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CONSTRUCTION OF ANY ANGLE WITHOUT PROTRACTOR

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Abstract

Geometry is one of the foremost significant chapters in mathematics. The construction of angles is one of the important topics of geometry. In geometry, the term construction refers to the precise sketching of shapes, lines, or angles using mathematical instruments. To construct angles, all you'll need is a compass, a ruler, and a pencil, as well as a protractor to measure the angles you've constructed. As there are no numbers involved, this is the simplest type of geometric building. We can only draw some special angles without using a protractor, such as 30, 60, 90, 120, 180, and non-special angles with 1 degree deviation from the needed angles, such as 83, 72, 68, and so on. In our lower secondary school, the method of constructing special angles is commonly taught. We can, however, draw any angle between 0 and 360 degrees using the approach we devised, with no variation from the needed angle. Construction of angle is essential in math because it may be applied to the construction of other geometric shapes, particularly triangles.

Keywords:
Angle; Intersection Points; Reference Arc; Primary arc

Introduction:

Geometry is one of the most important modules in mathematics. The prefix geo- stands for earth, and the suffix -metron stands for measurement. It is related to the distance, shape, size, and relative position of figures. It deals with dimensions 1D, 2D, and 3D. In this paper, we'll illustrate how to construct angles in two dimensions. Geometry is concerned with planes, angles, curves, and surfaces, among other things. Two rays start from a common point to form an angle. The vertex is the place where the two rays that make up the angle meet, and the arms or sides are the two rays that make up the angle. There are three different sorts of angles. They are the acute angle (less than 90 degrees), the obtuse angle (more than 90 degrees), and the right angle (exactly 90 deg.).

One of the most important concepts in geometry is the construction of angles. Now we'll look at how to construct angles without using a protractor. All we need is a ruler and a compass. As previously mentioned, we already have a method for drawing angles such as 30, 60, 90, and so on as stated in chapter Practical Geometry [2]. Let's consider these angles to be special. Other angles, such as 5, 37, 83, and so on, cannot be sketched using this method. We could draw angles like 60,120,180, 240, 300, and 360 using the already existing method described in [2, 3]. Because these are the bisectors of the previously mentioned angles, we can create angles like 30, 90, 150, 210, and so on. This bisection method of construction has been stated in [2, 3].

We can draw non-special angles without using the bisection approach, according to the available reference [1]. The final result will deviate by 1 degree from the required angle if the procedure described in [1] is used. Consider this deviation to be an error. The deviation, however, is negligible when using our method. Only a few tools are necessary, such as a compass and ruler for constructing the appropriate angle and a protractor for verifying the constructed angle. Construction of angles can also be applied to the creation of other geometric figures, primarily triangles [4].

In section 2, we describe our methodology. In section 3 we describe the two versions of our method. One is the general (complicated) version and the other is a simplified version. In section 4 we present our conclusion.

Methodology:

