

Comprehensive Review of Option Pricing Models and Their Practical Relevance

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Abstract - Options are imperative, yet complex instruments in financial markets, primarily deployed to hedge, manage risk and speculation. Pricing these instruments to near accuracy is critical for its use as mispricing can lead to unethical speculation and arbitrage, not to mention financial risk. Since the development of closed form models, numerous analytical, simulation, machine learning and stochastic approaches have been calibrated to improve valuations and address assumptions which may not align to the real world.

This paper presents a comprehensive review of pricing strategies; stemming from options and payoff fundamentals, then into exploring various traditional plus algorithmic methods addressing volatility in pricing, and buttressing the theoretical review with computational experiments, with payoff visualizations, and pricing simulations.

The objective of the study is to emphasize the advantages and constraints of various option pricing strategies, with an emphasis on their transdisciplinary significance in the fields of computational finance, risk management and applied mathematics.

Key Words: option pricing, derivatives, Black-Scholes, Numerical Methods, quantitative finance

1. INTRODUCTION

The management of financial instruments is a critical area of study in the larger horizon of gaining a cognizance of the various financial markets. An imperative segment of such is the modelling and pricing of derivatives. This area is an integrative crossroad between applied computational studies, statistical inference, and financial knowledge in applicative context. This process forms ground for risk management, investment strategy and the means to understand dynamic variables in the market. Research in this area primarily explores various models and processes to price and mediate already priced multiplex derivatives in order to assess behavioral undulations given various market precedents.

1.1 Importance of Option Pricing

Options give one the right; not obligation to exercise it, making them critical global market operand with controlled exposure. They are adept when used with research for purposes such as: hedging against risk, arbitrage opportunities in space and time, and speculation within an ethical framework. The insistence on accurate pricing is essential not only for traders and various other market operators, but also for risk managers, researchers, and regulatory bodies. The mispricing of options is not mere lack of tandem between information, but more so results on either ends of the pendulum being: significant arbitrage

opportunities or systematic risk if participants are misled by false valuations. Hence, improving existing methods with the infusion of more efficient parameter assessors, in particularly stressed market scenarios are central in ensuring market efficiency and stability.

1.2 Background and motivation

The primary objective of this initiative is to craft a comprehensive review of option pricing strategies, ranging from theoretical closed-form models to modern simulation or stochastic approaches. The purpose of this venture is to comprehend and contrast theoretical distinctions, applicative assumptions, and computational variances among approaches. Additionally, it seeks to demonstrate the significance of combination methods on historical market data through empirical analysis, given a simulation-backed implementation appropriated for applicative inference.

1.3 Interdisciplinary relevance

Derivative pricing stands at the crossroads of several disciplines such as: financial mathematics, applied mathematics, statistics, risk management, programming, and multiple other sub-subjects under each of the mention broad domains. For this paper, the following areas are of significant interest for comparative study:

- Computational Finance: practical study of models (stimulatory or empirical)
- Applied Mathematics: stochastic calculus, differentials, linear algebra, probabilities
- Statistics: visualizations of empirical journeys of instruments (dispersions, tendencies)
- Risk Management: sensitivities (Greeks), VaR, optimal hedging
- Programming: easing computational tediousness, machine learning approaches

1.4 Scope of paper

The rationale behind this paper stems from the evident need for robust, computationally feasible and realistically implementable option pricing mechanisms in an increasingly volatile and data-absorbent financial landscape. While many classical models provide neat solutions, the failure to capture smiles in volatility movement, jumps and liquidity constraints bends them into being non-amiable.

This paper aims to:

- Review foundational and incrementally advanced option pricing models
- Classify and compare them based on their mathematical structuring and use-case applicability
- Implement selected models computationally using Python with European option assumptions

4. Provide a reliable analysis of their accuracy, computational load, and amicability in assumptions taken to price in manner

The scope of models considered includes: analytical models (Black-Scholes, binomial trees), numerical models (differentiation and calculus methods), and simulation methods, with the extensions of machine learning parametric assessment, jump diffusion and stochastic volatility to reach as closer to real markets as within means.

