Confidence interval estimation for reliability of a two parameter Weibull distribution under complete and censored samples

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Estimating the reliability function of the two parameter Weibull distribution is a critical task in various fields, providing essential insights into product longevity and optimizing maintenance strategies. Due to its adaptable parameters namely scale parameter α and shape parameter β, the Weibull distribution models diverse failure behaviours, from early-life defects to wear-out periods. By evaluating reliability at key time phases namely early life ($t < 0.1\alpha$), useful life ($0.1\alpha \leq t \leq \alpha$), and end-of-life ($t > \alpha$), this estimation process supports quality control, operational forecasting, and end-of-life planning. Such reliability assessments help minimize operational disruptions, improve cost management, and support strategic planning across industries where dependability and lifecycle management are essential, such as in manufacturing, healthcare, and technology sectors. Although it holds practical significance, the estimation of confidence interval (CI) for the reliability function of two parameter Weibull distribution has been relatively underexplored in the literature. This paper presents a new approach for constructing CI for the reliability function of the Weibull distribution using the generalized variable (GV) technique, applicable to both complete samples and type II singly right-censored samples. The empirical evaluation of this method indicates that it provides coverage probabilities that are closely aligned with the nominal level, even when dealing with small uncensored samples (as small as 5) and censored samples where the proportion of censored observations can reach 70%. In comparison, traditional methods for the Weibull distribution tend to yield less reliable or widely varied coverage probabilities for complete samples. The findings are demonstrated through practical examples.

Keywords: Reliability, Confidence interval, Generalized Variable approach, Censoring.

1 Introduction

The two-parameter Weibull distribution has a wide range of applications across nearly all scientific disciplines. It is frequently employed to model data from various fields, including biology, environmental science, health, physical sciences, and social sciences. Additionally, this distribution is commonly used in meteorology and hydrology, establishing itself as a fundamental tool in reliability theory for analysing time-dependent failure data. Key references that underscore its relevance include works by Grace and Eagleson (1966), Crow (1982), Nathan and McMahon (1990), Selker and Haith (1990), Power (1992), Jiang et al. (1997), Duan et al. (1998), Jandhyala et al. (1999), Seshadri (1999), Aksoy (2000), Lun and Lam (2000), Seguro and Lambert (2000), Talkner and Weber (2000), Clarke (2002), Heo et al. (2001), Tan et al. (2007), Yang et al. (2007), Krishnamoorthy and Lin (2010), Kulkarni and Powar (2011), Jamdade and Jamdade (2012), J.I. McCool (2012) and Powar and Kulkarni (2015, among many others.

A continuous random variable (RV) X is said to follow a Weibull distribution with scale parameter α and shape parameter β if its probability density function (pdf) is given by,

$$f_X(x;\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} exp\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right); x > 0, \alpha > 0, \beta > 0.$$

We denote it as $X \rightarrow$ Weibull (α, β). The reliability function at t, $R(t)$, for Weibull (α, β) distribution is,

$$R(t,\alpha,\beta) = exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right); t > 0, \alpha > 0, \beta > 0.$$

The estimation of CI for $R(t,\alpha,\beta)$ plays a crucial role in understanding product longevity and optimizing maintenance strategies. For example, when assessing the reliability of light bulbs through Weibull (α, β) distribution, we can use a scale parameter, α, of 2000 hours and a shape parameter, β, of 1.5, which signifies a wear-out failure mode.

This analysis identifies three significant time periods for reliability evaluation: early life, useful life, and wear-out phases. In the early life stage ($t < 200$ hours), the model anticipates that failures primarily result from manufacturing defects, leading to a reliability function value of $R(100{,}2000{,}1.5) \approx 0.9889$. This indicates that roughly 98.89% of light bulbs are expected to function successfully during the first 100 hours.

As the bulbs transition into the useful life phase ($200 \leq t \leq 2000$ hours), they show minimal risk of failure, with a reliability function value of $R(1000{,}2000{,}1.5) \approx 0.7022$. This suggests that about 70.22% of bulbs are likely to remain operational beyond 1000 hours.

However, during the wear-out period ($t > 2000$ hours), the likelihood of failure begins to rise as the bulbs approach the end of their lifespan. In this phase, only 15.93% of bulbs are expected to last beyond 3000 hours, as indicated by $R(3000{,}2000{,}1.5) \approx 0.1593$.

These results emphasize the significance of reliability estimation across various time intervals, offering important insights for effective product lifecycle management, maintenance planning, and cost optimization strategies for lighting systems.

In various life-testing and reliability studies, researchers often encounter difficulties in collecting complete data on failure times for all experimental units. For example, in clinical trials, limited funding may result in participants discontinuing their involvement in the study. In industrial experiments, units may experience unexpected failures, or they might be deliberately removed before failure to save time and reduce costs. The data gathered from such experiments are classified as censored data. Among the different types of censoring, Type-I and Type-II censoring are the most commonly recognized.

Type-I censoring occurs when the duration of the experiment T is fixed, while the number of failures is variable. On the other hand, Type-II censoring involves a predetermined number of failures, referred to as r, with the duration of the experiment being variable. The GV method presented in this article is applicable to Type-II singly right-censored samples, as the pivotal quantities for the maximum likelihood estimators (MLEs) are still valid in this scenario.

Although the Weibull distribution is widely used across various fields, the estimation of CI for its reliability has been relatively underexplored in the literature, especially in the context of small sample sizes.

