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Characterizations of $(\alpha p)^*$- R_1 Spaces

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ABSTRACT: In this paper we introduce $(\alpha p)^*$-R_1- spaces and we study some characterization of $(\alpha p)^*$-R_1 Spaces. We analyse the relation between $(\alpha p)^*$- closed sets with already existing closed sets.

KEYWORDS: $(\alpha p)^*$- closed sets, $(\alpha p)^*$- open sets, $(\alpha p)^*$-closure, $(\alpha p)^*$- R_1 spaces.

I INTRODUCTION

Levine[7] introduced generalized closed sets (briefly g-closed sets) in topological spaces and studied their basic properties. O. Njastad[9] defined α - closed in 1965. N.Levine [6] introduced the class of semi-closed and semi-open sets in 1963. A. S. Mashhour[7] defined preopen and pre closed sets in 1982. L.Elvina Mary , R.Saranya [6]introduced $(\alpha p)^*$ - closed sets in 2017. The aim of this paper is to introduce a $(\alpha p)^*$ - R_1 spaces and we investigate some characterization of $(\alpha p)^*$ - R_1 - spaces.

II PRELIMINARIES

Definition 2.1 A subset A of a topological space (X,τ) is called

(i) a semi-open set if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆A,
(ii) a preopen set if A ⊆ int(cl(A)) and a preclosed set if cl(int(A)) ⊆ A,
(iii) an α- open set if A ⊆ int(cl(int(A))) and an α -closed set if cl(int(cl(A))) ⊆ A,
(iv) a semi-preopen set if A⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A))) ⊆A
(v) a regular open set if A=int(cl(A))and a regular closed set if cl(int(A))=A.

The semi-closure (resp.preclosure , semi-preclosure) of a subset A of a space (X,τ) is the intersection of all semi-closed(resp. preclosed , α-closed, semi-preclosed) sets that contain A and is denoted by scl(A) (resp.pcl(A), Acl(A), spcl(A)).

Definition 2.2 A subset A of a space (X,τ) is called

(i) a generalized closed (briefly g-closed) set[10] if cl(A)⊆U whenever A⊆U and U is open in (X,τ); the compliment of a g-closed set is called a g-open set,
(ii) a semi-generalized closed (briefly sg-closed) set[2] if scl(A)⊆U whenever A⊆U and U is semi-open in(X,τ); the compliment of sg-closed set is called a sg-open set,
(iii) a generalized semi-closed (briefly gs-closed) set if scl(A)⊆U whenever A⊆U and U is open in (X,τ)
(iv) an α-generalized closed (briefly αg-closed) set[3] if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ),
(v) a generalized α-closed (briefly gα-closed) set [3] if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ)
(vi) a g*- closed set [10] if cl(A)⊆U whenever A⊆U and U is g-open in (X,τ),

(vii) a g^{**}-closed set [8] if cl(A)⊆U whenever A⊆U and U is g*-open in (X,τ),
(viii) a generalized preclosed(briefly gp-closed) setif pcl(A)⊆U whenever A⊆U and U is open in (X,τ),
(ix) a generalized semi-preclosed (briefly gsp-closed) set [5] if spcl(A)⊆U whenever A⊆U and U is open in(X,τ)
(x) a generalized pre regular closed (briefly gpr-closed) set [6] if pcl(A)⊆U whenever A⊆U and U is regular open in (X,τ),
(xi) a $g^{\#}$-closed set [11] if cl(A)⊆U whenever A⊆U and U is αg-open in (X,τ),
(xii) a generalized α**-closed (briefly gα**-closed) set [3] if αcl(A)⊆int(cl(U)) whenever A⊆U and U is α-open in (X,τ),
(xiii) a μ*-closed set [4] if cl(A)⊆U whenever A⊆U and U is gα**-open in (X,τ),
(xiv) a g*s- closed set [9] if scl(A)⊆U whenever A⊆U and U is gs-open in (X,τ).

The compliment of the above mentioned sets are called their respective open sets.

