preserve the distinctive area in the entropy functional if a classical solution has an area. This result has relevance for statistical mechanics since if a theory is presented that defines a thermodynamic state function, it does not imply a specific microscopic model of counting.

Geometric entropy is not some arbitrary construction. It is the quantity that makes the variational principle of the gravitational action assume the form of the first law of thermodynamics. However, geometry in itself does not determine Ω . The result regarding Noether charge specifies the macroscopic entropy for a theory, while statistical mechanics inquires what quantum states of a theory are responsible for the macroscopic states and why in a number of theories, the leading contribution is an area.

2.3 Statistical and Microscopic Interpretations

The equation $S = k_B \ln \Omega$ requests a microscopic perspective. Initial attempts viewed the entropy either as thermal bath entropy or as the degrees of freedom close to the horizon. Such approaches tend to generate area scaling because of the short-distance correlations being concentrated at the boundary, and while they also introduce ultraviolet divergences and dependences on renormalization, entanglement entropy would be absorbed by the renormalization of Newton's constant. However, this by itself does not determine what are the fundamental degrees of freedom (Solodukhin, 2011).

Strominger and Vafa (1996) achieved what is known to be one of the first breakthroughs by counting the D-brane bound states that correspond to one class of five-dimensional supersymmetric extremal black holes. In the weak coupling framework, the degeneracy is calculable and, through the use of protected quantities, one can compare the two extremes of the strong coupling black hole. The logarithm of the degeneracy coincides with the Bekenstein-Hawking entropy. This is compelling evidence that gravitational entropy is capable of counting quantum states, but it does not provide a universal counting of states for the nonextremal astrophysical Kerr black holes.

Other attempts place the degrees of freedom at or near the horizon, quantize the geometry of the horizon, or use near-horizon conformal symmetry. Typically, these methods provide leading terms that are area proportional and predict subleading terms of a logarithmic order. The differences between these approaches are mainly the Hilbert space upon which they are based, the treatment of gauge fixing, and the overall dependence on the freedom of choice during the quantization process. The frequent occurrences of area law across different approaches and descriptions suggest that the actual picture is dominated by symmetry, and entanglement and constraint principles of gravity rather than a literal picture of a microscopic world.

2.4 Entanglement, Holography, and Quantum Information

Holography provides a geometric realisation of quantum entanglement. Ryu and Takayanagi (2006) derived a relation between the entropy of a boundary region in holographic quantum field theory and the area of a bulk minimal surface. Quantum corrections require an extremisation of the generalized entropy, which can also be interpreted as a surface area, giving rise to quantum extremal surfaces (Engelhardt & Wall, 2015). The aim is to connect black hole entropy and subsystems of entanglement, thus making the recovery of information a problem of reconstructing bulk regions from quantum degrees of freedom.

The Page curve serves as the primary diagnostic. The entropy of a system in a pure state increases due to the system being partially in a black hole. The span of time until the system becomes fully in the black hole is the Page time (Page 1993). The dominant contribution to black hole evaporation in the semiclassical theory is a continual increase in black hole radiation. In contrast, part of the black hole entanglement wedge loops, and the Page curve is reproduced, as a result of the quantum extremal surfaces (Penington, 2020; Almheiri et al., 2020) switching dominance around Page time. Almheiri et al. (2021) in their later review consolidated the methods of replica wormholes and semiclassical generalized entropy, while providing a more clear explanation of their model-dependent approach.

2.5 Extended and Non-Equilibrium Thermodynamics

Thermodynamic stability and Phase Transition of black holes can be studied when boundary conditions in anti-de Sitter (AdS) space are applied. When the cosmological constant (Λ) is varied, in conjunction with the first law of thermodynamics, we interpret the mass of the black hole as enthalpy, and (differential) volume as the thermodynamic volume. In the presence of charge, AdS black holes show first order small/large black hole phase transitions, and do exhibit a critical point as do van der Waals-type liquids (Kubizňák et al., 2017). The Hawking-Page transition describes the phase change between thermally stable AdS space and the large AdS black hole and is related to the confinement and deconfinement phase transitions in quantum chromodynamics (QCD).

The absence of equilibrium is intrinsic to black holes. The processes of accretion and mergers break the stationary horizon condition. Quasi-local trapping and dynamical horizons lead to the formulation of balance laws for non-equilibrium thermodynamics. Using dynamical horizons, the first law is extended for black holes that are far from equilibrium by Ashtekar, Paraizo, and Shu (2026). Mann (2026) reviews this work, which positions the first law of black hole thermodynamics within the existing gaps and unresolved contradictions of black hole thermodynamics.

2.6 Synthesis and Remaining Tensions

The existing literature can inform a multi-faceted conclusion. On a grand scale, horizon mechanics, quantum radiation, Euclidean action, and generalized entropy can be described thermodynamically. On a finer scale, state counts can be described as either powerful or restricted, and if defined, subsystem and gravitational factorization can be described as entanglement-based. The third law manifests as sensitive along the boundary of extremality, and controlled entropy exhibits the same behavior as dynamical entropy. There are outstanding issues about the end stage of evaporation and the precise way of encoding information from the interior. These issues prompt the need for a framework that contrasts each result via approximation, ensemble, and domain as opposed to the assumption that all black holes behave the same way in a given statistical system.

