Comparison Between Poisson and Telegraph Models

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Abstract

We investigate the impact of finite-speed gravitational interactions on the formation of cosmic structure using N-body simulations modified to include telegraph-type corrections to the standard Poisson equation. By applying persistent homology to particle distributions, we extract Betti numbers and persistence diagrams to quantify the emergence and evolution of topological features such as voids and filaments. Our analysis reveals that simula- tions incorporating finite-speed gravity suppress the early formation of H (void) features compared to traditional Poisson-based simulations. This suggests that delayed gravitational response alters the morphological evolution of cosmic structure in measurable ways. Our results highlight the utility of topological data analysis in probing non-standard gravitational dynamics in cosmological models.

1 Introduction

For generations, physicists have relied on the Poisson equation to model cosmic growth [4]. This mathematical framework connects the clumping of matter to gravity's pull, assuming gravitational effects are instantaneous. However, nothing in the universe can exceed the speed of light, making the Poisson equation a shortcut that overlooks important truths.

Enter the telegraph equation, a modern enhancement that recognizes gravity's influence propagates at a finite speed rather than instantly. This delay, though seemingly minor, could dramatically change structural evolution [3], making the universe's development more dynamic and wave-like.

Traditional tools like power spectra and correlation functions measure the extent of structure but not its shape. This is where topology comes in [2]; topological data analysis (TDA) allows us to mathematically explore the universe's shape [1].

Our question is: if gravity doesn't act instantly, does the universe's skeleton grow differently? By examining Betti numbers over time, we aim to determine whether a universe shaped by slower gravity would reveal a sparser network or a complex tapestry reflecting delayed construction. This exploration probes whether the universe's design evidences gravity's speed limits.

2 The Telegraph equation: A more realistic Look

The Poisson equation is a most common method in n body simulation for describing how a field responds to a source, given by $\nabla^2\phi = \delta$. It assumes that changes in the source instantaneously affect the field everywhere. This assumption is good for systems that evolve slowly and are non-relativistic. However, it does not account for the finite speed at which gravity actually propagate in universe.

To take account for the speed of gravity waves, the Telegraph equation introduces a more comprehensive framework . It is given by,

$$\frac{1}{c_g^2}\frac{\partial^2\phi}{\partial t^2} = \nabla^2\phi - \delta$$

This form has a wave term which allows waves to propagate at a finite speed. Under the circumstances. These characteristics render it a better model for scenarios with wave-like phenomena.

The Telegraph equation is representing effects spreading in time and not appearing all at once. It permits the simulation of transient dynamics, oscillations, and energy dissipation, which are the very things required to describe complex physical phenomena such as wave propagation, reflection, or interference. This makes it particularly valuable in cosmology and other research areas studying modified gravity or coupling with more complex time dependent behavior. In a way, while the Poisson equation represents a static, instantaneous view of field dynamics, the Telegraph equation represents a dynamic and realistic view of how fields evolve, move, and dissipate over time and space.

We start with the telegraph equation, Assuming a quasi-static limit where time derivatives are negligible or contribute as effective constants, the equation simplifies to:

$$-\nabla^2\phi + \frac{1}{\tau^2 c_g^2}\phi = -\delta(\vec{x}) \quad (1)$$

Transforming this into Fourier space gives,

$$k^2\phi_k - \frac{1}{\tau^2 c_g^2}\phi_k = -\delta_k \quad (2)$$

Solving for ϕ_k, we obtain the modified Poisson equation in Fourier space as,

$$\phi_k = -\frac{\delta_k}{k^2 - \frac{1}{\tau^2 c_g^2}} \quad (3)$$

This result shows how damping and finite propagation speed introduce a scale-dependent suppression in the potential response, reducing to the standard Pois- son equation when $\tau \to \infty$.