III .$(\alpha p)^*$ - R_1 Spaces

Definition 3.1. Let (X,τ) be a topological space and $A\subset X$.Then the $(\alpha p)^*$ - R_1 kernel of A,denoted by $(\alpha p)^*$ - $Ker(A)$ is defined to be the set $(\alpha p)^*$ $Ker(A)=\cap\{U\in(\alpha p)^*$ $O(X,\tau)| A\subset U\}$.
Lemma 3.1. Let (X, τ) be a topological space and x ∈ X. Then,y ∈ $(\alpha p)^*$ $Ker(\{x\})$ if and only if x ∈ $(\alpha p)^*$ -$Cl(\{y\})$. **Proof:**Assume that y ∉ $Ker(\{x\})$ Then there exist a $(\alpha p)^*$ -open set containing x such that y ∉ V . Therefore, we have x ∉ $(\alpha p)^*$ -$Cl($ y $)$. The converse is similarly shown.
Lemma 3.2. Let (X,τ) be a topological space and A a subset of X. Then, $(\alpha p)^*$ - $Ker(A)$ = $\{x \in X \mid (\alpha p)^*$ -$Cl(\{x\}) \cap A \neq \emptyset\}$. **Proof.** Let x ∈ $(\alpha p)^*$ -$Ker(A)$ and $(\alpha p)^*$ -$Cl(\{x\}) \cap A = \emptyset$. Therefore, x ∉ X- $(\alpha p)^*$ -$Cl(\{x\})$ which is a $(\alpha p)^*$ -open set containing A. But this is impossible, since $x \in (\alpha p)^*$- $Ker(A)$. Consequently, $(\alpha p)^*$- $Cl(\{x\}) \cap A \neq \emptyset$. Now, let x ∈ X such that $(\alpha p)^*$- $Cl(\{x\}) \cap A \neq \emptyset$. Suppose that x ∉ $(\alpha p)^*Ker(A)$. Then, there exists a $(\alpha p)^*$-open set U containing A and x ∉ U. Let y ∈ $(\alpha p)^*$-$Cl(\{x\}) \cap A$. Thus, U is a $(\alpha p)^*$-neigbourhood of y such that x ∉ U. By this contradiction x ∈$(\alpha p)^*$- $Ker(A)$.

Lemma 3.3. The following statements are equivalent for any points x and y in a topological space (X, τ) : (1) $(\alpha p)^*$-$Ker(\{x\}) \neq (\alpha p)^*$-$Ker(\{y\})$;
(2) $(\alpha p)^*$-$Cl(\{x\}) \neq (\alpha p)^*$-$Cl(\{y\})$. **Proof. (1) ⇒ (2)** : Let$(\alpha p)^*$-$Ker(\{x\}) \neq (\alpha p)^*$-$Ker(\{y\})$. Then there exists a point z in X such that z ∈ $(\alpha p)^*$-$Ker(\{x\})$ and z ∉ $(\alpha p)^*$-$Ker(\{y\})$. From z ∈ $(\alpha p)^*$-$Ker(\{x\})$ it follows that $\{x\} \cap (\alpha p)^*$-$Ker(\{z\}) \neq \emptyset$ which implies x ∈ $(\alpha p)^*$-$Cl(\{z\})$. By z ∉ $(\alpha p)^*$-$Ker(\{y\})$, we have $\{y\} \cap (\alpha p)^*$-$Cl(\{z\}) = \emptyset$. Since x∈$(\alpha p)^*$-$Cl(\{z\}$, $(\alpha p)^*$-$Cl(\{x\}) \subset (\alpha p)^*$-$Cl(\{z\})$ and $\{y\} \cap (\alpha p)^*$-$Cl(\{x\}) = \emptyset$. Therefore it follows that $(\alpha p)^*$-$Cl(\{x\}) \neq (\alpha p)^*$-$Cl(\{y\})$Now $(\alpha p)^*$-$Ker(\{x\}) \neq (\alpha p)^*$-$Ker(\{y\})$ implies that $(\alpha p)^*$-$Cl(\{x\}) \neq (\alpha p)^*$-$Cl(\{y\})$.
(2) ⇒ (1) : Suppose that $(\alpha p)^*$-$Cl(\{x\}) \neq (\alpha p)^*$-$Cl(\{y\})$. Then there exists a point z in X such that z ∈ $(\alpha p)^*$-$Cl(\{x\})$ and z ∉ $(\alpha p)^*$-$Cl(\{y\})$. It means that there exists a $(\alpha p)^*$-open set containing z and therefore x but not y, i.e., y ∉ $(\alpha p)^*$-$Ker(\{x\})$ and hence $(\alpha p)^*$-$Ker(\{x\}) \neq (\alpha p)^*$-$Ker(\{y\})$. **Definition 3.2.** A topological space (X,τ) is said to be a $(\alpha p)^*$-R_1 space if every $(\alpha p)^*$-open set contains the $(\alpha p)^*$- closure of each of its singletons.
Proposition 3.1. For a topological space (X, τ), the following properties are equivalent: (i)(X,τ) is$(\alpha p)^*$-R_1 space;
(2)For any $F \in (\alpha p)^*C(X,\tau)$, x ∉ F implies $F \subset U$ and x ∉ U for some $U\in(\alpha p)^*O(X,\tau)$;
(3)For any $F \in (\alpha p)^*C(X,\tau)$, x ∉ F implies $F \cap (\alpha p)^*Cl(\{x\}) = \emptyset$; (4)For any distinct points x and y of X, either $(\alpha p)^*$-$Cl(\{x\}) = (\alpha p)^*$-$Cl(\{y\})$ or $(\alpha p)^*$-$Cl(\{x\}) \cap (\alpha p)^*$-$Cl(\{y\}) = \emptyset$. **Proof. (1) ⇒ (2)** : Let F ∈