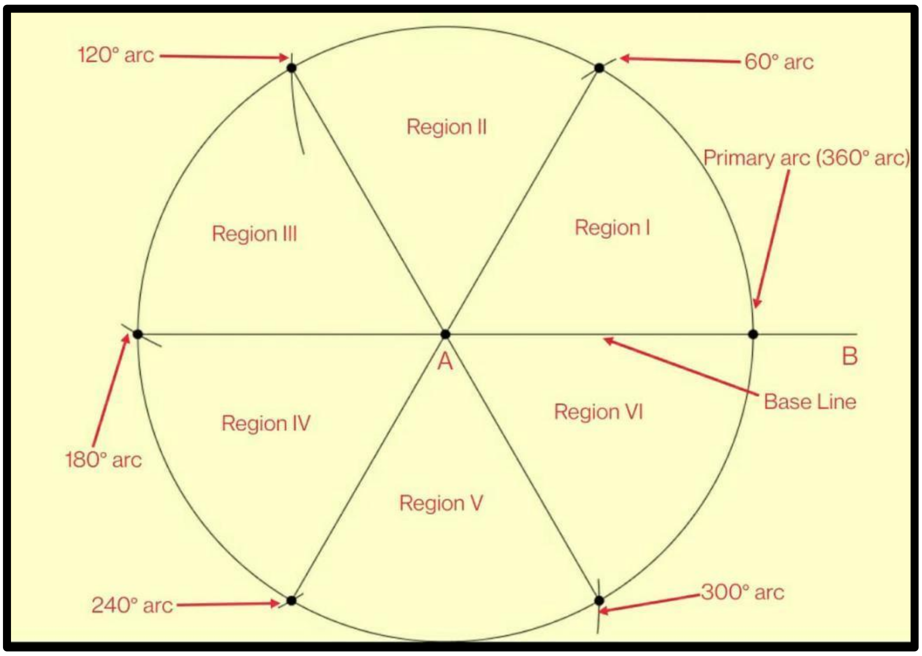

Fig. 2.1 Regions

To get farther in our process, we must learn how to construct angles such as 60, 120, 180, and so on. Draw 60, 120, 180, 240, 300, 360-degree angles using this procedure, which will serve as our reference angles.

1. Assume that the desired angle is *n* degrees.
2. Draw a line AB with a length of x cm. The vertex is Point A.
3. With the compass pointer at point A, cut an arc with a radius more than half the length of AB which cuts an arm AB at O. Let this arc be the primary arc and consider the radius as *y* cm.
4. Cut an arc on the primary arc using O as the centre and y as the radius. Let this arc be 60 degrees arc. Draw another arc in the same direction, with the intersection point of the 60 degrees arc and the primary arc as the centre and y cm as the radius, results in 120 degrees arc. Draw a new arc with the intersection point of the 120 degrees arc and the primary arc as the centre and y cm as the radius using the intersection point of the 120 degrees arc and the primary arc as the centre and y cm as the radius, resulting in 180 degrees arc. To make 240,300, and 360 degrees arcs, repeat the process. These arcs are the reference arcs. Draw the reference arcs according to the requirements. When all of the reference arcs are taken into account, we get six areas, as illustrated in *figure 2.1*. Currently, there are six regions in this sketch:
   - Region I (0-60 deg.) – 0,60 deg. are extreme angles of this region
   - Region II (60-120 deg.) – 60,120 deg. are extreme angles of this region

- Region III (120-180 deg.) – 120,180 deg. are extreme angles of this region
- Region IV (180-240 deg.) – 180,240 deg. are extreme angles of this region
- Region V (240-300 deg.) – 240, 300 deg. are extreme angles of this region
- Region VI (300-360 deg.) – 300, 360 deg. are extreme angles of this region

5. Assume that the needed angle is n degrees. Now locate the area in which the required angle exists. For example, angle n be 83 degrees which lies in region II 60<83<120 degrees, draw the reference arcs as needed and the mark intersection points of reference arcs and primary arcs as E1, E2, E3, and so on.
6. We must find the differences between the required angle (n) and extreme angles. In this instance, the differences are 83-60=23 and 120-83=37. All of the reference arcs drawn so far have a radius of y cm, resulting in regions with arc lengths of 60 degrees.

$$60^\circ \rightarrow y\ cm\ (\ convert\ it\ into\ mm)$$

$$1^\circ \rightarrow ?\ mm$$

$$\Rightarrow \frac{?}{y}\ mm = \frac{1}{60}^\circ$$

$$\Rightarrow ?\ mm = \frac{y}{60}$$

$$Let\ us\ denote\ the\ ?\ with\ d$$

$$\therefore d = \frac{y}{60}$$

7. Next, calculate the products of differences and d, which are the needed lengths for drawing the arcs on the primary arc with extreme angle arcs as centres (E1, E2 here). For 60 degrees arc and 120 degrees arc, the product is d*23 mm and d*37 mm, respectively.
8. Let p1 and p2 be the intersection points. Draw arcs above or below the primary arc with p1 and p2 of as centres, respectively, with any length an as radius. We'll now get a p3 intersection point. Connect the vertex A with the point p3 now. As a result, we'll get angle OAp3, which is the desired angle.