This work is aimed at students, researchers, and industry professionals in quantitative finance, applied mathematics/statistics, and computational engineering who wish to broaden their academic knowledge, as well as their judgements in implementations of traditional and modern option pricing methods.

2. REVIEW OF LITERATURE

To assess the evolution of option pricing, the study must thread to a little more than a century spanned. In the 1900, Bachelier introduced the critical Brownian motion to frame stock prices [1]. His interpretations were mathematically satisfying; assuming normality in distribution, however, it failed to reflect real-world market positive scale of prices. The Black-Scholes-Merton model (1973), also still known as the biggest breakthrough in option pricing, introduced risk-neutral valuation and a closed-form equation for exclusively European Options [2]. This proposal was transformatory in pricing the complexities that options bear and was awarded the Nobel Prize in Economics.

Assuming no transactional costs, in addition to other such constant variables which limit practical scope led to the emergence of classical discrete-time models. Binomial option pricing was introduced by Cox, Ross and Rubinstein in 1979 [4] which was a lattice approach that converges Black-Scholes price under finer partitions. Trinomial pricing [3] extended this using a middle state facilitating stability and efficiency. Additionally, PDE-based models associated the heat equation to the pricing problem [6], offering a bridge between theory into boundary conditionality and hedging.

An unaddressed issue was however volatility seen as a constant. Advanced models aimed to bridge this gap. Heston introduced stochastic volatility through his model in 1993 [5] to capture smiles, the Merton jump-diffusion model [9] scaled jerks in price movements. As accuracy favoring as these models were, they introduced computational strain. The 2000s saw a leaning towards machine learning approaches that leverage neural networks and reinforcement learning for parameter estimation and hedging. [10]. The most recent development being the proposal of quantum computing-based models; although not as accessible as machine learning models, to solve higher dimensional derivatives in a much shorter time span.

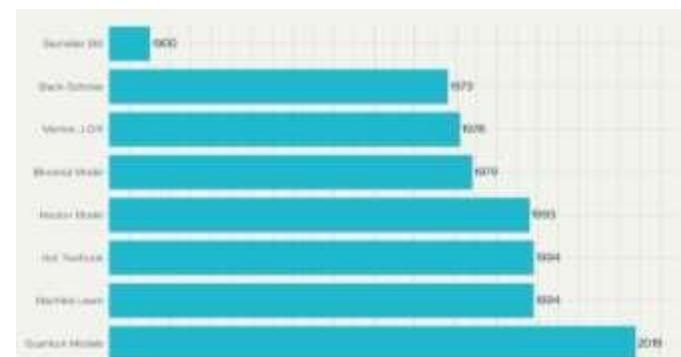


Fig -1: Timeline of major option pricing model developments from Bachelier (1900) to Quantum Finance (2019)

As active research progresses, the challenge of computational efficiency and accessibility remains for more complex pricing scenarios (e.g. clubbed option trading strategies). Anomalies such as volatility skew, liquidity, and tail risks remain explained in partiality. Moreover machine learning and quantum approaches are still in the infancy stage in terms of interpretability, robustness, real-time usage and accessibility to the common trader.

3. FUNDAMENTALS OF OPTIONS

A derivative is a financial instrument or contract whose value is “derived” from the price of an underlying asset such as a stock, index, currency, commodity or even credit.