$(\alpha p)^*C(X,\tau)$ and $x \notin F$. Then by (1) $(\alpha p)^*Cl(\{x\}) \subset X - F$. Set $U = X - (\alpha p)^*Cl(\{x\})$, then $U \in (\alpha p)^*O(X,\tau)$, $F \subset U$ and $x \notin U$.

(2) => (3) : Let Let $F \in (\alpha p)^* C(X,\tau)$ and $x \notin F$. There exists $U \in (\alpha p)^*O(X,\tau)$ such that $F \subset U$ and $x \notin U$. Since $U \in (\alpha p)^*O(X,\tau) \cap (\alpha p)^*Cl(\{x\}) = \emptyset$and $F \cap (\alpha p)^*Cl(\{x\}) = \emptyset$.

(3) => (4) : Let $(\alpha p)^*Cl(\{x\}) \neq (\alpha p)^*Cl(\{y\})$ for distinct points $x,y \in X$. so there exists $z \in (\alpha p)^*Cl(x)$ such that $z \notin (\alpha p)^*Cl(y)$ (or $z \in (\alpha p)^*Cl(y)$. such that $z \notin (\alpha p)^*Cl(x)$). There exists $V \in (\alpha p)^*O(X,\tau)$. such that $y \notin V$ and $z \in V$; hence $x \in V$. Hence,we have $x \notin (\alpha p)^*Cl(y)$.By(3), we obtain $(\alpha p)^*Cl(x) \cap (\alpha p)^*Cl(\{y\}) = \emptyset$.The proof for otherwise is similar

(4) => (1) : Let $V \in (\alpha p)^*O(X,\tau)$ and $x \in V$. For each $y \notin V$, $x \neq y$ and $x \notin (\alpha p)^*Cl(\{y\}$.This shows that $(\alpha p)^*Cl(\{x\}) \neq (\alpha p)^*Cl(\{y\})$. By (4) $(\alpha p)^*Cl(\{x\}) \cap (\alpha p)^*Cl(\{y\}) = \emptyset$ for each $y \in X - V$ and hence $(\alpha p)^*Cl(\{x\}) \cap (\bigcup_{y \in X-V} (\alpha p)^*Cl(\{y\}) = \emptyset$.On the other hand ,since $V \in (\alpha p)^*O(X,\tau)$ and $y \in X-V$,we have $(\alpha p)^*Cl(\{y\}) \subset X - V$ and hence $X - V = \bigcup_{y \in X-V} (\alpha p)^*Cl(\{y\}$.Therefore we obtain $(X - V) \cap (\alpha p)^*Cl(\{x\}) = \emptyset$.and $(\alpha p)^*Cl(\{x\} \subset V$. This shows that (X, τ) is a $(\alpha p)^*$-R_1 space.

Corollary 3.1. A topological space (X,τ) is a $(\alpha p)^*$-R_1 space if and only if for any x and y in X, $(\alpha p)^*Cl(\{x\}) \neq (\alpha p)^*Cl(\{y\}) => (\alpha p)^*Cl(\{x\}) \cap (\alpha p)^*Cl(\{y\}) = \emptyset$

Proof. It follows from Proposition 3.1.