Verification and Results:

General version:

1. Let us consider the required angle *n* = 83 deg.

2. Draw a line AB of length x = 10cm.
3. Taking A as center and y as radius draw an arc where y must be greater than half the length of x (y = 8cm here). The arc drawn is the primary arc.
4. Let O be the intersection point of the primary arc and line AB. Now with the same radius y and O as the center, cut a *60deg. arc* on the primary arc.
5. Since n = 83 deg., it lies in region II whose extreme angles are 60 and 120 deg. As we have drawn *60 deg. arc* already, we have to draw *120 deg. arc* with y as radius and the intersection point of *60 deg. arc* and primary arc as the center. Let the intersections be E1 and E2.
6. The differences of extreme angles and required angles are (83-60 = 23) and (120-83 = 37). Calculate d as below:

$$\Rightarrow d = \frac{y}{60}$$

$$\Rightarrow d = \frac{80}{60} = 1.33mm \left[\because y = 8cm = 80mm\right]$$

7. Now calculate the products of differences and d value. In this case, the products are going to be 1.33*23 = 30.59 mm and 1.33*37= 49.21mm respectively. So now taking these values as radius draw arcs on the primary arc within region II with E1, E2 as centres respectively.
8. Now the arcs intersect the primary arc at p1, p2. Now take some random length as radius and centres as p1, p2 draw arcs which will intersect at a point p3. Now join the p3 and A. Our required angle is OAp3 = 83 deg.

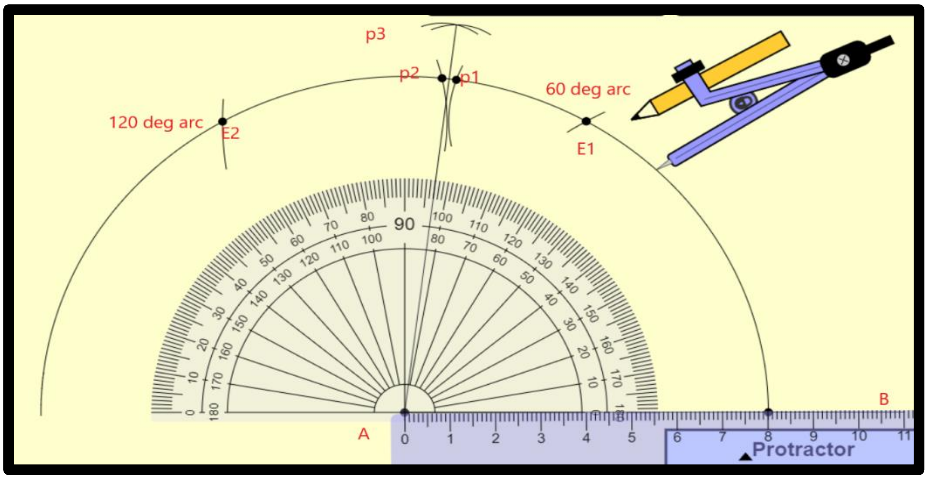

Fig. 3.1 Final Result of General Version

Simplified version:

In the earlier version *d* value was 1.33 since *y*=80mm which made the calculation bit complicated as the values were decimal. To make this easier, consider *y* as 60mm, then the *d* value becomes 1 mm which makes the further calculation and sketching easier as the values are integers.