Yang et al. (2007) proposed a method for constructing a CI for $R(t,\alpha,\beta)$ by recognizing that a Weibull RV raised to the power of its shape parameter β acts as an Exponential RV with a mean α^{β}. This initial CI is referred to as the naive CI. To enhance the performance of the naive CI, especially regarding coverage probabilities for small sample sizes, the authors derived an analytical adjustment, leading to what they termed the analytically adjusted naive (AAN) CI for $R(t,\alpha,\beta)$.

When the shape parameter β is unknown, they suggested using estimators from maximum likelihood (ML) and modified maximum likelihood (MML) methods. Their results demonstrated that the MML-based CI consistently outperformed the one based on ML. Therefore, in our comparative analysis, we chose to employ the MML-based CI for our evaluation.

The empirical analysis presented in this article highlights notable inconsistencies between the estimated coverage probabilities and the nominal coverage probabilities of existing methods for CI estimation of $R(t, \alpha, \beta)$. These inconsistencies are particularly evident for various values of t when working with uncensored samples, especially under small sample sizes. This situation is frequently encountered in healthcare research, where factors such as the high costs of laboratory testing for contaminant levels often limit sample sizes.

Since regulatory requirements may necessitate the estimation of reliability at larger values of t using small to moderate sample sizes, addressing this challenge is essential. Therefore, the aim of this article is to introduce a method for estimating CI for the reliability of the widely used Weibull distribution. Our proposed method seeks to ensure that the coverage probabilities are closely aligned with nominal values, even when dealing with small sample sizes and across both uncensored and censored data scenarios for all values of t.

This article focuses on the statistical challenge of estimating a CI for the reliability function of the Weibull distribution, using the GV approach pioneered by Tsui and Weerahandi (1989) and further refined by Weerahandi (1993). For a detailed exploration of the GV approach and its diverse applications, we recommend the texts by Weerahandi (1995, 2004). Furthermore, Hannig et al. (2006) offer insightful examples that illustrate the practical use of the GV approach. This methodology enables the creation of a generalized pivotal quantity (GPQ), which is essential for deriving CIs for various parametric functions of interest.

Unlike traditional pivotal quantities, a GPQ is constructed from observed statistics and random variables, and does not rely on unknown parameters. A key benefit of the GV approach is its capability to directly derive a GPQ for a function of parameters by substituting the GPQs corresponding to the individual parameters (Krishnamoorthy et al. (2009)). The objective of this article is to introduce a GV approach for formulating two-sided CI for the reliability function of any distribution that has GPQs for its parameters. We assess the performance of the proposed method for the Weibull distribution through numerical simulations involving both uncensored and Type-II singly right-censored samples.

The organization of this paper is as follows: Section 2 provides an overview of the essential preliminaries related to GPQ and introduces the proposed method. Section 3 details the construction of CI for the reliability function of the Weibull distribution. Section 4 reviews current methods in the literature for estimating CIs for Weibull reliability function, comparing these with the proposed approach through simulation studies focusing on CI coverage probabilities. Section 5 examines the applications of the proposed CI method, and Section 6 presents concluding remarks.

2 Confidence Interval Based on GPQ Methodology:

A GPQ, denoted as G_θ for a parameter θ, is defined as a RV $T_\theta(X; x)$, where X is a RV with a distribution dependent on the parameter of interest θ and an additional nuisance parameter δ. The observed value of X is represented by x, and $T_\theta(X; x)$ adheres to the following two conditions.

1. The value of $G_\theta = T_\theta(X; x)$ at $X = x$, is free from the nuisance parameter δ. For most of the cases, $G_\theta = \theta$.
2. The distribution of $G_\theta = T_\theta(X; x)$ for given $X = x$ is free from any unknown parameters.

2.1 The proposed CI for a population reliability:

Consider a random sample $X_1, X_2, \dots, X_n$ of size n from a distribution with pdf $f_X(x; \underline{\theta})$, where $\underline{\theta} = (\theta_1, \theta_2, \dots, \theta_k)$ is a vector of unknown parameters. It is assumed that a GPQ is available for each component of $\underline{\theta} \in \Theta \subseteq \Re^k$, with the vector of GPQs denoted as $G_{\underline{\theta}} = \left(G_{\theta_1}, G_{\theta_2}, \dots, G_{\theta_k}\right)$. Let $R\left(t, \underline{\theta}\right)$ be the reliability function of *X*. Although $R\left(t, \underline{\theta}\right)$ may not always have a closed-form expression, it can be numerically determined for specific values of *t* and $\underline{\theta}$. A GPQ for $R\left(t, \underline{\theta}\right)$ can be expressed as:

$$G_{R_t} = R\left(t, G_{\underline{\theta}}\right) \quad (1)$$

where G_{R_t} has a distribution independent of $\underline{\theta}$. Thus, a two-sided CI for $R\left(t, \underline{\theta}\right)$ at a confidence level of (1−*α*)×100%, based on the GPQ G_{R_t}, can be constructed by the following process:

1. For observed data $\underline{x}$ and maximum likelihood estimates (or other equivariant estimators) $\widehat{\underline{\theta}_0}$ of $\underline{\theta}$, repeat the following steps N times (for example, N=100,000):
   i. Calculate GPQs $G_{\underline{\theta}} = \left(G_{\theta_1}, G_{\theta_2}, \dots, G_{\theta_k}\right)$ for $\underline{\theta} = (\theta_1, \theta_2, \dots, \theta_k)$, potentially using the method suggested by Iyer and Patterson (2002).
   ii. Calculate G_{R_t} using the expression (1) above.
2. The (100×*α*/2)th and 100×(1−*α*/2)th, 0 ≤ *α* ≤1, percentiles of the generated N values of G_{R_t} serve as the lower (L) and upper (U) bounds of the two-sided (1−*α*)×100% CI for $R\left(t, \underline{\theta}\right)$, denoted as [L,U]. This interval will be referred to as a "Generalized Confidence Interval (GCI)" for $R\left(t, \underline{\theta}\right)$.

