CONNECTION BETWEEN SPECIAL PYTHAGOREAN TRIANGLES AND DISARIUM NUMBER

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Abstract - We present Special Pythagorean Triangles in connection with the Disarium numbers. Also we present the number of primitive and non-primitive Pythagorean triangles and some special cases are also discussed. A few interesting results are obtained.

***Key Words*:** Pythagorean Triangles, Disarium Number.

1.INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number theory is one of the largest and oldest branches of Mathematics. The main goal of Number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In Number theory, Pythagorean triangles have been a matter of interest to various mathematicians. For an extensive variety of fascinating problems one may refer [1-5]. Apart from the polygonal numbers we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [6-9]. In [10-11], special Pythagorean triangles connected with Polygonal numbers and Nasty numbers are obtained. Recently in [12-13], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 and Jarasandha Numbers are obtained. In this communication, we present Special Pythagorean Triangles in connection with the Disarium numbers. Also we present the number of primitive and non-primitive Pythagorean triangles and some special cases are also discussed. A few interesting relations between the numbers and some special number patterns are presented.

2. Basic Definitions:

Definition 1

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as **Pythagorean** equation where x, y and z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$ Also, in Pythagorean triangle $T(x, y, z): x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2

Most cited solution of the Pythagorean equation is $x = m^2 - n^2,\ \ y = 2mn,\ \ z = m^2 + n^2$ where $m > n > 0$. This solution is called **primitive**, if m, n are of opposite parity and gcd(m, n) = 1.

Definition 3

A number will be called **"DISARIUM"** if sum of its digits powered with their respective positions equal to the original number.

3. METHOD OF ANALYSIS:

Special pythagorean triangles

Case 1: When m,n are of Disarium number,we get 28 pythagorean triangles.

Table 1: Pythagorean Triangles with m and n of Disarium Number

m	n	x^2	y^2	$z^2 = x^2 + y^2$
135	89	106172416	577440900	683613316
175	89	515471616	970322500	1485794116
518	89	67809722409	8501577616	76311300025
598	89	122278200489	11330325136	133608525625
1306	89	2882236221225	54041371024	2936277592249
1676	89	7845909113025	88999595584	7934908708609
2427	89	34602723878464	186629184036	34789353062500
175	135	153760000	2134440000	2102222500
518	135	62549509801	19560819600	71998305625
598	135	115178105641	26069331600	141247437241
1306	135	28473558	12434086	2971696747321

		82921	4400	
1676	135	7788291144001	204774350400	7993065494401
2427	135	34481605386816	429404984100	34911010370916
518	175	56500814601	32869690000	89370504601
598	175	106915266441	43806490000	150721756441
1306	175	2809150306704	208940410000	3014602260121
1676	175	7719234279201	344099560000	8063333839201
2427	175	34336130967616	721565302500	35057696270116
598	518	7970918400	383814942784	387318277801
1306	518	2065865785344	1830652296256	3896518081600
1676	518	6454912585104	3014862704896	9408040966009
2427	518	31606940220025	6322066554384	37929006774409
1306	598	1817190273024	2439769024576	4256959297600
1676	598	6009224682384	4018004214016	10027228896400
2427	598	30611045925625	8425620846864	39036666772489
1676	1306	1217359155600	19164362354944	20381721510544
2427	1306	17511655504249	40187028776976	57698684281225
2427	1676	9494736310609	66183171172416	75677907483025

Thus it is seen that there are 28 pythagorean triangles. Of these 28 pythagorean triangles,18 is a primitive triangle and other 10 is non-primitive triangle.

Case 2: When $x = 89$ (2 digit Disarium number)

Table 2: Pythagorean Triangles with x = 89 (2-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
45	44	7921	15681600	15689521

Thus it is seen that one pythagorean triangle is primitive.

ii) When $x = 135$ (3 digit Disarium number)

Table 3: Pythagorean Triangles with x = 135 (3-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
68	67	18225	83028544	83046769
24	21	18225	1016064	1034289
12	3	18225	5184	23409
16	11	18225	123904	142129

Thus it is seen that there are 4 pythagorean triangles. Of these 4 pythagorean triangles, 2 is a primitive triangle and other 2 is non-primitive triangle.

iii) When $x = 175$ (3 digit Disarium number)

