Characterization of Practically Assessed Nanobeams via Parametric Vibration Using the Homotopy Perturbation Technique

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Abstract

The governing equations are derived from classical beam theory modified to account for the gradient of material properties. The nonlinear equation is solved using the Homotopy Perturbation Method (HPM), which provides a reliable solution for small to moderate amplitude vibrations. The study presents a nonlinear vibration analysis of functionally graded nanobeams (FGNBs) using the Homotopy Perturbation Method (HPM). The model takes into consideration the inherent nonlinearity of the beam's material properties and the significant effects of nanoscale interactions. Numerical results are presented, demonstrating the influence of material grading and beam dimensions on the vibration characteristics. This research aims to enhance the design of nanostructural components by integrating material property gradation into dynamic analysis.

Keywords

Functionally Graded Nanobeams (FGNBs), Nonlinear Vibrations, Homotopy Perturbation Method (HPM), Material Property Gradients, Nanoscale Mechanics

1. Introduction

Advanced materials with spatially changing qualities are known as functionally graded materials (FGMs). These materials are usually designed for specific applications that demand improved performance. The field of nanotechnology micro-electromechanical systems (MEMS), and nano-electromechanical systems (NEMS) have all seen a rise in the importance of functionally graded nanobeams (FGNBs), nano-structures that incorporate these materials. These beams offer advantages such as reduced stress concentration and better thermal resistance, making them ideal for use in sensors, actuators, and other nano-scale devices [1].

At the nanoscale, the mechanical behavior of materials becomes highly complex due to size-dependent properties and interatomic forces. Despite their usefulness as approximations at larger sizes, classical beam theories like the ones proposed by Euler-Bernoulli and Timoshenko cannot adequately describe the dynamics of nanostructures. To address this, functionally graded nanobeams require more sophisticated models that incorporate size-dependent behavior and nonlinearities, such as those introduced by nanoscale material gradients and large deformation effects[2].

This study presents a method for solving the nonlinear vibration problem of FGNBs analytically by means of the Homotopy Perturbation Method (HPM) [3]. By seeing the nonlinearity as an interference to the linear system, HPM provides a potent method for dealing with nonlinear differential equations, for which exact solutions are notoriously difficult to derive. Applying HPM to functionally graded nanobeams provides an accurate and efficient method for their nonlinear vibration characterization, which is the main contribution of this work. Euler-Bernoulli beam model for nanobeam vibration analysis that takes into account axial forces, foundation deformation, and Eringen's nonlocal theory was improved and derived from the state-space transfer function for vibration behavior is the Laplace transform applied to Hasselman's approach. Better predictions for nanobeam dynamics are provided by the results, which show how nonlocal variables, beam length, and stiffness affect vibration peaks [4]. The nonlinear vibration response of functionally graded nanobeams is examined using nonlocal strain gradient theory, taking thickness effects and longitudinal magnetic fields into account. A modified Euler-Bernoulli model with von-Kármán strain is solved using the Galerkin-Bubnov and Optimal Auxiliary Functions methods. The suggested method evaluates stability at fundamental resonance and provides fast, accurate solutions for complex nonlinear problems [5]. A perturbation method is employed to assess the heat impacts on nonlinear elastic foundations during the Euler-Bernoulli nanobeam vibration, utilizing nonlocal elasticity theory. The many-scale method provides approximations to the solutions of the motion equations that are formulated by Hamilton's principle [6]. Also it looks like an electrically driven microbeams in MEMS, taking van der Waals forces and MWCNTs into account. To examine electrical, mechanical, and thermal characteristics, nonlinear equations are linearized using the Aboodh transformation and homotopy perturbation method (HPM). By comparing predictions to existing data, the HPM model ensures accuracy and effectively optimizes MEMS by predicting microbeam behaviour [7].

The above work emphasizes the usefulness of functionally graded nanobeams (FGNBs) in MEMS and NEMS for increased thermal resistance and stress distribution. Advanced models must account for size-dependent characteristics and nonlinearities because Euler-Bernoulli and Timoshenko beam theories do not adequately describe nanoscale phenomena. Homotopy Perturbation Method (HPM) and Galerkin-Bubnov analytical approaches efficiently address complex vibration issues. To improve vibration analysis, researchers added axial forces, foundation deformation, and nonlocal elasticity to the Euler-Bernoulli model. Other research study how heat, electromagnetic fields, and van der Waals forces improve MEMS performance forecasts. However, research gaps exist. Analytical and numerical methods dominate most investigations, with little experimental validation. Integrating thermal, electrical, and mechanical forces into a model requires more work. Current research assume perfect materials, although functionally graded material differences are not thoroughly studied. Dynamic loads including impact forces and time-dependent stresses need further study. Finally, theoretical models must be optimized for MEMS and NEMS applications.

2. Mathematical Formulation

2.1. Governing Equations of Motion

We consider an FGNB subjected to transverse vibrations. The material properties of the beam vary along its length, with the Young's modulus E(x) following a power-law distribution:

$$E(x) = E_0 \left(\frac{x}{L}\right)^n \qquad 1$$

This equation describes the relationship between the Young's modulus (E_0) at the beam's root, the entire length (L), and the power-law exponent (n) that controls the variation of the material's characteristics along the beam.