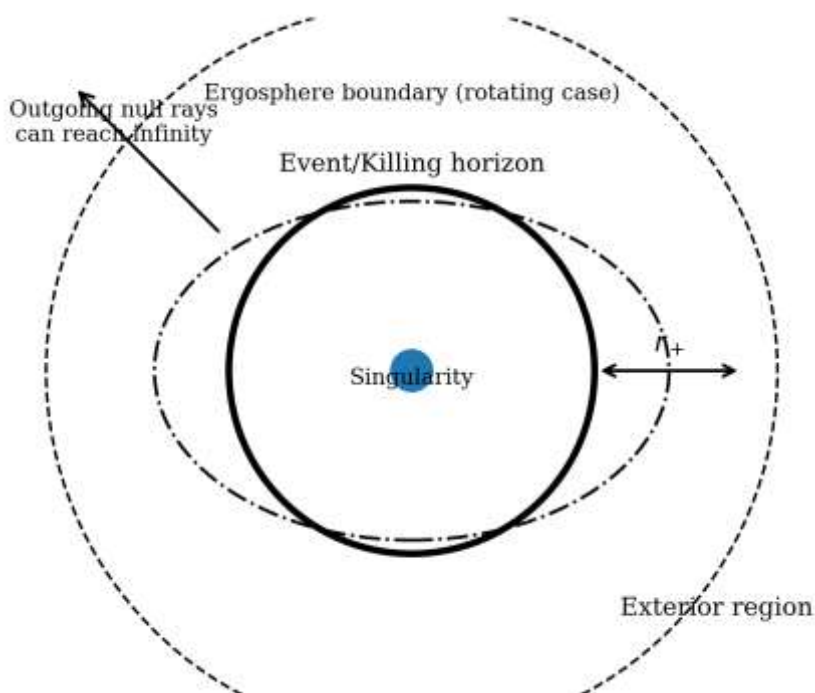


Figure 1. Geometric anatomy of a stationary black hole.

Source: Author's illustration based on Bardeen et al. (1973) and standard Schwarzschild/Kerr geometry.

3. Theoretical Background

3.1 Representative Black-Hole Geometries

A horizon must be defined in context. An event horizon is the global boundary of the causal past of future null infinity and thus requires the future evolution of all of spacetime. A Killing horizon is a null hypersurface that is generated by a Killing vector that is null on the horizon; this is the natural place for stationary thermodynamics. Apparent and trapping

horizons are defined by the expansion of null congruences, and a dynamical horizon is a spacelike hypersurface that is foliated by marginally trapped surfaces. These last concepts are more applicable to evolving systems.

For a Schwarzschild black hole of mass M , the event horizon is located at the Schwarzschild radius r_s . The area of this horizon is a consequence of the surface of a sphere. Charge alters the horizon positions of the Reissner–Nordström solution. Angular momentum induces inner and outer horizons and also an angular velocity of the horizon, Ω_H . In the Kerr solution an ergoregion is outside the event horizon and permits negative-energy states compared to infinity. The Kerr–Newman family encompasses rotation and charge. AdS black holes differ globally because the bounded nature of the timelike boundary confines emissions and effectively allows stable canonical equilibrium for sufficiently large black hole horizons.

$$r_s = \frac{2GM}{c^2} \quad (1)$$

$$A = 4\pi r_s^2 \quad (2)$$

From equations (1) and (2) in Schwarzschild geometry, we can derive $A \propto M^2$. This scaling accounts for the mass-squared entropy relation and the black hole entropy distinction from the extensive entropy relation of ordinary volume.

3.2 Four Laws of Black-Hole Mechanics

The zeroth law fixes κ over a stationary Killing horizon under appropriate assumptions. The first law for a stationary Einstein–Maxwell black hole relates neighboring solutions through

$$dM = \frac{\kappa}{8\pi} dA + \Omega_H dJ + \Phi_H dQ \quad (3)$$

in natural units $G = c = \hbar = k_B = 1$. Here Ω_H is the horizon angular velocity and Φ_H is the electric potential at the horizon. With $T = \frac{\kappa}{2\pi}$ and $S = \frac{A}{4}$, Equation (3) takes the conventional thermodynamic form $dE = T dS + \Omega_H dJ + \Phi_H dQ$. The coefficients are intensive variables conjugate to the extensive or conserved quantities.

The classical second law says that provided the equations of motion and appropriate energy conditions are satisfied, the area of an event horizon will never decrease. However, quantum fields can violate the local energy conditions and allow for evaporation, thus for our purposes the statement we will be using is the generalized second law of thermodynamics which states that $S_{\text{gen}} = A/(4G\hbar) + S_{\text{out}}$. There are a number of different ways of formulating the third law of thermodynamics; the most common way is to state that it is impossible to reach a state of absolute zero ($\kappa = 0$) in a finite number of physical operations. There are complications in drawing a direct analogy to the Nernst formulation due to extremal solutions, because an extremal solution may have a nonzero area but also a zero Hawking temperature.

Table 1. Correspondence Between the Laws of Thermodynamics and Black-Hole Mechanics

Law	Conventional thermodynamics	Black-hole statement	Variable mapping	Qualifications
Zeroth	Temperature is uniform in equilibrium.	Surface gravity κ is constant on a stationary Killing horizon.	$T \leftrightarrow \kappa/2\pi$	Requires stationarity and regularity conditions.
First	$dE = T dS +$ work terms.	$dM = (\kappa/8\pi)dA + \Omega_H dJ + \Phi_H dQ$.	$S \leftrightarrow A/4$; work $\leftrightarrow$ rotation/charge	An identity between nearby stationary solutions; extended form may add VdP.
Second	Total entropy does not decrease.	Classical horizon area does not decrease; quantum theory uses generalized entropy.	$S_{\text{total}} \leftrightarrow A/4G\hbar + S_{\text{out}}$	Classical theorem requires energy conditions; event horizon is teleological.

Law	Conventional thermodynamics	Black-hole statement	Variable mapping	Qualifications
Third	Absolute zero is unattainable; entropy behavior depends on formulation.	Zero surface gravity cannot be reached by finite operations.	$T = 0 \leftrightarrow \kappa = 0$	Extremal horizons make the Nernst interpretation subtle.