GPQ-based inference is known to produce exact results; see, for example, Roy and Bose (2009).

3 The proposed CI for $R(t, \alpha, \beta)$ of Weibull (*α*, *β*) distribution:

For a complete sample, the MLE $\hat{\beta}$ for β is the solution to the equation:

$$\frac{1}{\hat{\beta}} - \frac{\sum_{i=1}^{n} x_i^{\hat{\beta}} \, log(x_i)}{\sum_{i=1}^{n} x_i^{\hat{\beta}}} + \frac{1}{n} \sum_{i=1}^{n} log(x_i) = 0 \quad (2)$$

with $\hat{\alpha} = \left(\sum_{i=1}^{n} x_i^{\hat{\beta}} / n\right)^{1/\hat{\beta}}$.

For a Type-II singly right-censored sample, in which we observe only the smallest *r* observations, $x_{(1)} \le x_{(2)} \le \dots \le x_{(r)}$, the MLE for β is found by solving the equation:

$$\frac{1}{\hat{\beta}} - \frac{\sum_{i=1}^{n} x_{iu}^{\hat{\beta}} \, log(x_{iu})}{\sum_{i=1}^{n} x_{iu}^{\hat{\beta}}} + \frac{1}{r} \sum_{i=1}^{r} log(x_{iu}) = 0 \quad (3)$$

and $\hat{\alpha} = \left(\sum_{i=1}^{n} x_{iu}^{\hat{\beta}} / n\right)^{1/\hat{\beta}}$.

Here, $x_{iu} = x_{(i)}$ denotes the observed values in ordered form for *i* =1, 2,…,*r* and $x_{iu} = x_{(r)}$ for *i*=*r*+1,…,*n*. The Newton–Raphson method can be applied to iteratively solve the equations (2) and (3), and softwares such as R and MINITAB provides tools to estimate these parameters directly.

3.1 GPQs for parameters α, β, and $R(t, \alpha, \beta)$:

Krishnamoorthy et al. (2009) introduced GPQs for parameters α and β as follows. Let $\widehat{\alpha_0}$ and $\widehat{\beta_0}$ denote the observed values of the MLEs $\hat{\alpha}$ and $\hat{\beta}$, respectively. Then, the GPQs for α and β can be defined by:

$$G_\alpha = \widehat{\alpha_0} \left(\frac{\alpha}{\hat{\alpha}}\right)^{\hat{\beta}/\widehat{\beta_0}} = \widehat{\alpha_0} \left(\frac{1}{\tilde{\alpha}}\right)^{\tilde{\beta}/\widehat{\beta_0}} \quad (4)$$

and

$$G_\beta = \frac{\beta}{\tilde{\beta}}\,\widehat{\beta_0} = \frac{\widehat{\beta_0}}{\tilde{\beta}} \tag{5}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ represent the MLEs of α and β based on a censored or uncensored sample from a Weibull (1,1) distribution. Using equation (1), the GPQ for $R(t,\alpha,\beta)$ can be expressed as:

$$G_{R_t} = R(t, G_\alpha, G_\beta) = exp\left(-\left(\frac{t}{G_\alpha}\right)^{G_\beta}\right) = exp\left(-\left(\frac{t(\tilde{\alpha})^{\tilde{\beta}/\widehat{\beta_0}}}{\widehat{\alpha_0}}\right)^{\frac{\widehat{\beta_0}}{\tilde{\beta}}}\right) \tag{6}$$

To compute a two-sided (1−α)100% GCI for $R(t,\alpha,\beta)$ with $t > 0$, based on a complete sample, the following algorithm can be used. This method also applies to Type-II singly right-censored samples, using the relevant MLEs and GPQs.

Steps of the Algorithm:

1. Calculate the MLEs $\widehat{\alpha_0}$ and $\widehat{\beta_0}$ for the parameters α and β from a sample $x_1, x_2, \dots, x_n$ of size n, assuming a Weibull (α, β) distribution.

2. Given the values $\widehat{\alpha_0}$ and $\widehat{\beta_0}$, repeat the following process N times (e.g., N=100,000):

    i. Generate n independent random values $x_{1_{11}}, x_{2_{11}}, x_{3_{11}}, \dots, x_{n_{11}}$ from a Weibull(1,1) distribution, then estimate $\tilde{\alpha}$ and $\tilde{\beta}$, the MLEs for α and β from this generated data.
    ii. Use Equations (4) and (5) to compute the GPQs, G_α and G_β.
    iii. Use Equation (6) to determine G_{R_t}, the GPQ for $R(t,\alpha,\beta)$.

The (1−α)×100% GCI for $R(t,\alpha,\beta)$ with $t > 0$ can be expressed as follows:

$$\left[G_{R_t;\alpha/2},\ G_{R_t;1-\alpha/2}\right] \tag{7}$$

where $G_{R_t;\alpha}$ represents the (100×α)th percentile of G_{R_t}.

4 A comparative study

The two-parameter Weibull distribution is extensively used in manufacturing, healthcare, and technology sectors, with applications demonstrated in studies by Lun and Lam (2000), Krishnamoorthy and Lin (2010), and Jamdade and Jamdade (2012). This research conducts a comparative analysis of the proposed GCI with existing methods, focusing on complete sample cases.