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International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 09 Issue: 05 | May - 2025 SJIF Rating: 8.586 ISSN: 2582-3930

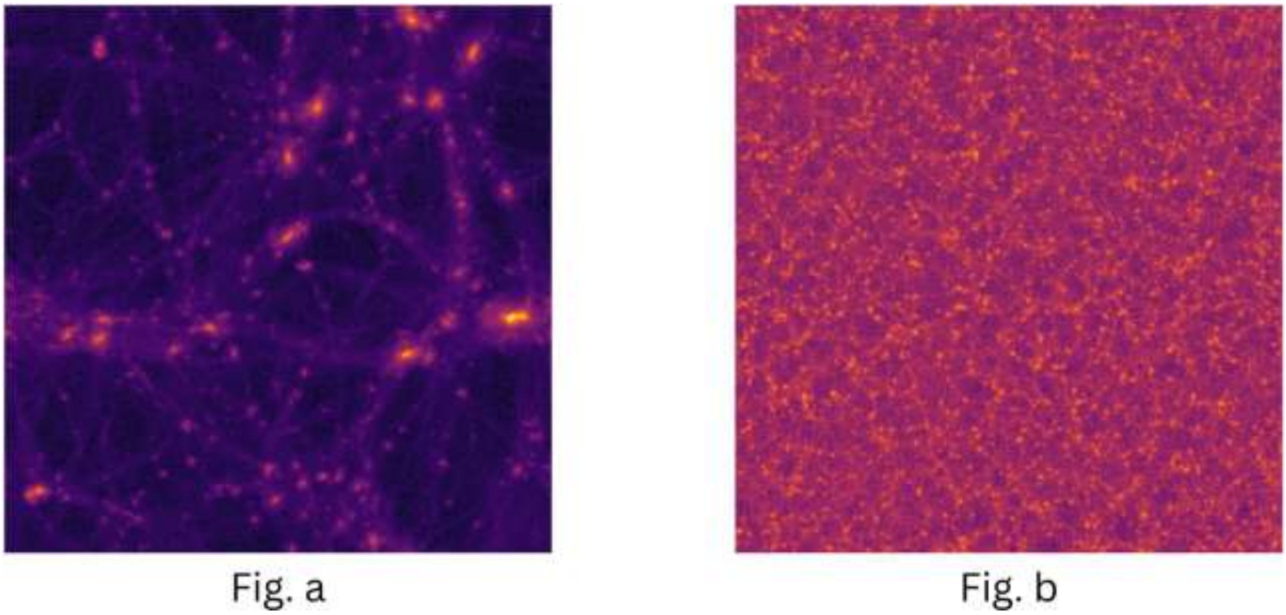

Figure 1: Fig. shows the final particle distributions of the two different method used at redshift z = 0. In fig. a poisson method is used and in fig. b improved poisson method is used. (grid size = 128, box size = 17.92)

3 Result

In this work i apply Toplogical Data analysis(TDA) to the particle distribution shown in fig 1. Using Persistence Topology i examine the topological features present in the particle cloud for both cases. A subset of 10,000 particles are randomly sampled from the particle distribution. We construct a Vietoris–Rips complexes on this point cloud across a range of distance thresholds determined by the average k-nearest neighbor spacing. Persistent homology is then computed up to 2 dimension using the Ripser library, and the results are visualized as persistence diagrams, which plot the birth and death scales of each topological feature as the filtration progresses. This approach allows us to track how the underlying topology of the structure emerges and evolves across scales.

Persistence diagrams reveal topological features in various dimensions. Zero-dimensional Holes(H_0) indicate disconnected clusters that merge with increasing thresholds, reflecting particle distribution fragmentation. One-dimensional holes (H_1) represent loops or filamentary structures, important for understanding galaxy filaments in the cosmic web. Two-dimensional holes(H_2) identify voids or cavities surrounded by particles, akin to cosmic voids with low matter density. Analyzing these features provides insights into the geometric structure of matter in the universe, offering a scale-invariant framework for understanding complex spatial patterns.

In our topological analysis for poisson simulation, we observe two prominent H_2 features that appear at very small distance thresholds and persist across the entire filtration range without ever disappearing. This persistence suggests the presence of large, stable voids within the spatial distribution of particles regions that remain topologically significant even as the scale of analysis increases. Such features are likely correspond to underdense structures in the particle simulation. The longevity of these H_2 components highlights the strength of persistent homology in identifying and characterizing the large-scale, three-dimensional architecture of matter in the universe.