Theorem 3.1. A topological space (X, τ) is a $(\alpha p)^*$- R_1 space if and only if for any points x and y in X, $(\alpha p)^*Ker(\{x\}) \neq (\alpha p)^*Ker(\{y\}) => (\alpha p)^*Ker(\{x\}) \cap (\alpha p)^*Ker(\{y\}) = \emptyset$.

Proof. Suppose that (X, τ) is a $(\alpha p)^*$-R_1 space, for any points x and y in X if $(\alpha p)^*Ker(\{x\}) \neq (\alpha p)^*Ker(\{y\})$ then $(\alpha p)^*Cl(\{x\}) \neq (\alpha p)^*Cl(\{y\})$.We prove that $(\alpha p)^*Ker(\{x\}) \cap (\alpha p)^*Ker(\{y\}) = \emptyset$.Let $z \in (\alpha p)^*Ker(\{x\}) (\alpha p)^*Ker(\{y\})$. By $z \in (\alpha p)^* Ker(\{x\})$, it follows that $x \in (\alpha p)^*Cl(\{z\})$. Since $x \in (\alpha p)^*Cl(\{x\})$, $(\alpha p)^*Cl(\{x\}) = (\alpha p)^*Cl(\{z\})$.Similarly, we have $(\alpha p)^*Cl(\{y\}) = (\alpha p)^*Cl(\{z\}) = (\alpha p)^*Cl(\{x\})$. This is a contradiction and therefore,we have $(\alpha p)^*Ker(\{x\}) \cap (\alpha p)^*Ker(\{y\}) = \emptyset$ Conversely , let for any points x and y in X $(\alpha p)^*Ker(\{x\}) \neq (\alpha p)^*Ker(\{y\})$ implies $(\alpha p)^*Ker(\{x\}) \cap (\alpha p)^*Ker(\{y\}) = \emptyset$.$(\alpha p)^*Cl(\{x\}) \neq (\alpha p)^*Cl(\{y\})$, then , $(\alpha p)^*Ker(\{x\}) \neq (\alpha p)^*Ker(\{y\})$.Therefore $(\alpha p)^*Ker(\{x\}) \cap (\alpha p)^*Ker(\{y\}) = \emptyset$ which implies $(\alpha p)^*Cl(\{x\}) \cap (\alpha p)^*Cl(\{y\}) = \emptyset$.Since $z \in (\alpha p)^*Cl(\{x\})$ implies that $x \in (\alpha p)^*Ker(\{z\})$ and therefore $(\alpha p)^*Ker(\{x\}) \cap (\alpha p)^*Ker(\{z\}) \neq \emptyset$. By hypothesis, we have $(\alpha p)^*Ker(\{x\}) = (\alpha p)^*Ker(\{z\})$. Then $z \in (\alpha p)^*Cl(\{x\}) \cap (\alpha p)^*Cl(\{y\})$implies that $(\alpha p)^*Ker(\{x\}) = (\alpha p)^*Ker(\{z\}) = (\alpha p)^*Ker(\{y\})$. But this is a contradiction. Therefore, $(\alpha p)^*Cl(\{x\}) \cap (\alpha p)^*Cl(\{y\}) = \emptyset$ and (X, τ) is a $(\alpha p)^*$-R_1 space.

Theorem 3.2. For a topological space (X, τ), the following properties are equivalent :

(1) (X, τ) is a $(\alpha p)^*$-R_1 space;

(2) For any nonempty set A and $G \in (\alpha p)^*o(X, \tau)$ such that $A \cap G \neq \emptyset$, there exists $F \in (\alpha p)^*o(X, \tau)$ such that $A \cap F \neq \emptyset$ and $F \subset G$;

(3) Any $G \in (\alpha p)^*$ -$o(X, \tau)$, $G = \cup\{F \in G \in (\alpha p)^*$-$o(X, \tau)| F \subset G\}$;

(4) Any $F \in (\alpha p)^*$-$o(X, \tau)$, $F = \cap\{G \in (\alpha p)^*$-$o(X, \tau), | F \subset G\}$;

(5) For any $x \in X$, $(\alpha p)^*$-$Cl(\{x\}) \subset (\alpha p)^*$-$Ker(\{x\})$

Proof. (1) ⇒ (2) : Let A be a nonempty set of X and $G \in (\alpha p)^*$-$o(X, \tau)$, such that $A \cap G \neq \emptyset$. There exists $x \in A \cap G$. Since $x \in G \in (\alpha p)^*$-$o(X, \tau)$, μ^{**}-$Cl(\{x\}) \subset G$. Set $F = (\alpha p)^*$-$Cl(\{x\})$, then $F \in (\alpha p)^*$ -$o(X, \tau)$, $F \subset G$ and $A \cap F \neq \emptyset$.