3.3 Hawking Temperature and Bekenstein–Hawking Entropy

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \quad (4)$$

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar} = \frac{4\pi k_B G M^2}{\hbar c} \quad (5)$$

Larger Schwarzschild black holes are colder, as shown in Equation (4). A solar mass black hole has a black hole temperature T of approximately 10^{-8} K which is much less than the current temperature of the background cosmic microwave radiation. While larger black holes are colder, sufficiently small black holes are hot and evaporate in a very short amount of time. Equation (5) shows that black holes have substantially more entropy than other forms of matter of the same mass. Thermodynamic models of black hole entropy indicate that black holes have a vast number of microstates, and black holes are the maximum-entropy systems that can exist for a given boundary (the black hole's event horizon).

3.4 Statistical Ensembles, Partition Functions, and Stability

For a canonical ensemble, the partition function is given by $Z(\beta) = \text{Tr} \exp(-\beta H)$. An expression for the Helmholtz free energy can then be constructed as $F = -k_B T \ln Z$. The entropy S is given by $S = -\frac{\partial F}{\partial T}$, and the heat capacity is defined as $C = \frac{\partial E}{\partial T}$. The gravity Euclidean path integral is the approximation of classical saddles which means that $\ln Z$ is directly related to the on-shell Euclidean action. When the temperature, angular potential, and electric potential are fixed, we select the boundary data, and therefore, we select an ensemble.

For Schwarzschild geometry, $E = M c^2$ and $T_H \propto \frac{1}{M}$. Differentiation gives

$$C_{\text{Sch}} = \frac{d(M c^2)}{dT_H} = -\frac{8\pi k_B G M^2}{\hbar c} \quad (6)$$

The negative sign indicates that an isolated black hole will heat up as it loses energy. With contact to an infinite heat bath a small fluctuation can be boosted: a black hole will cool and absorb energy if it absorbs energy and heats and emits energy if it emits energy. Thus, the asymptotically flat canonical ensemble is unstable. This is not a contradiction to the first law; this is a consequence of long-range gravity and ensemble inequivalence.

The conclusion changes due to confinement. A reflecting cavity can fix a finite thermal environment. In AdS, the redshift and boundary structure allow a positive heat capacity large black holes branch. Charged and rotating solutions can also have locally stable parameter regions. Global stability requires evaluating free energies, not heat capacities: a black hole can be locally stable and have a higher free energy than a competing thermal phase, thus be globally unstable.

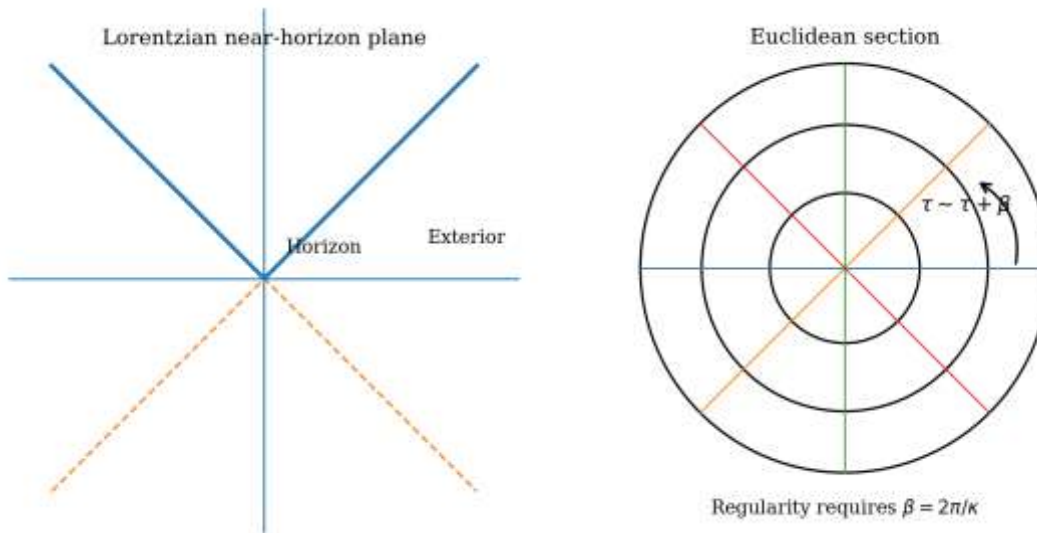


Figure 2. Euclidean periodicity and the origin of Hawking temperature.

Source: Author's illustration based on Gibbons and Hawking (1977).

3.5 Microscopic Entropy and Degeneracy

$$S = k_B \ln \Omega \quad (7)$$

Ω is defined to be the number of microstates corresponding to a given macrostate in Equation (7). For black holes, the macrostate can be specified by the mass (M), angular momentum (J), charge (Q), asymptotic boundary conditions, and other conserved charges. A valid microstate formulation needs to include a quantum Hilbert space, constraints, a control on the approximation relating weak to strong coupling, and calculate the leading and, preferably, the subleading corrections to the entropy.

String-theory counts perform the above tasks in some of the extremal and near-extremal cases. Unlike them, horizon-state and loop-quantum-gravity calculations focus on the quantization of either the geometric or gauge data corresponding to a boundary. Entanglement entropy is the measure of the correlations between the degrees of freedom designated to the exterior and interior domains. In terms of gravity and gauge theory, Hilbert-space factorization poses a challenge. Holography and the AdS/CFT correspondence interpret gravitational entropy as the entropy of a quantum system with no gravity, thereby providing a unitary account. The Noether-charge construction is still at the macroscopic level; it prescribes the entropy counting for a particular gravitational Lagrangian and does not provide a microscopic account.