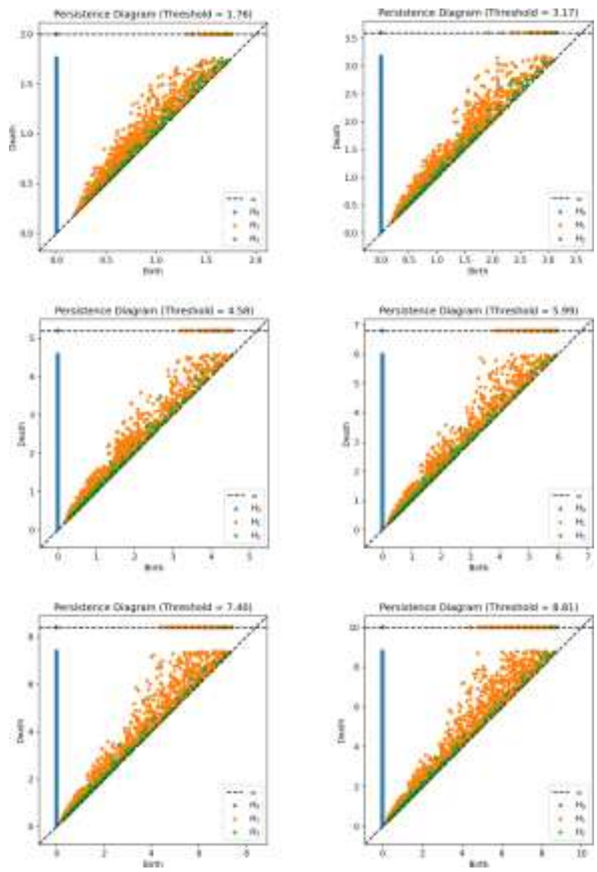

Fig 2: For Poisson equation solver. (grid size = 128, box size = 17.92)

In the improved Poisson simulation, we find that the prominent H_2 features emerge only at comparatively larger distance thresholds. This indicates that the formation of stable voids or enclosed cavities in the particle distribution requires a greater spatial scale in this case. Unlike the earlier scenario where voids were present and persistent even at finer resolutions, the delayed appearance of H_2 features suggests a more homogeneous or tightly packed structure at smaller scales. This shift in the birth scale of H_2 features reflects how the global structure of the simulation has changed, potentially due to the inclusion of more realistic physical dynamics in the improved model.

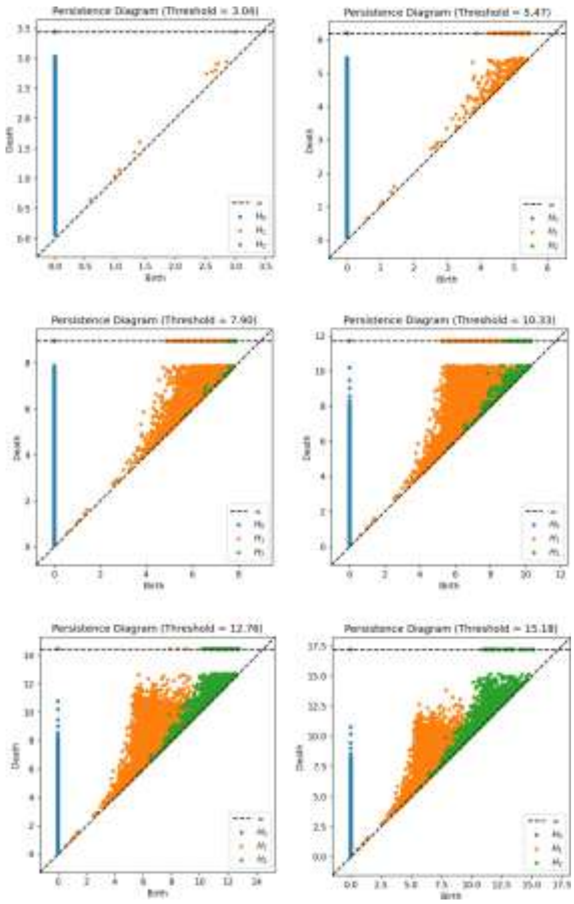

Fig 3: For Telegraph equation solver. (grid size = 128, box size = 17.92MPch)

4 Conclusion

The topological analysis of both the original and improved Poisson simulations reveals meaningful differences in the structure and evolution of voids. In the original simulation, persistent H_2 features that appear at very low density thresholds and persist indefinitely suggest the early formation of large, stable voids that remain prominent throughout the filtration process. In contrast, the improved simulation exhibits H_2 features that only emerge at larger distance thresholds, imply- ing that void formation is suppressed at smaller scales and becomes significant only when probing the structure at coarser resolutions. This contrast indicates a shift toward a more uniform or compact distribution in the improved model, with large-scale voids being less promi- nent or more delayed in their appearance. These observations highlight how persistent homology can capture subtle structural changes induced by physical modeling improvements in N-body simulations.

References

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