(2) ⇒ (3) : Let $G \in (\alpha p)^*$ -$o(X, \tau)$, then $G \supset \cup\{F \in (\alpha p)^*$-$o(X, \tau),) | F \subset G\}$. Let x be any point of G. There exists $F \in (\alpha p)^*$-$o(X, \tau)$, such that $x \in F$ and $F \subset G$. Hence, we have $x \in F \subset \cup\{F \in (\alpha p)^*$-$o(X, \tau), | F \subset G\}$hence $G = \cup\{F \in (\alpha p)^*$-$o(X, \tau), | F \subset G\}$.

(3) ⇒ (4) : Straightforward.

(4) ⇒ (5) : Let x be any point of X and $y /\in (\alpha p)^*Ker(\{x\})$. There exists $V \in (\alpha p)^*$-$o(X, \tau)$, such that $x \in V$ and $y \notin V$; hence $(\alpha p)^*$-$Cl(\{y\}) \cap V = \emptyset$. By (4) $(\cap\{G \in (\alpha p)^*$-$o(X, \tau),| (\alpha p)^*$-$Cl(\{y\}) \subset G\}) \cap V = \emptyset$ and there exists $G \in (\alpha p)^*$-$o(X, \tau)$,such that $x \notin G$ and $(\alpha p)^*$-$Cl(\{y\}) \subset G$. Therefore, $(\alpha p)^*$-$Cl(\{x\}) \cap G = \emptyset$ and $y /\in$ **v** -$Cl(\{x\})$. Consequently, we obtain $(\alpha p)^*$-$Cl(\{x\}) \subset (\alpha p)^*$-$Ker(\{x\})$. (5) ⇒ (1) : Let $G \in (\alpha p)^*$-$o(X, \tau)$, and $x \in G$.Let $y \in (\alpha p)^*$-$Ker(\{x\})$,then $x \in (\alpha p)^*Cl(\{y\})$ and $y \in G$. This implies that $Ker(\{x\}) \subset G$. Therefore, we obtain x ∈ $(\alpha p)^*$- Cl({x}) ⊂ $(\alpha p)^*$-Ker({x}) ⊂ G. This shows that (X, τ) is a $(\alpha p)^*$-R_1 space

Corollary 3.2. For a topological space (X, τ), the following properties are equivalent : (1) (X, τ) is a $(\alpha p)^*$-R_1 space; (2) $(\alpha p)^*$- Cl({x}) = $(\alpha p)^*$-Ker({x}) for all x ∈ X. **Proof.** (1) ⇒ (2) : Let (X, τ) be a $(\alpha p)^*$-R_1 space. It follows that $(\alpha p)^*$- Cl({x}) ⊂$(\alpha p)^*$-Ker({x}) for each x ∈ X. Suppose y ∈$(\alpha p)^*$-Ker({x}), then x ∈ $(\alpha p)^*$-Cl({y}) and $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}).Therefore, y ∈ $(\alpha p)^*$-Cl({x}) and hence $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Cl({x}).This shows that $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Ker({x}). (2) ⇒ (1).