3.6 Generalized Entropy, Page Curve, and Islands

$$S_{\text{gen}} = \frac{A}{4G\hbar} + S_{\text{out}} \quad (8)$$

$$S(R) = \min \text{ext}_I \left[\frac{\text{Area}(\partial I)}{4G\hbar} + S_{\text{matter}}(R \cup I) \right] \quad (9)$$

In Equation (8), S_{out} refers to the von Neumann entropy concerning quantum fields situated beyond a particular surface. A quantum extremal surface is one that extremizes S_{gen} . The island rule for the radiation region R is schematically represented in Equation (9). Candidate islands I are incorporated, the generalized entropy is extremized, and the minimum saddle is chosen. In the early period, the no-island saddle prevails while the radiation entropy continues to increase. A new saddle can dominate after the Page time. The contribution of the island stops the growth of the radiation entropy and corresponds to the required declining branch of the entropy curve in accordance with unitary evolution.

Although the island formula is one of the more important semiclassical results, it is still not a fully microscopic description. It uses gravitational path integrals to compute the radiation entropy and signals a modification to the entanglement-wedge reconstruction. However, it does not show a well-defined practical dimensional decoding map of four-dimensional radiation nor address how to make sense of a singular semiclassical configuration when replica wormholes bridge the gaps in an entropy computation.

3.7 Extended Phase Space and Black-Hole Chemistry

$$P = -\frac{\Lambda}{8\pi G}, \quad dM = T dS + V dP + \Omega_H dJ + \Phi_H dQ \quad (10)$$

For negative Λ , a positive pressure P has V as the variable conjugate to P , and M is better interpreted as H rather than as internal energy. According to dimensional analysis, the Smarr relation works. The equation of state for charged AdS black holes is analogous to a van der Waals fluid with subcritical oscillations and a supercritical region, where there is a critical inflection point and a monotonic branch.

The analogy is similar to a van der Waals fluid. The horizon radius is the specific volume and the small/large black hole phase transition is the liquid/gas phase transition. The extended phase space is a new interpretation of the black hole as a thermodynamic system in relation to the curvature of the space-time background, and is related to multiple critical points, reentrant phase transitions, and thermodynamic geometry.

Table 2. Thermodynamic Quantities of Representative Black-Hole Solutions (Natural Units)

Solution	Horizon radius	Temperature	Entropy	Charge s	Heat capacity	Stability feature
Schwarzschild	$r_+ = 2M$	$T = 1/(8\pi M)$	$S = \pi r_+^2$	M	$C < 0$	Unstable in asymptotically flat canonical ensemble; stable with suitable confinement.
Reissner–Nordström	$r_+ = M + \sqrt{M^2 - Q^2}$	$T = (r_+ - r_-)/(4\pi r_+^2)$	$S = \pi r_+^2$	M, Q	Sign changes by parameter region	Near extremality $T \rightarrow 0$; ensemble choice and fixed Q or Φ matter.
Kerr/Kerr–Newman	$r_+ = M + \sqrt{M^2 - a^2 - Q^2}$	$T = (r_+ - r_-)/[4\pi(r_+^2 + a^2)]$	$S = \pi(r_+^2 + a^2)$	M, J, Q	Branch-dependent	Rotation introduces Ω_H and superradiant/boundary stability issues.
Schwarzschild-AdS / RN-AdS	Root of $f(r_+)=0$	Includes AdS and charge terms	$S = \pi r_+^2$	M, Q, P	Large branch often $C > 0$	Allows Hawking–Page or van der Waals-like phase structure.

4. Research Objectives

1. To explain the correspondence between the laws of black-hole mechanics and conventional thermodynamics.
2. To derive and compare temperature, entropy, heat capacity, free energy, and thermodynamic volume for representative black-hole solutions.
3. To evaluate major statistical-mechanical interpretations of black-hole entropy and identify the regimes in which microscopic state counting reproduces the area law.
4. To assess the implications of generalized entropy, Page curves, quantum extremal surfaces, and islands for the information problem.
5. To identify how extended phase space and non-equilibrium horizon frameworks broaden black-hole thermodynamics and define future priorities.

5. Methodology (Theoretical/Review-Based)

This study employs a framework of critical review with a preference for a non-chronological style. Evidence was assigned to one of six analytical domains: horizon laws and geometry; quantum radiation; entropy and partition functions; counting the states of a microscopic system; entanglement and information; and extended or dynamical thermodynamics. This framework caters for the comparison of different levels of approximation, and allows for the differentiation between evidence of string counts, semiclassical radiation, classical area theorems, and schematic Page curves.

The intended search coverage was 1973-2026. Major works were located using Web of Science, Scopus, INSPIRE-HEP, APS Journals, SpringerLink and various publisher platforms. Search terms included, but were not limited to, black hole thermodynamics, Bekenstein-Hawking entropy, black hole statistical mechanics, entropy of Hawking radiation, microstate counting, Page curve islands, black hole chemistry and entropy of dynamical horizons. Foundational works were preserved even when outside the preferred recency of the search, since the four laws, the entropy proposal, the derivation by Hawking, the Euclidean action, Noether charge entropy and the first D-brane count, are of primary importance and cannot be replaced by later works.

Eligible sources were those which covered horizon thermodynamic laws, entropy, ensembles, microscopic degeneracy, phase entanglement, and the non-equilibrium horizon information gap. Among these works, the peer-reviewed versions were favored over their preprints, and records with DOI were preferred. Plain language accounts and works for which black hole thermodynamics was but a side note were excluded. Recent reviews were used to integrate results of highly specialized research, but claims of primary importance were traced to their original sources.