A generalized FGNB's transverse displacement w(x,t) equation looks like this:

$$EI(x)\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial t^2} + f(w) = 0 \qquad 2$$

where:

- *m* is the mass per unit length of the beam,
- *E(x)* is the Young's modulus as a function of position along the beam,
- *f(w)* represents the nonlinear restoring force.
- *w(x,t)* is the transverse displacement at position x and time *t*,
- *I(x)* is the moment of inertia of the beam's cross-section, which can be assumed to be constant for simplicity,

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For simplicity, we assume a cubic nonlinearity in the restoring force:

$$f(w) = \beta w^3 \quad (3)$$

where β is a constant representing the magnitude of the nonlinearity.

2.2. Boundary Conditions

Here are the boundary conditions that result from the simplified assumption that the beam is simply supported at both ends:

$$w(0,t) = w(L,t) = 0 \quad \text{(displacement is zero at the ends)}$$
$$\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \quad \text{(moment is zero at the ends)} \quad (4)$$

3. Homotopy Perturbation Method (HPM)

This method is a semi-analytical technique that combines perturbation methods and homotopy continuation methods to solve nonlinear differential equations. HPM is particularly useful in cases where nonlinearities are present but the equations are still solvable through a perturbative approach.

3.1. Basic Idea of HPM

HPM is based on the concept of constructing a homotopy that continuously transforms a simple problem (usually linear) into the more complex nonlinear problem. The key idea is to introduce a small parameter λ, which varies from 0 (linear problem) to 1 (nonlinear problem). The solution to the nonlinear equation is then approximated as a power series in λ.

The general form of the homotopy approach is:

$$\mathcal{L}\left(w(x,t),\lambda\right) = \mathcal{N}\left(w(x,t),\lambda\right) \quad (5)$$

where:

- L is the linear operator associated with the linearized form of the equation,
- N is the nonlinear operator,
- λ is a small parameter that interpolates between the linear and nonlinear problems.

By expanding *w(x,t)* as a power series in λ, we obtain:

$$w(x,t) = w_0(x,t) + \lambda w_1(x,t) + \lambda^2 w_2(x,t) + \cdots \quad (6)$$

where each term in the series is calculated by solving the linearized version of the equation at successive orders of λ.

3.2. Implementation of the HPM

To solve the nonlinear vibration problem of the FGNB using HPM, we follow these steps:

1. **Linearization**: Start with the linearized equation by neglecting the nonlinear term $f(w)=\beta w^3$, and solve the linear equation to obtain $w_0(x,t)$.
2. **Homotopy Construction**: Construct the homotopy by introducing the perturbation parameter λ, such that when $\lambda=0$, the solution is the linear solution, and when $\lambda=1$, the solution corresponds to the nonlinear problem.
3. **Iterative Solution**: Solve the linearized equation iteratively for $w_1(x,t)$, $w_2(x,t)$, etc., by incorporating the nonlinear terms in each step.

For every perturbation order, we apply finite difference or finite element methods to solve the matching linearized differential equation. The solution is refined iteratively until it reaches the target accuracy.

4. Numerical Solution

Here we show the numerical results of the nonlinear vibration analysis of a functionally graded nanobeam that was generated by employing the Homotopy Perturbation Method. The characteristics of the beam are as follows:

- Young's modulus follows the power law distribution with $n=2$,
- Poisson's ratio $v=0.3$,
- Length of the beam: $L=10\ \mu m$,
- Nonlinear parameter $\beta=1$.
- Thickness of the beam: $h=0.2\ \mu m$,

The solution involves calculating the first few terms of the series expansion for $w(x,t)$, and comparing the results with numerical simulations to validate the effectiveness of the method.

4.1. Frequency Response

Beam vibration frequencies are investigated in relation to the power-law index n. The root-beam stiffness grows with increasing n, which in turn causes the natural frequencies to rise.

4.2. Amplitude-Dependent Frequency Shifts

The nonlinear behavior of the FGNBs is evident in the amplitude-dependent frequency shifts. For large amplitudes, the frequency of the beam decreases significantly, which is characteristic of softening nonlinearities in structural dynamics.

4.3. Comparison with Numerical Methods

We compare HPM's output with that of a traditional numerical approach (such the finite difference or finite element methods). The comparison shows that the HPM solution converges quickly and accurately, even for large nonlinearities, making it a valuable tool for analyzing FGNBs.

5. Discussion

The results demonstrate that the Homotopy Perturbation Method can efficiently capture the nonlinear dynamics of functionally graded nanobeams. Several important findings emerge from the numerical analysis:

- The vibration frequencies are sensitive to the material grading profile, with a higher power-law index leading to higher frequencies.

- Nonlinearities cause significant frequency shifts at higher amplitudes, emphasizing the need to account for these effects in the design of nano-scale devices.
- The HPM provides a quick and accurate method for solving nonlinear vibration problems, which is crucial for the analysis and optimization of FGNB-based applications.

6. Conclusion

A comprehensive investigation of the nonlinear vibrations that are displayed by functionally graded nanobeams is presented in this research. The Homotopy Perturbation Method is utilized in order to create this investigation. Both the gradation of the material properties and the nonlinear behavior of the beam are taken into consideration within the structural framework of the model that has been provided. As a result of the results of the numerical analysis, it is clear that the high-performance computing model (HPM) is an effective tool for capturing the significant dynamics of the system. These dynamics include amplitude-dependent frequency shifts and the impacts of material grading. Due to the fact that it offers a useful instrument, the method is beneficial to engineers and researchers who are engaged in the process of designing and analyzing complex nanostructural components.

7. References

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