4.1Existing method

Analytically adjusted naïve CI based on modified ML estimator (AANMML):

Yang et al. (2007) introduced a two-sided (1−α)×100% CI for $R(t,\alpha,\beta)$ by applying a Weibull-to-Exponential transformation. This interval is expressed as:

$$\left[exp\left(-\frac{t^{\breve{\beta}}\chi^{2*}_{2n,\alpha/2}}{2S(\breve{\beta})}\right), exp\left(-\frac{t^{\breve{\beta}}\chi^{2*}_{2n,1-\alpha/2}}{2S(\breve{\beta})}\right)\right] \tag{8}$$

where $S(\breve{\beta}) = \sum_{i=1}^{n} x_i^{\breve{\beta}}$ and $\breve{\beta}$ is the MML estimate, determined by solving:

$$\frac{n-2}{\breve{\beta}} - \left(n\sum_{i=1}^{n} X_i^{\breve{\beta}} \, logX_i\right)\left(\sum_{i=1}^{n} X_i^{\breve{\beta}}\right)^{-1} + \sum_{i=1}^{n} logX_i = 0$$

Here, $\chi^{2*}_{2n,\delta} = c\chi^2_{2n,\delta} - 2n(c-1)$ with $c^2 = 1 + 0.6079\left(0.4226 - \breve{\beta} log(t/\breve{\alpha})\right)^2$, $\breve{\alpha} = \left(\frac{\sum_{i=1}^{n} X_i^{\breve{\beta}}}{n}\right)^{1/\breve{\beta}}$ and $\chi^2_{n,\delta}$ is the δth quantile of the chi-square distribution with *n* degrees of freedom. The adjusted CI based upon $\breve{\beta}$ has been shown to consistently provide better precision than the standard MLE $\hat{\beta}$.

4.2 Evaluation and comparison of the CIs:

The following empirical analysis was carried out to evaluate the proposed method in comparison to existing one. In this study, we set a significance level of $\alpha = 0.05$. We generated 10,000 samples from the Weibull distribution, utilizing scale parameters α=0.5,1,2,…,6 and shape parameters β=0.3,0.5,1,5,7,9,10 and considering values of *t* such that reliability function takes the values 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99 across various sample sizes of *n*=5,15,25,50. The resulting lower and upper limits Li, and Ui (i=1,2,…,10,000) for the two-sided CIs were computed using Equations (7) and (8). Finally, we calculated the coverage probability, which represents the proportion of intervals that included the true value of $R(t, \alpha, \beta)$ for each confidence interval assessed.

The main objective of this study is to evaluate the effectiveness of CI estimators for various sample sizes (*n*) and various values of *t*. We present boxplots that display the percentage of coverage probabilities for all combinations of *n*, α, β, and R(*t*) associated with the above CIs in Figures 1 and 2. The results highlight the superior performance of the proposed GV method when compared to existing method. Specifically, the boxplots indicate that the coverage probabilities from the GV method closely match the nominal level and are tightly concentrated. This outcome is anticipated, given that the conventional methods are often asymptotic or approximate. In contrast, the GV method, as noted in the findings of Roy and Bose (2009), is regarded as exact. To our knowledge, this proposed method is unique in its exactness, and we recommend its application in practical scenarios.

For Type II censored samples, the proportion of censored observations, denoted as PC = $P(X > X_{(r)})$, is selected at values of 0.3, 0.5, and 0.7. To ensure clarity and brevity, the results are illustrated through graphical representations in Figures 3 and 4 for the proportions PC=0.3, 0.5, 0.7. The visual data in Figures 1 to 4 demonstrate that the proposed method consistently achieves coverage probabilities that are closely aligned with 0.95, even for small uncensored sample sizes, such as 5. This accuracy is preserved for censored samples, as long as the proportion of censored observations remains up to 0.70.

5. Applications in Environmental Monitoring: Focus on Groundwater and Air Quality

This section presents an analysis of two real-world datasets to illustrate how interval estimation of reliability, based on the commonly used Weibull distribution, can be applied in hydrology and environmental science.

5.1 Analysis and Trends in Groundwater Contamination Levels

Vinyl chloride ranks among the top fifty chemicals produced globally, with annual production nearly doubling over the last 20 years to an estimated 27 million tons. Due to its toxicity and carcinogenic nature, elevated vinyl chloride levels in water are linked to serious health risks such as cancer and liver damage, making it a critical contaminant in groundwater. This study examines vinyl chloride levels from cleanup-gradient monitoring wells. Previous research by Krishnamoorthy et al. (2009) found that these concentrations fit well with a Weibull distribution model. The vinyl chloride data, measured in micrograms per liter (μg/L), includes values: 5.1, 2.4, 0.4, 0.5, 2.5, 0.1, 6.8, 1.2, 0.5, 0.6, 5.3, 2.3, 1.8, 1.2, 1.3, 1.1, 0.9, 3.2, 1.0, 0.9, 0.4, 0.6, 8.0, 0.4, 2.7, 0.2, 2.0, 0.2, 0.5, 0.8, 2.0, 2.9, 0.1, and 4.0. Using a Kolmogorov–Smirnov test, we found a two-tailed P value of 0.94, supporting the Weibull distribution model for this dataset. MLEs of the parameters for this data are $\hat{\alpha} = 1.89$ and $\hat{\beta} = 1.01$. The U.S.

Environmental Protection Agency (USEPA) suggests a safe concentration range between 2.0 and 2.4 µg/L. Based on our confidence interval at 95%, the threshold of 2.4 µg/L is reached at 0.4142, indicating that approximately 41% of the concentrations are likely to exceed this threshold, underscoring the importance of monitoring in these wells.