The analysis for each source included extraction of the black-hole class, boundary conditions, approximation, ensemble, conserved quantities, principal equations, entropy interpretation, stability conclusion, information-theoretic implications, and limitations. For the purpose of comparison, the equations were normalized in natural units, and the SI constants were restored in context to the Schwarzschild temperature and entropy. The four datasets that were plotted, were solely based on the equations: $\frac{T}{T_0} = \left(\frac{M}{M_0}\right)^{-1}$, $\frac{S}{S_0} = \left(\frac{M}{M_0}\right)^2$, the reduced charged-AdS equation of state, and a schematic comparison between Hawking radiation and unitary entropy. These datasets are plots of a theory and are not empirical data.

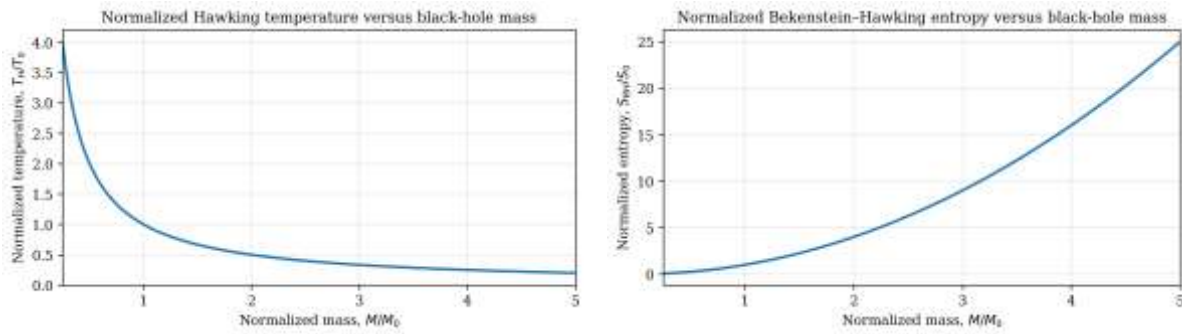
The reliability of the theory was improved by referring to independent derivations of the main hypotheses. For instance, the formulations of entropy via Hawking radiation, the euclidean action, noether charge, microscopic counting, and entanglement were all considered. The divergence of the hypotheses was classified as classical, semiclassical, holographic, and protected sectors as well as being schematic. The main constraints were ideal stationary solutions, a semiclassical treatment of quantum fields over a classical background, indeterminacy in the gravitational path integral, and the finite range of exact microscopic computations. There were no observational or experimental measurements made. Therefore ethics was not a concern, and informed consent was not applicable.

6. Results and Discussion

6.1 Thermodynamic Correspondence Is Structurally Robust

Table 1 shows that correspondence is not based on simple resemblance. Each law has a counterpart in horizons, and variable mapping is fixed quantitatively by Hawking radiation. Prior to the advent of quantum theory, κ and A could be scaled to T and S at will. The thermal spectrum sets $T = \frac{\hbar\kappa}{2\pi ck_B}$ and the first law sets the quarter-area coefficient. The gravitational action and Euclidean regularity yield the same entropy and periodicity. This convergence is more than an analogy of equations.

Even the strongest correspondence has limits. Teleological and global event horizons contrast with the local or quasi-local nature of most thermodynamic state variables. The presence of Extremal black holes complicates the validity of the Third Law. The replacement of area with Wald entropy is further evidence of the need for rapid evolution. Dynamical and trapping horizons, and quantum fields require a generalized form of entropy and are not purely geometric. These boundaries of thermodynamics do not eliminate the presence of thermodynamics, but rather indicate the appropriate horizon and entropy concept for each of these domains.



Charts 1 and 2. Normalized Hawking temperature (left) and Bekenstein–Hawking entropy (right) versus mass.

Source: Author's calculation from Equations (4) and (5), based on Bekenstein (1973) and Hawking (1975).

6.2 Schwarzschild Scaling Reveals Unusual Thermodynamics

Chart 1 shows an inverse relation between mass and temperature. When $\frac{M}{M_0} = 0.25$, normalized temperature is 4. When $\frac{M}{M_0} = 5$, it is 0.2. This suggests that loss of energy increases temperature. Since Hawking emission increases with temperature, this causes a runaway effect at the end of the evaporation process within the limits of the semiclassical approximation. This also explains why stellar and supermassive black holes are very cold, and why we expect microscopic black holes to be rather active thermally.

Chart 2 shows $\frac{S}{S_0} = \left(\frac{M}{M_0}\right)^2$. Massively increasing the mass of a black hole doubles its entropy. If black holes become five times more massive, their entropy increases by a factor of twenty-five. The area of the event horizon is greater than the volume of the black hole. This does not imply that the black hole's interior does not have degrees of freedom. It means the variety of external degrees of freedom is primarily determined by the event horizon. The relations T is inversely proportional to M and S is proportional to M^2 satisfy $dE = T dS$ and produce the negative heat capacity in (6).

6.3 Stability and Phase Behavior Depend on the Ensemble

Table 2 shows that no single stability statement applies to all black holes. The case with Schwarzschild black holes in asymptotically flat space has $C < 0$. For charged and rotating black holes, C has different branches depending on the parameters, and extremality drives the black hole temperature to zero. In AdS, heat capacity is positive for large black hole horizons. Local stability is about the Hessian of the thermodynamic potential and global stability is about the competing phases of the system with the lowest free energy. Therefore, changing from constant charge to constant potential, and from constant angular momentum to constant angular velocity can alter the phase diagram.

Chart 3 shows the application of the reduced equation $p = \frac{8\tau}{3v} - \frac{2}{v^2} + \frac{1}{3v^4}$. When $\tau = 0.8$, the isotherm shows a region of non-monotonic behavior which is associated with the coexistence of phases, following the appropriate Maxwell construction. When $\tau = 1$, the curve shows the critical inflection points, and when $\tau = 1.2$, the curve is monotonic. The analogy to a van der Waals fluid helps to organize the phase behavior, but does not suggest that ordinary van der Waals molecules exist behind the black hole horizons. The degrees of freedom that are relevant are still an interpretive question.