Table 4: Pythagorean Triangles with x = 175 (3-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
88	87	30625	234457344	234487969
20	15	30625	360000	390625
16	9	30625	82944	113569

Thus it is seen that there are 3 pythagorean triangles. Of these 3 triangles,2 is a primitive and other 1 is non-primitive triangle.

iv) When $x = 518, 598$ and 1306, which is impossible as x is even.

v) when $x = m^2 - n^2 = 2427$ (4 digit Disarium number)

Table 5: Pythagorean Triangles with x = 2427 (4-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
1214	1213	5890329	8673990986896	8673996877225
406	403	5890329	107083399696	107089290025

Thus it is seen that there are 2 Pythagorean triangles. Both the Pythagorean triangles are primitive.

Case 3 : When y = Disarium number, then y = 2mn .

i) Since we had taken only the Disarium numbers 89,135,175 & 2427. All these numbers are odd, so for y = Disarium number we get no Pythagorean triangles for these numbers.

ii) when $y = 598$ (3 digit Disarium number)

$$y = 2mn = 598$$

Table 6: Pythagorean Triangles with x = 135 (3-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
299	1	7992360000	357604	7992717604
23	13	129600	357604	487204

Thus it is seen that there are 2 Pythagorean triangles. Both the Pythagorean triangles are rimitive.

iii) when $y = 518$

Table 7: Pythagorean Triangles with x = 518 (3-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
259	1	4499726400	268324	4499994724
37	7	1742400	268324	2010724

Thus it is seen that there are 2 Pythagorean triangles. Both the Pythagorean triangles are primitive.

iv) when y=1306

Table 8: Pythagorean Triangles with x = 1306 (4-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
653	1	181823782464	1705636	181825488100

Thus it is seen that above Pythagorean triangle is primitive.

v) when y=1676

Table 9: Pythagorean Triangles with x = 1676 (4-digit Disarium Number)

m	n	x^2	y^2	$z^2 = x^2 + y^2$
838	1	493145231049	2808976	493148040025
419	2	30821313600	2808976	30822015844

Thus it is seen that there are 2 Pythagorean triangles. Both the Pythagorean triangle is primitive.

Case 4: When $z = m^2 + n^2$ Disarium number, then we get pythagorean triangles only for theDisarium number 89 and 1306

a) $z = m^2 + n^2 = 89$ we get one pythagorean triangle

b) $z = m^2 + n^2 = 1306$ we get one pythagorean tiangle.

Table 10: Pythagorean Triangles with z = 89 & 1306 Disarium Numbers

m	n	x^2	y^2	$z^2 = x^2 + y^2$
8	5	1521	6400	7921
35	9	1308736	396900	1705636

Thus it is seen that there are 2 Pythagorean triangles. Both the Pythagorean triangles are primitive.

Case 5.

Hypotenuse and one leg are consecutive and the other leg equals Disarium number .when hypotenuse and one leg are consecutive ,then either $z = x + 1$ or $z = y + 1$

i) If $z = y + 1$ we get

$$m^2 + n^2 = 2mn + 1$$

$$m^2 + n^2 - 2mn = 1$$

$$(m - n)^2 = 1$$

$$m = n + 1$$

$$\therefore \quad x = 2n + 1, \; y = 2n^2 + 2n, \; z = 2n^2 + 2n + 1$$

If $z = x + 1$ then we get $2n^2 = 1$

Which gives n as irrational number, which is not possible.

Taking x=Disarium number and y,z are consecutive we have the following table.

Table 11: Pythagorean Triangles with x = Disarium number, and z = y +1

m	n	x^2	y^2	$z^2 = x^2 + y^2$
45	44	7921	15681600	15689521
68	67	18225	83028544	83046769
88	87	30625	234457344	234487969
1214	1213	5890329	8673990986896	8673996877225

Thus it is seen that there are 4 Pythagorean triangles. These 4 Pythagorean triangles are non-primitive.

4. OBSERVATIONS

(i) $y + z \,;\, z - y \;\&\; z^2 - x^2$ are perfect square

(ii) $12(x + z)$ is nasty number.

5. CONCLUSION

To conclude, one may search for the connections between the Pythagorean triangles and other Disarium numbers of higher order and other number patterns.

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