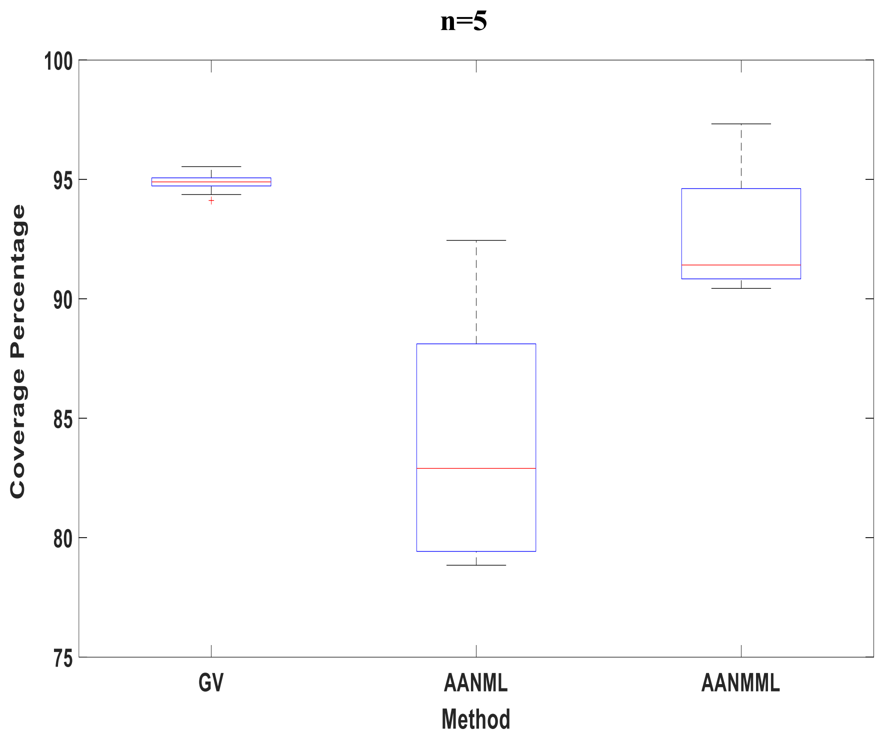
5.1 Analysing Lead Concentration Data in Air Quality

Lead, a soft and easily shaped metal, is present in the atmosphere as tiny particulate matter. Natural processes such as soil erosion, volcanic eruptions, sea spray, and wildfires introduce lead into the air, while human activities including smelting, mining, waste incineration, battery recycling, and lead product manufacturing also contribute significantly to airborne lead. When people breathe in or ingest lead-laden dust or fumes, it accumulates in the body and can cause symptoms like joint pain, muscle aches, anaemia, gastrointestinal distress, sleep disruption, concentration issues, headaches, and hypertension. Children exposed to lead can experience developmental challenges with motor skills, memory, and attention, as well as colic and stomach discomfort. Children, especially young ones, are particularly susceptible due to their developing bodies. For pregnant women, any lead exposure is concerning, as it can impact the foetus, leading to outcomes like premature birth, low birth weight, miscarriage, or even stillbirth. This underscores the importance of careful regulation of lead levels in air quality.

In this study, lead concentrations in the air (measured in µg/m³) were sampled by the National Institute of Occupational Safety and Health (NIOSH) across 15 different areas of a facility, as part of a health hazard assessment (Krishnamoorthy & Mathew (2009)). Concentration levels recorded included values of 200, 120, 15, 7, 8, 6, 48, 61, 380, 80, 29, 1000, 350, 1400, and 110. Using the Kolmogorov–Smirnov test, the data closely aligned with a Weibull distribution (P-value of 0.95), indicating its suitability as a model. MLEs for Weibull parameters were $\hat{\alpha} = 176.6$ and $\hat{\beta} = 0.63$. The Occupational Safety and Health Administration (OSHA) designates 50 µg/m³ as the occupational exposure limit (OEL) for lead, which corresponds to an upper limit at a threshold level 0.8075 in this analysis. Findings indicate that around 81% of these sampled concentrations may surpass the threshold level with 95% confidence, highlighting the need for ongoing air quality monitoring in workplace environments.

6 Overall conclusion

This article introduces a method for constructing confidence intervals (CIs) for the reliability function of a two-parameter Weibull distribution using generalized variable approach. The method is applicable to both complete and Type-II censored data and is straightforward to implement. Simulation results demonstrate that the proposed CIs achieve coverage probabilities close to nominal values, even for small sample sizes down to 5 observations in uncensored cases and for Type-II right-censored samples with censoring levels up to 70%. Real-world datasets are analysed to illustrate the method's practical use in assessing health risks associated with environmental exposure to chemicals and microbes.