Theorem 3.3. For a topological space (X, τ), the following properties are equivalent : (1) (X, τ) is a $(\alpha p)^*$-R_1 space; (2) If F is $(\alpha p)^*$-closed, then F = $(\alpha p)^*$-Ker(F); (3) If F is $(\alpha p)^*$-closed and x ∈ F, then $(\alpha p)^*$-Ker({x}) ⊂ F; (4) If x ∈ X, then $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Cl({x}). **Proof. (1) ⇒ (2)** : Suppose that F is $(\alpha p)^*$-closed and x /∈ F. Thus X − F is $(\alpha p)^*$-open and x ∈ X − F. Since (X, τ) is $(\alpha p)^*$ − R1 μ^{**}-Cl({x}) ⊂ X − F. Thus $(\alpha p)^*$-Cl({x}) ∩ F = ∅ and x ∉ $(\alpha p)^*$-Ker(F). Therefore $(\alpha p)^*$-Ker(F) = F.
(2) ⇒ (3) : In general, A ⊂ B implies $(\alpha p)^*$-Ker(A) ⊂ $(\alpha p)^*$-Ker(B). Therefore, it follows from (2) that $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Ker(F) = F. **(3)<=> (4)** : Since x ∈$(\alpha p)^*$-Cl({x}) and $(\alpha p)^*$Cl({x}) is $(\alpha p)^*$-closed, by (3) $(\alpha p)^*$-Ker({x}) ⊂$(\alpha p)^*$-Cl({x}).
(4) <=> (1) : Let x ∈$(\alpha p)^*$-Cl({y}). Then y ∈ $(\alpha p)^*$-Ker({x}). Since x ∈ $(\alpha p)^*$-Cl({x}) and $(\alpha p)^*$-Cl({x}) is $(\alpha p)^*$-closed, by (4) we obtain y ∈ $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Cl({x}). Therefore x ∈$(\alpha p)^*$Cl({y}) implies y ∈ $(\alpha p)^*$-Cl({x}). The converse is obvious and (X, τ) is $(\alpha p)^*$-R_1 space. **Definition 3.3.** A topological space (X, τ) is $(\alpha p)^*$-symmetric if for x and y in X, x ∈ $(\alpha p)^*$-Cl({y}) implies y ∈ $(\alpha p)^*$-Cl({x}).

Definition 3.4. A subset A of a topological space (X, τ) is called a ($(\alpha p)^*$, $(\alpha p)^*$)closed set (briefly ($(\alpha p)^*$, $(\alpha p)^*$)closed) if $(\alpha p)^*$-Cl(A) ⊂ U whenever A ⊂ U and U is $(\alpha p)^*$-open in (X, τ).

Lemma 3.4. Every μ^{**}-closed set is ($(\alpha p)^*$, $(\alpha p)^*$) - closed. **Theorem 3.4.** A topological space (X, τ) is $(\alpha p)^*$-symmetric if and only if {x} is ($(\alpha p)^*$, $(\alpha p)^*$) - closed for each x ∈ X. Proof. Assume that x ∈ $(\alpha p)^*$-Cl({y}) but y ∉ $(\alpha p)^*$-Cl({x}). This means that the complement of $(\alpha p)^*$-Cl({x}) contains y. Therefore the set {y} is a subset of the complement of $(\alpha p)^*$-Cl({x}). This implies that $(\alpha p)^*$Cl({y}) is a subset of the complement of $(\alpha p)^*$-Cl({x}). Now the complement of $(\alpha p)^*$-Cl({x}) contains x which is a contradiction. Conversely, suppose that {x} ⊂ E ∈ $(\alpha p)^*$-O(X, τ) but $(\alpha p)^*$-Cl({x}) is not a subset of E. This means that $(\alpha p)^*$-Cl({x}) and the complement of E are not disjoint. Let y ∈ ($(\alpha p)^*$-Cl({x}) ∩ Ec). Now we have x ∈ $(\alpha p)^*$-Cl({y}) ⊂ E^c and x ∉ E. But this is a contradiction.

Definition 3.5. A topological space (X, τ) is called $(\alpha p)^*$-T_0 if for any distinct pair of points x and y in X, there is a $(\alpha p)^*$-open U in X containing x but not y and a $(\alpha p)^*$-open set V in X containing y but not x.