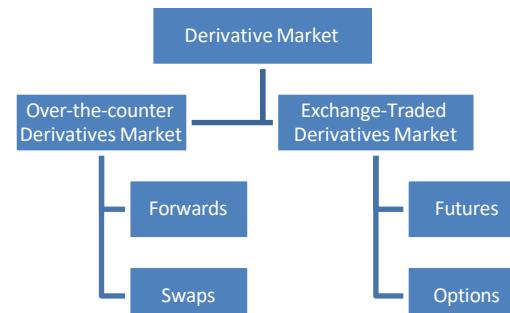


Fig -2: Types of derivatives based on type of trade

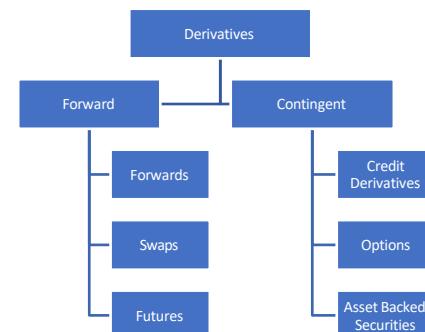


Fig -3: Types of derivatives based on nature of contract

1.1 Types of options

Options are instruments that give the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined strike price on or before the specified expiration date. While the call option gives the holder the right to buy the underlying asset, the put option gives the holder the right to sell it. They are exercised on fixed terms as they are exchange traded and contingent derivatives. Based on how they are exercised, options are further classified as European, American,

and Exotic options. American options can be exercised at any time from inception to maturity, European options can only be exercised on maturity, and exotic options have different treatments and are exercised on a case-by-case basis.

1.2 Payoff structures

The payoff from an option is the net result of the transaction in positive or negative reflective balance that is directly correlated to the movement of the underlying asset price.

Call Option Payoff:

$$\text{Payoff} = \max(S_T - K, 0)$$

Put Option Payoff:

$$\text{Payoff} = \max(0, K - S_T)$$

Here, S_T stands for the underlying price of the asset at maturity while K is the strike price. No option entails a negative pricing; setting aside mathematical theory, if the underlying asset cannot be in negative scale, then an option having a negative scale would be meaningless. Thus, we use the max. function with 0 as the least possible payoff on any option type.

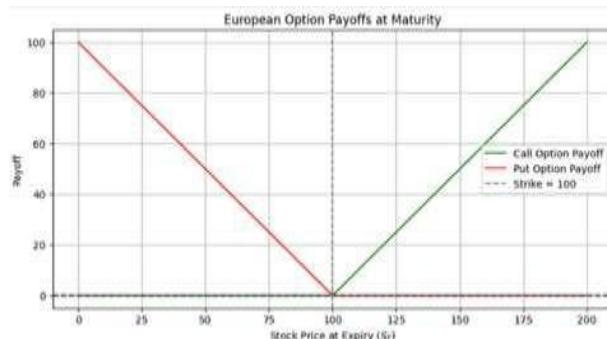


Fig -4: European option payoff structure

1.3 Option strategies

Options are combined into trading strategies that can benefit the holder. A simple buy/sell is a vanilla form of exercising an option and is not always the most effective when it comes to stressful market scenarios i.e. the optimum juncture to exercise the option.

1. **Covered Call:** Selling a call option on the stock held for premium income (downside protection, caps upside potential)
2. **Protective Put:** Buying a put option on stock own to limit losses (downside insurance)
3. **Bull Call Spread:** Buying call at lower strike and selling another call at higher strike for same expiry (reduces costs and limits profits and losses to fixed range)
4. **Bear Put Spread:** Buying put option at higher strike and selling one at lower strike within the same expiry (profit from expected deadlines with limited losses)
5. **Straddle:** Buying a call and a put at the same strike and expiration, betting on higher volatility (high upside potential on movement, limited losses to premium paid)
6. **Strangle:** Buying call option above and put option below stock price respectively, with the same expiration (high upside potential on movement, limited losses to premium paid)
7. **Butterfly Spread:** Buy one at lower strike, sell two at middle strikes and buy one at higher strike for the same expiry (neutral strategy; can be exercised for either option)