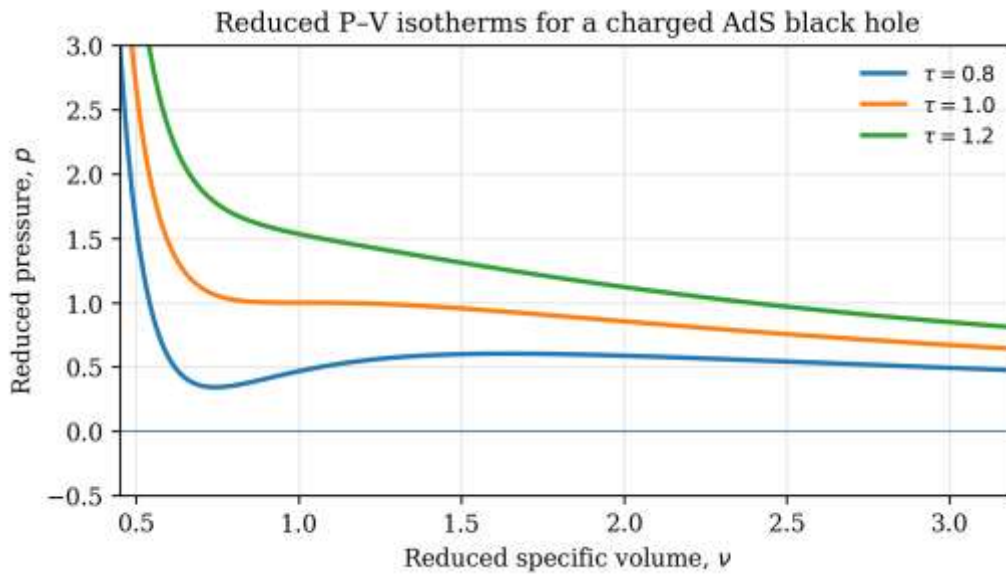


Chart 3. Reduced pressure–volume isotherms for a charged AdS black hole.

Source: Author’s calculation from the reduced equation of state summarized by Kubizňák et al. (2017) and Mann (2026).

6.4 Microscopic Explanations Reproduce the Area Law in Different Domains

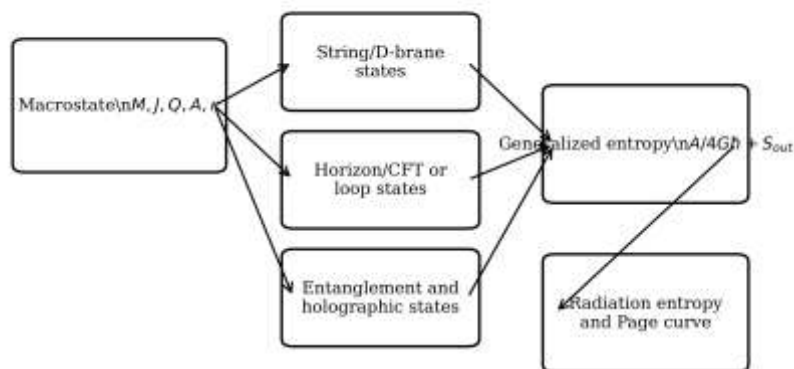
Table 3. Comparison of Statistical-Mechanical Accounts of Black-Hole Entropy

Approach	Counted/represented object	Principal achievement	Domain limitation or	Unresolved issue
String/D-brane counting	Protected bound-state degeneracy	Exact leading match for selected supersymmetric extremal/near-extremal holes	Restricted charges, dimensions, and supersymmetry	Extension to generic nonextremal Kerr and realistic collapse
Entanglement entropy	Correlations across a horizon or entangling surface	Natural area scaling and relation to renormalized gravitational coupling	UV divergence and gravitational factorization	Regulator-independent universal state interpretation
Holography/QES	Boundary quantum degrees and extremal generalized-entropy surfaces	Geometric entropy and Page-curve recovery in controlled AdS models	Relies on holographic setting and semiclassical saddles	Realistic asymptotically flat evaporation and explicit decoding
Horizon/CFT or loop states	Boundary excitations, punctures, or near-horizon symmetry sectors	Area law and possible logarithmic corrections	Choice of boundary variables and quantization details	Framework-independent Hilbert space and coefficient
Noether-charge geometry	Gravitational fields and symmetry charge	Correct entropy functional for covariant Lagrangians	Macroscopic, not a direct microstate count	Quantum states that universally realize the charge entropy

Two conclusions can rather simultaneously be drawn from Table 3. The first is that methods with very different starting points reproduce the same leading area law. The second is that the meanings of the counted states are context dependent. The Strominger–Vafa count is the most convincing example of black hole entropy equating to the logarithm of a true quantum degeneracy. However, the count is performed in a context with supersymmetry which is not justified to be

extended to black holes in general. There are more reasons to think that the boundary scaling is justified, but the calculations also carry ambiguities of regularization, factorization, or quantization.

The convergence may indicate a similar situation at the level of coarse grained approximations. Near-horizon symmetry, gravitational gauge restrictions, and quantum entanglement all represent a loss of information that would be counted as independent in a naive counting of the volume. A future universal explanation would most likely be able to integrate these conceptualizations rather than having to pick one to be literally true in all cases. Most of the relevant area law corrections should be of the same order, and can be treated as leading corrections, while different models can vary in lower order contributions.



Agreement: leading area law | Open question: universal microscopic identity and realistic evaporation

Figure 3. From black-hole macrostate to candidate microstates and radiation entropy.