Theorem 3.5. A topological space (X, τ) is $(\alpha p)^*$-T_0 if and only if the singletons are $(\alpha p)^*$-closed sets. **Proof.** Suppose that (X, τ) is $(\alpha p)^*$-T_0 and x ∈ X. Let y ∈ $\{x\}^c$. Then x ≠ y and so there exists a $(\alpha p)^*$-open set U_y such that y ∈ Uy but x ∉ U_y. Consequently y ∈ U_y ⊂ $\{x\}^c$ i.e., $\{x\}^c$ = ∪{ U_y /y ∈ $\{x\}^c$} which is $(\alpha p)^*$-open. Conversely. Suppose that {p} is $(\alpha p)^*$-closed for every p ∈ X. Let x, y ∈ X with x ≠ y . Now x ≠y implies y ∈ $\{x\}^c$. Hence$\{x\}^c$is a$(\alpha p)^*$-open set containing y but not x. Similarly $\{y\}^c$ is a $(\alpha p)^*$-open set containing x but not y. Accordingly X is a $(\alpha p)^*$-T_0 space. **Theorem 3.6**. For a topological space (X, τ) the following are equivalent: (1) (X, τ) is $(\alpha p)^*$-R_1 ; (2) (X, τ) is $(\alpha p)^*$-symmetric. **Proof. (1) ⇒ (2).** If x ∉ $(\alpha p)^*$-Cl({y}). Then there exist a $(\alpha p)^*$-open set U containing x such that y ∉ U. Hence y ∉ $(\alpha p)^*$-Cl(U). The converse is similarly shown. **(2) ⇒ (1):** Let U be a $(\alpha p)^*$-open set and x ∈ U. If y ∉ U, then x ∉$(\alpha p)^*$-Cl({y}) and hence y∉ $(\alpha p)^*$-Cl({x}) This implies that $(\alpha p)^*$-Cl({x}) ⊂ U. Hence (X, τ) is $(\alpha p)^*$-R_1. **Definition 3.6.** A filterbase F is called $(\alpha p)^*$-convergent to a point x in X, if for any

$(\alpha p)^*$-open set U of X containing x, there exists B in F such that B is a subset of U. **Lemma 3.5.** Let (X, τ) be a topological space and let x and y be any two points in X such that every net in X $(\alpha p)^*$-converging to y μ**-converges to x. Then x ∈ $(\alpha p)^*$-Cl({y}). **Proof.** Suppose that x_n = y for each n ∈ N. Then $\{x_n\}_n$ ∈N is a net in $(\alpha p)^*$-Cl({y}). Since $\{x_n\}_n$ ∈N $(\alpha p)^*$-converges to y, then $\{x_n\}_n$ ∈N $(\alpha p)^*$-converges to x and this implies that x ∈ $(\alpha p)^*$-Cl({y}).

Theorem 3.7. For a topological space (X, τ), the following statements are equivalent: (1) (X, τ) is a $(\alpha p)^*$-R_1 space; (2) If x, y ∈ X, then y ∈ $(\alpha p)^*$-Cl({x}) if and only if every net in X converging to y $(\alpha p)^*$-converges to x.

Proof. (1)→(2): Let x, y ∈ X such that y ∈ $(\alpha p)^*$-Cl({x}). Suppose that $\{x_\alpha\}_{\alpha\in\wedge}$ be a net in such that $\{x_\alpha\}_{\alpha\in\wedge}$ $(\alpha p)^*$-converges to y. Since y ∈ $(\alpha p)^*$-Cl({x}), we have $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}). Therefore x ∈ $(\alpha p)^*$-Cl({y}). This means that $\{x_\alpha\}_{\alpha\in\wedge}$ $(\alpha p)^*$-converges to x. Conversely, let x, y ∈ X such that every net in X $(\alpha p)^*$-converging to y $(\alpha p)^*$-converges to x. Then x ∈ $(\alpha p)^*$-Cl({y}) by Lemma l3.2. By Theorem 3.5, we have $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}). Therefore y ∈ $(\alpha p)^*$-Cl({x}).

(2)→(1): Assume that x and y are any two points of X such that $(\alpha p)^*$-Cl({x}) ∩ $(\alpha p)^*$-Cl({y}) 6= ∅. Let z ∈ $(\alpha p)^*$-Cl({x}) ∩ $(\alpha p)^*$-Cl({y}). So there exists a net $\{x_\alpha\}_{\alpha\in\wedge}$ in geCl({x}) such that {xα}α∈∧ -converges to z. Since z ∈ $(\alpha p)^*$-Cl({y}), then $\{x_\alpha\}_{\alpha\in\wedge}$ $(\alpha p)^*$converges to y. It follows that y ∈ $(\alpha p)^*$-Cl({x}). By the same taken we obtain x ∈ $(\alpha p)^*$-Cl({y}). Therefore $(\alpha p)^*$Cl({x}) = $(\alpha p)^*$Cl({y}) and (X, τ) is $(\alpha p)^*$-R_1

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