1.4 Arbitrage-free pricing

Options are exercised on the pretext of no arbitrage, meaning that there should be no riskless profit opportunities on temporal/spatial basis in an efficient market scenario. This adherence is used to define bounds for the fair pricing of options. The put-call parity encapsulates this theory which states that for European options (on a non-dividend paying stock – non-adjusted formula) the price of a call and the present value of the strike must sum the price of the put and the spot price. Mathematically,

$$C - P = S_0 - K e^{-rT}$$

$$C + K e^{-rT} + PV(D) = P + S_0$$

C is the call option

P is the put option

S_0 is the spot price

$K e^{-rT}$ is the present value of the strike price (strike price by exponent of minus rate by time)

$PV(D)$ is the present value of divided (second formula is adjusted for dividend)

4. METHODOLOGY AND MODELS REVIEWED

This study systematically reviews the principal approaches to option modelling, ranging from classical analytical models to more modern integrative approaches; helping ascertain the advantages, disadvantages, and gaps to be addressed in each implementation. Each approach is explained and differentiated based on approach and application.

4.1 Classical Models

4.1.1 Black-Scholes-Merton (BSM) Model

Till date, it stands as the benchmark for European option pricing [2]. Assuming lognormal asset pricing, frictionless markets, and constant volatility, it can be represented as such:

$$C(S, t) = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$\text{where } d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

N = CDF of normal distribution

t = time to maturity

4.1.2 Binomial Tree Model

The binomial option pricing model is a discrete-time lattice framework where option values are obtained through backward propagation [4]. It is computationally the easiest method and unlike the BSM model, adaptable to American options as well.

4.2 Numerical and Simulation Approaches

4.2.1 Monte Carlo Simulation

This simulation type is a random-sampling method used to generate as many possible outcomes for complex, uncertain events providing a range of possible results. Iterations are typically 1000-10000. Thousand such price paths under risk-neutral dynamics are generated, this is effective for exotic or complex option strategies.

4.2.2 Finite Difference Methods (FDMs)

For continuous price generation, the BSM model is usually seen as a PDE. The finite difference method discretizes this PDE as a system of equations to solve iteratively. This is suitable for boundary-sensitive options.

4.3 Advanced Stochastic Methods

4.3.2 Heston Model

Introduces the concept of randomness via probabilistic volatility which leans more towards statistical attesting. The square-root formulation is used for semi-closed solutions and a better empirical pricing fit [5].

4.3.3 Merton Jump-Diffusion Model

This model is a practical extension with the theoretical base set as the BSM model with jumps forming the Poisson distribution. This captures discontinuous movements, useful for assessing idiosyncratic risk [9].

4.4 Machine Learning and Hybrid Models

Neural networks and deep learning (perceptron) models are used to estimate parameters based on empirical price movements without closed-form equations or predefined notions; simply on movement of market data [7]. Reinforcement learning methods build on this to optimize hedging strategies. Hybrid models are amalgamation tests that combine stochastic PDE with machine learning frameworks to improve accuracy while retaining interpretation ease. Recent developments explore quantum computing for high-dimensional derivative pricing, using physical science and amplitude implications; still at the infant stage [10].

5. RESULTS AND COMPUTATIONS

Practical differences between analytical and simulation-based pricing methods can be illustrated by sampling a BSM model solution and Monte Carlo simulation for a European call option. The BSM model provides a deterministic price based on fixed assumptions of constant volatility and risk-free interest (no transactional costs), while the simulation generates an empirical estimate using average payoffs across many paths randomly framed by it, considering more movement possibilities.

Consider a European call option using textbook parameters:

Underlying price $S_0 = 100$

Strike price $K = 100$

Time to maturity $T = 1$ year

Risk-free rate of interest $r = 5\%$

Volatility $\sigma = 20\%$

The results from Fig -5 and Fig -6 in the charts given below indicate that simulation estimates converge to the theoretical BSM price as the number of simulations increase. For 10000 paths, the error margin relative to the BSM benchmark is less than 1%. However, with around nearer to 1000 paths, noticeable deviations occurred, attesting the trade-off between efficiency and accuracy.