Source: Author's synthesis based on Strominger and Vafa (1996), Solodukhin (2011), Harlow (2016), and Almheiri et al. (2021).

6.5 The Page Curve Reframes Black-Hole Statistical Mechanics

Chart 4 shows two entropy curves and highlights how the Hawking curve and the Page curve differ. Hawking's evaporation curve rises almost linearly throughout black hole evaporation and post-black hole evaporation because the terminals of the Hawking pairs remain connected and therefore entangled. When a black hole completely evaporates and the final state of the black hole is a black hole, the last evaporated particles will be entangled. The Hawking curve ends at the Page time when the final state is completely pure.

The entropy curve does not change the local Hawking flux. Before the Page time, the generalized entropy of the no-island saddle is greater. After the transition, the localized QES and the island reduce the generalized entropy. The island means that the interior of the black hole no longer preserves information, therefore it is reconstructable, and means that the information paradox does not exist.

To clarify, in black holes that are at least somewhat tractable, the Page curve and the reconciliation of the two approaches are the result of the gravitational replica trick and of QES. The biggest unresolved issues remain the incorporation of black holes into the framework of a unitary theory, the extension of the theory of black holes to four-dimensional flat rotating black holes, and the practical retrieval of information from highly encoded and nearly maximally entropic radiation. The islands are a significant breakthrough in the entropy calculation, but are not a complete explanation of every aspect of evaporation.

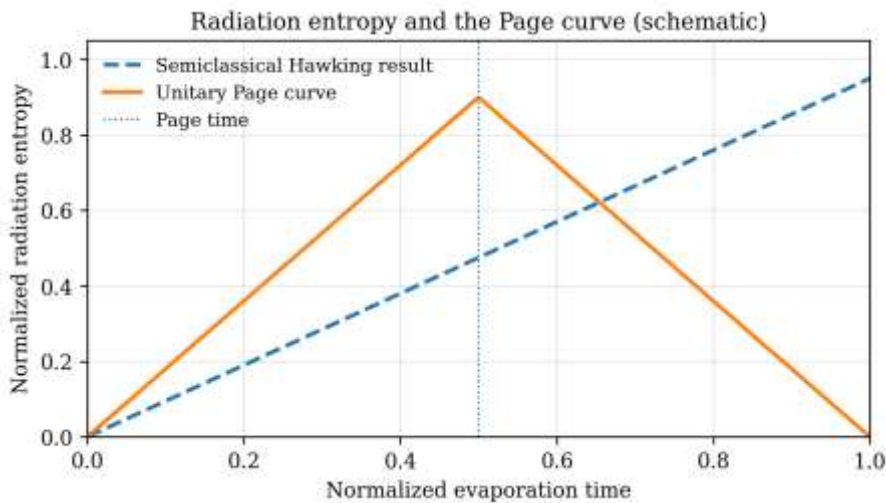


Chart 4. Schematic comparison of semiclassical Hawking radiation entropy and a unitary Page curve.

Source: Author's schematic based on Page (1993), Penington (2020), Almheiri et al. (2020), and Almheiri et al. (2021).

6.6 Thermodynamics Extends Beyond Stationary Horizons

Stationary thermodynamics describes equilibrium systems. A merger or accreting black hole describes a finite process where the energy and angular momentum transit a horizon that is dynamically changing. Quasi-local horizons sidestep the teleological nature of event horizons and allow a flux balance to be expressed with respect to fields on the changing surface. It is then possible to describe the entropy change in terms of a flux of matter and gravitational radiation across the horizon.

The 2026 far-from-equilibrium first law (Ashtekar et al., 2026) claims the black hole thermodynamic structure is sustained at the boundary of equilibrium. The black hole first law should be viewed as a differential identity of a family of exact stationary metrics. The black hole first law is a step beyond a stationary metric. The black hole first law is a complete framework for non-equilibrium black hole thermodynamics. The framework is incomplete as the definition of temperature and equilibrium is yet to be established in the context of black hole horizons. Likewise, the concepts of entropy production and fluctuation theorems are still in their infancy as applied to black hole horizons.

6.7 Integrated Discussion, Limitations, and Open Questions

Table 4. Evidence–Consensus–Limitation Matrix

Theme	Strongest evidence	Current consensus	Limitation	Future test/direction
Area law	Classical area theorem, Hawking radiation, Euclidean action, Noether charge, protected counts	Leading entropy is geometrical and universal in Einstein gravity	Microstate identity and quantum corrections remain framework-dependent	Compare subleading corrections across quantum-gravity models and observations
Hawking temperature	Quantum-field derivation and Euclidean periodicity	Nonextremal stationary horizons are thermal at $T \propto \kappa$	Backreaction and endpoint exceed fixed-background calculation	Self-consistent evaporation and analogue/indirect tests
Canonical stability	Negative Schwarzschild heat capacity; stable AdS/cavity branches	Stability depends on ensemble and boundary conditions	Global gravitational ensemble may be nonstandard	Map phase structure and kinetics beyond equilibrium
Microstates	Exact D-brane counts; horizon and entanglement approaches	Entropy can represent quantum degeneracy	No universal count for generic realistic holes	Derive generic Kerr/nonextremal degeneracy

Theme	Strongest evidence	Current consensus	Limitation	Future test/direction
Page curve	QES, islands, replica saddles in controlled models	Generalized entropy can recover unitary curve	Model dependence and interpretation of gravitational replicas	Four-dimensional asymptotically flat calculation and decoding
Dynamical entropy	Horizon balance laws and far-from-equilibrium first law	Thermodynamic relations extend beyond stationarity	Temperature and entropy production are not fully settled	Non-equilibrium constitutive relations for mergers and evaporation

The integrated evidence creates a hierarchy of confidence. There are several independent semiclassical and geometric arguments supporting the existence of black hole temperatures and entropies. Less ubiquitous are the arguments for the microscopics of entropy, although special situations do allow for the counting of states. The information paradox deals with the problem of having local radiation, which appears to be thermal, coexist with a globally unitary evolution. Generalized entropy shows that the existence of such a case is possible using gravitational path integrals with a high level of control.