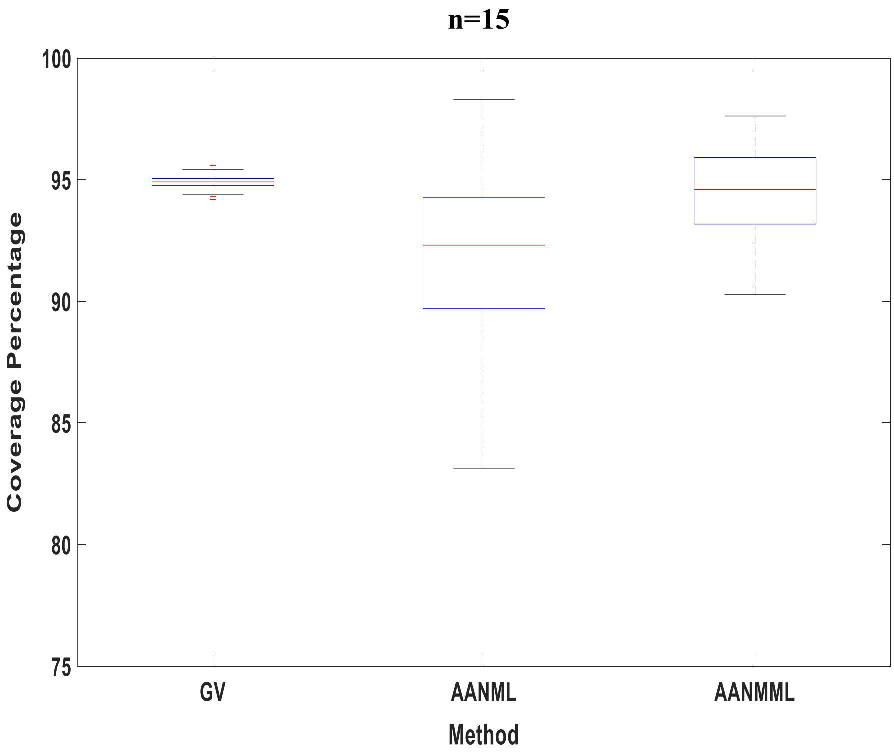
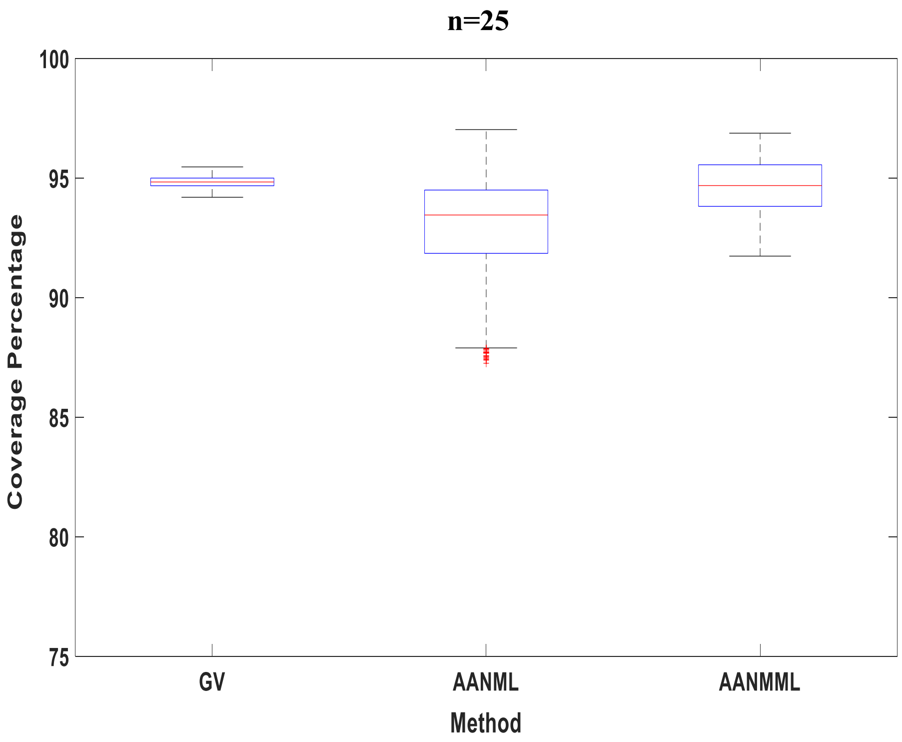


Fig.1 Box plots of simulated expected coverage probabilities (in percentage) for 95% CIs based upon GV, AANML and AANMML methods for sample sizes n=5,15 over the range of $\alpha = 0.5,1,2,\ldots,6, \beta = 0.3,0.5,1,5,7,9,10$ and R(t) = 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99.