Practically, BSM is applicable for more liquid and standard options. However, stressful market scenarios indicating uncertainty and complex derivatives require simulations to encompass random price paths outside of a closed-form

framework. For hedging, BSM provides closed-form Greeks that traders use directly, while Monte Carlo requires additional simulations to compute sensitivities, which limit their real-time applicability in fast moving markets. In computational finance, it is critical that models are buildable and can be used as base for hybrid models that can tread one closer to market movements.

Table -1: Result: Option Pricing Models and Use Cases

Model	Category	Assumption	Strength	Limitation/Gap	Use Case
BSM	Parametric	Constant Volatility	Closed-form, efficient	Poor fit for volatility smiles	European option
Binomial Model	Parametric	Discrete time	American options included	Ineffective for long maturity periods	American and Barrier options
Monte Carlo	Non-Parametric	Simulated path	Flexibility	Slow convergence	Exotic option
Heston	Parametric	Stochastic volatility	Captures smiles	Complex computation	Equity/FX option
Jump Diffusion	Parametric	Poisson jumps	Models price peak	Discontinuous movement	Idiosyncratic risk
Hybrid ML	Non-Parametric	Data-driven	Adapts to market	Interpretation	Real-time pricing

Overall, the comparison illustrates the importance of choosing the right model to develop on. Analytical methods would always provide superior results and variations over standard instruments; however, standard instruments are more buildable to integrate into hybrid structures.

Charts

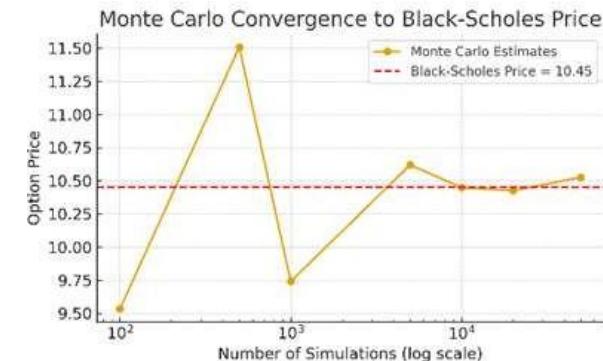


Fig -5: Monte Carlo simulation estimates converging to Black-Scholes analytical price as the number of simulations increases

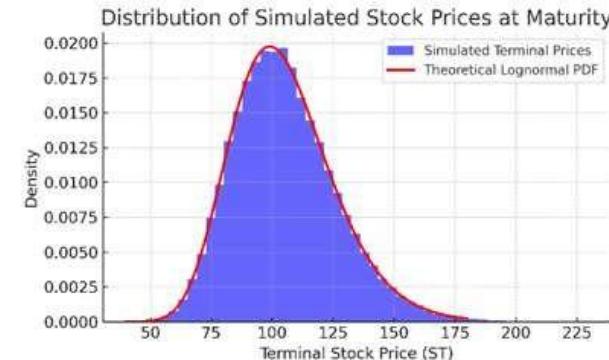


Fig -6: Distribution of simulated terminal stock prices (Monte Carlo) with overlayed theoretical lognormal density

6. CONCLUSIONS

6.1 Key Findings

- Classical models are efficient when limited to standard options
- Simulation methods offer flexibility and more price path scenarios for stress-built market environments and suit exotic options, but cost more computationally

- Advanced models bridge realistic scenario by stochastic volatility and jumps, addressing limitations of classical and simulation methods

6.2 Limitations

- Results are based on textbook data
- Real-world factors such as volatility smiles, liquidity and transaction costs are not captured

6.3 Future Directions

- Empirical validation with live option chain data
- Integration of machine learning and AI for volatility forecasting and dynamic hedging
- Exploration of quantum computing methods for efficiency

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