Gravitational systems are non-additive and lead to inequivalent ensembles. Gauge constraints create a division that is not easily overcome between the inner and outer regions of a system. Negative heat capacity can be described in a microcanonical ensemble and does not constitute a failure of the system when in contact with a canonical bath of infinite size. In a similar fashion, the entropy that is described to exist at a horizon is possibly the result of edge modes, entanglement, or a coarse-grained level of description of states that are not local in a conventional sense.

There are four main limitations to the review. Most derivations assume symmetry in some form. The semiclassical computation of Hawking Radiation does not provide a complete description of the quantum-gravitational endpoint. The calculations of Holographic islands are best described in AdS or lower dimensional models with more controlled dual descriptions. Finally, the data sets that are presented in the review are illustrations of equations derived from theory and do not provide a direct observation of nature. These limitations do not allow for the claim that the information paradox has been solved at the experimental level or that a universal model has been confirmed.

The synthesis may have some low-level problems, but it answers the research questions. When quantum effects are added, the laws of mechanics and thermodynamics coincide. With the right approximation, it's easy to visualize how temperature, entropy, and stability are related to mass, charge, rotation, pressure, and boundary conditions. Approaches at a microscopic level show the area law of black hole entropy in different contexts. Generalized entropy theory is a modern approach to the understanding of the flow of information. Within the extended framework, black hole thermodynamics is a correlating principle within many dynamic/extended contexts of black hole physics.

Methodologically, the focus of a proposed microscopic model cannot only be the reproduction of a leading order approximation of $A/4$. This is due to the principle of universality that states that this is a necessary condition to pass, but far from being sufficient. This is a necessary condition but far from being sufficient. Surely, the correct dependence on conserved quantities, higher-order curvature terms, logarithmic corrections in the near-extremal spectrum, response coefficients, and the evolution of entropy during physical processes should account for strong discrimination. Conversely, within a given context, a law of macroscopic thermodynamics may remain valid while its realization at the microscopic level changes. Black hole thermodynamics is mature and incomplete at the same time for these reasons. The equations of state are well established in many different contexts. The quantum ontology is still under active research.

7. Conclusion

In semiclassical gravity, black holes are real thermodynamic systems. The statement is not based on a single analogy, but rather on a web of results: horizon thermodynamics, Hawking radiation, Euclidean periodicity, gravitational partition functions, the generalized second law, and correct microscopic counts in the protected sectors. For Schwarzschild black holes, $T_H \propto M^{-1}$, $S_{BH} \propto M^2$, and $C < 0$. These relations account for the black holes in the universe and the black holes in the universe's asymptotic region.

The stability, phase behavior, and the rest are all environmental and ensemble dependent. Black holes with charge, rotation, finite cavities, and AdS boundary conditions distinguish branches with different heat capacities and free energies. When considering the AdS boundary as a pressure, liquid–gas-like behavior and critical phenomena emerge. This strongly suggests that black-hole thermodynamics is a rich and complex system beyond landscape constraining.

At a global level, the microscopic problem remains open. String/D-brane counting provides a connection between the Bekenstein–Hawking entropy and the logarithm of quantum level degeneracy, but this is mainly applicable to certain extremal or near-extremal cases. The horizon state, entanglement, and the rest provide different approaches for the area law and through various state spaces. Their convergence supports the area law, while their differences indicate that for astrophysical black holes, the underlying degrees of freedom have not been determined.

With respect to the information problem, generalized entropy, quantum extremal surfaces and islands, have been helpful in understanding the problem as a whole. These concepts, in controlled semi-classical models, recapture the unitary Page curve and show how the interior black hole can be included within the radiation entanglement wedge. Recent work done well outside of equilibrium has shown that reasoning of the first law can be applied to dynamical horizons. Black holes are still the best places to study how the quantum of spacetime and the statistical nature of spacetime, as well as the structure of entanglement and information, might be defined.

8. Future Scope

A main goal is to derive something fundamental from first principles that applies beyond supersymmetric, extremal, and AdS-controlled regions. Generic Kerr black holes are the most relevant targets because astrophysically motivated black holes will include rotation and thus will include superradiance, inner-horizon structure, and ensembles. Agreement should take place beyond the leading area term and include quantum corrections and the dynamical response.

Islands and generalized-entropy calculations should be performed for four dimensions, including rotation, charge, and asymptotic flatness, and evaporation with backreaction. The relationship between replica saddles, a single unitary history, and operational decoding is an open question. Perhaps the tools of quantum information theory will help explain the transformation of locality from the realm of semiclassical theory into the nonlocal domain of the gravitational degrees of freedom.

Non-equilibrium thermodynamics is another of the major frontiers. Mergers, accretion, and evaporation require quasi-local definitions of entropy, effective temperature, and the use of fluctuation relations out of equilibrium. Gravitational wave astronomy will help test the validity of the classical result that the area of a black hole will increase in the case of a merger, and possibly stimulate research into the relaxation of the event horizon and the ringdown phase. While these will help understand the black hole evaporation process, they cannot be used to directly measure Hawking radiation. An extended phase space will help understand black holes and related phenomena, including multicriticality and beyond equilibrium thermodynamics. Advances in this area may help understand which phenomena of black hole thermodynamics are non-local effects of quantum gravity and which are due to boundary effects and microscopic realizations.

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