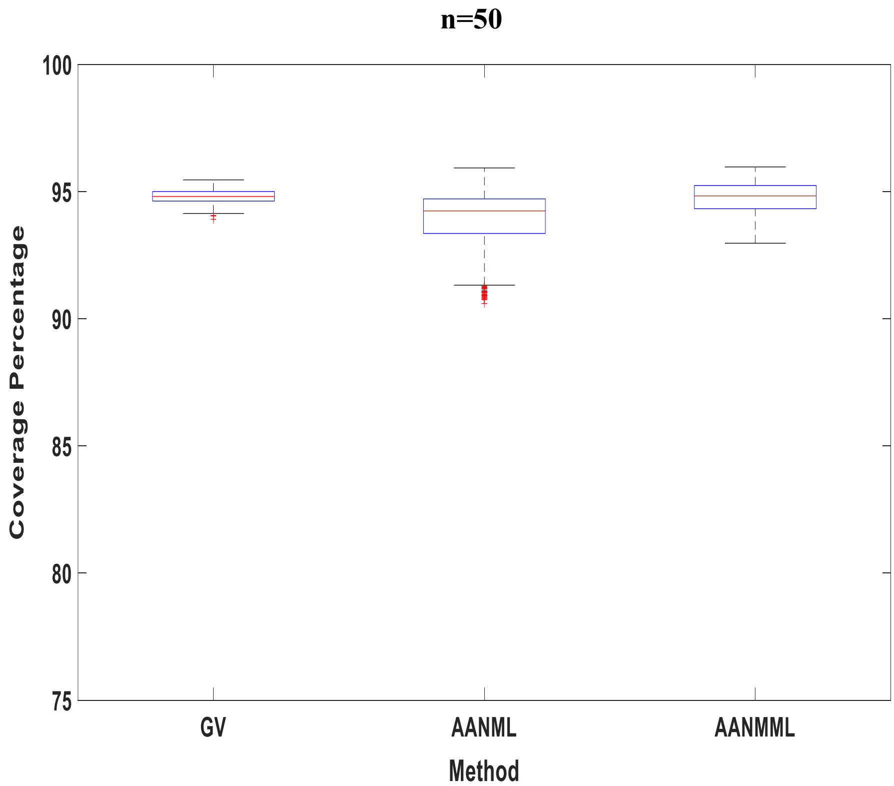
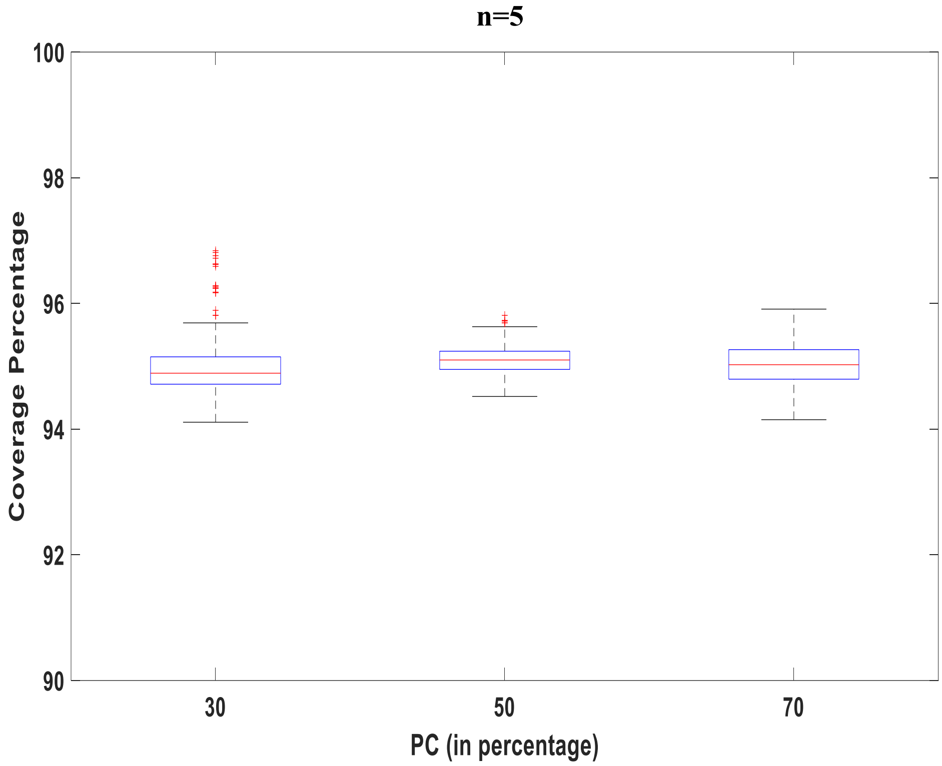


Fig.2 Box plots of simulated expected coverage probabilities (in percentage) for 95% CIs based upon GV, AANML and AANMML methods for sample sizes n=25,50 over the range of $\alpha = 0.5, 1, 2, \ldots, 6$, $\beta = 0.3, 0.5, 1, 5, 7, 9, 10$ and R(t) = 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99.



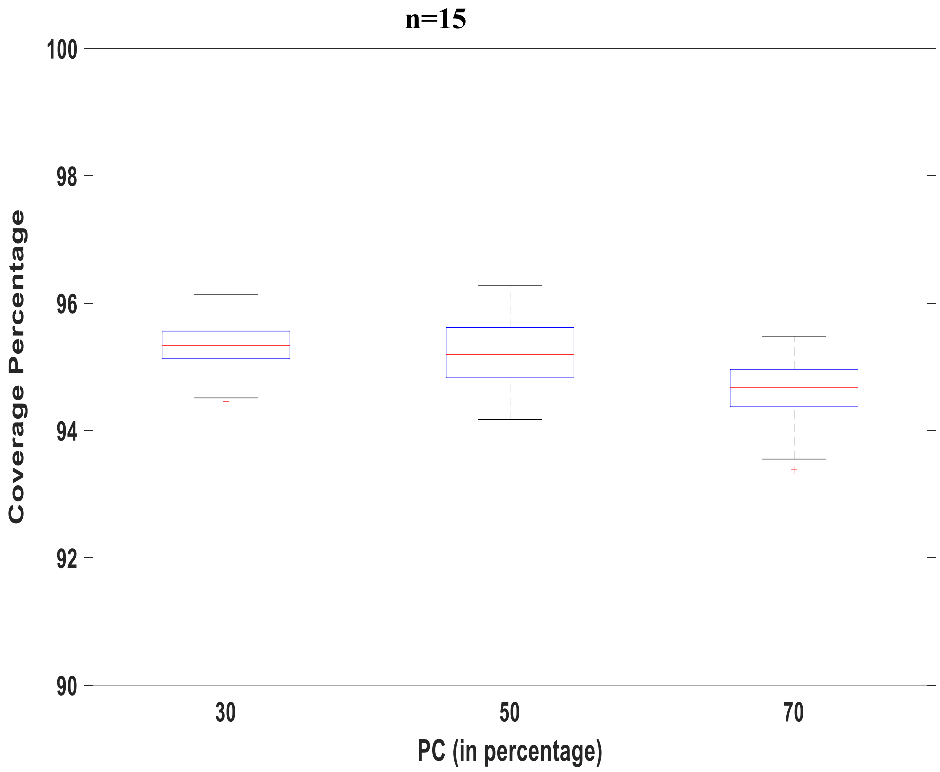
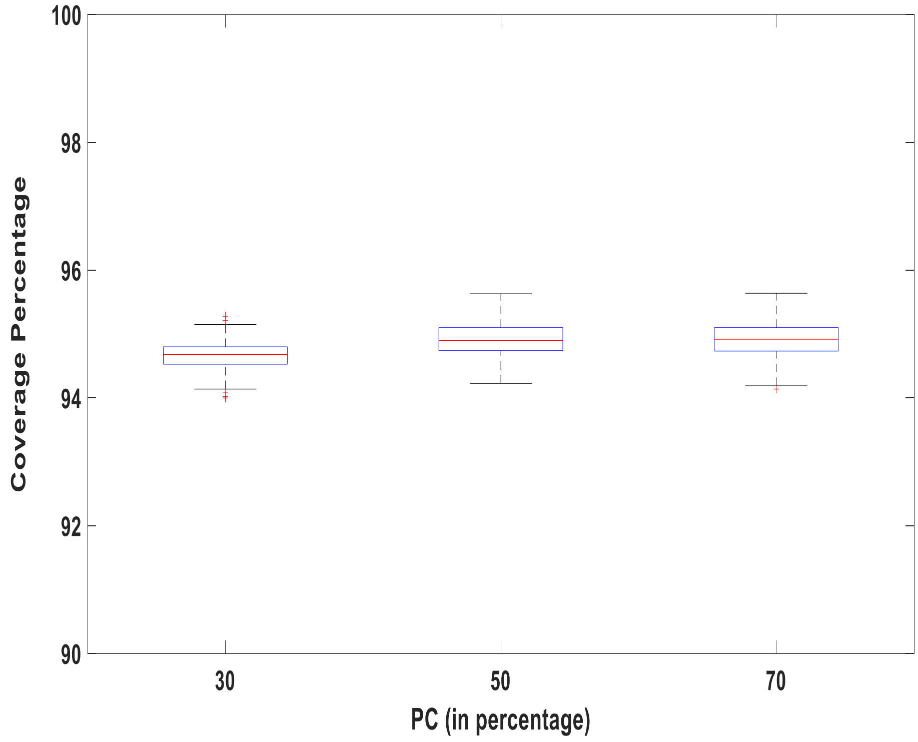


Fig.3 Box plots of simulated expected coverage probabilities (in percentage) for 95% GCI based upon GV method for various sample sizes n=5, 15 over the range of $\alpha = 0.5,1,2,\ldots,6,$ $\beta = 0.3,0.5,1,5,7,9,10$ and R(t) = 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99 for Type-II censored samples with proportion of censoring (in percentage) PC= 30%, 50%, 70%.

n=25



n=50

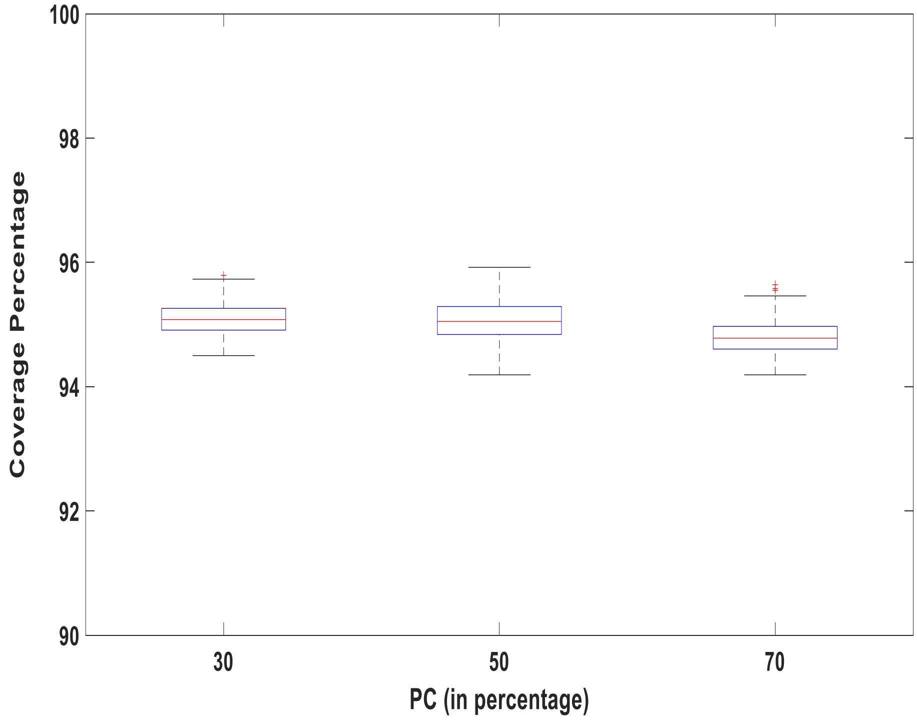


Fig.4 Box plots of simulated expected coverage probabilities (in percentage) for 95% GCI based upon GV method for various sample sizes n=25, 50 over the range of $\alpha = 0.5,1,2,\ldots,6$, $\beta = 0.3,0.5,1,5,7,9,10$ and R(t) = 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99 for Type-II censored samples with proportion of censoring (in percentage) PC= 30%, 50%, 70%.

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