1. Let *n*=83 deg. be our required angle.
2. Draw a line AB of length *x*=10cm. Length of the length *x* should be always more than 6 cm.
3. Take A as the centre and *y*=6cm draw a primary arc and the intersections point of AB and arc will be O.
4. Now with the same radius *y* cm and O as centre draw *60 deg. arc* on the primary arc.
5. Since the required angle is 83 deg. It lies in region II whose extreme angles are 60,120 deg. So now draw 120 deg. arc on primary arc with E1 as centre and *y* as the radius.
6. Now the differences of extreme angles are 83-60 = 23 and 120-83 = 37 deg.

   Calculate d as below:

   $$\Rightarrow d = \frac{y}{60}$$

   $$\Rightarrow d = \frac{60}{60} \left[\because y = 6cm = 60mm\right]$$

   $$\Rightarrow d = 1mm$$

7. Now calculate the products of differences and d value. In this case, the products are going to be 1*23 = 23 mm and 1*37= 37mm respectively. So now taking these values as radius draw arcs on primary arc within region II with E1, E2 as centres respectively.

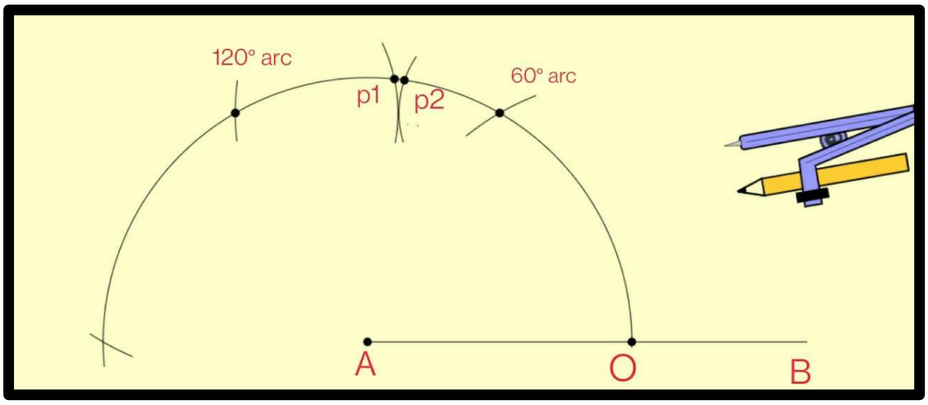

Fig. 3.2 Intermediate state of simplified version

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8. Now the arcs intersect the primary arc at p1, p2. Now take some random length as radius and centres as p1, p2 draw arcs which will intersect at a point p3. Now join the p3 and A. Our required angle is OAp3 = 83 deg.

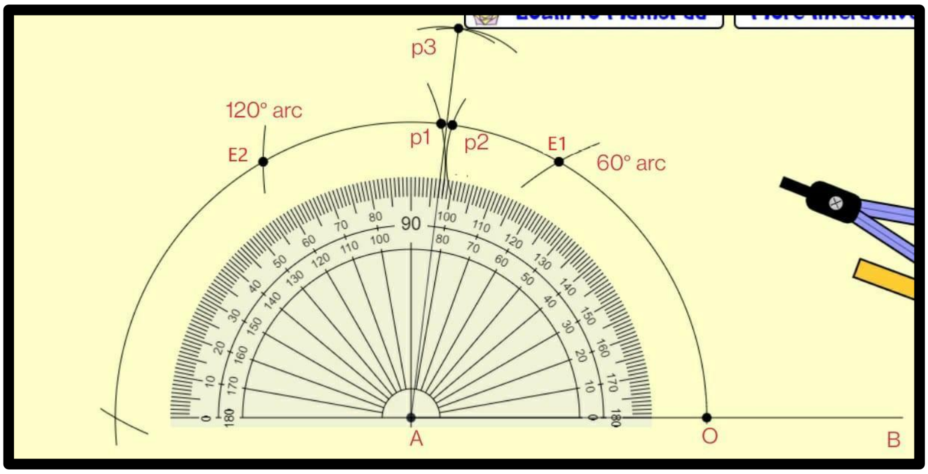

Fig. 3.3 Final Result of Simplified Version

Conclusion:

Construction of angles without the use of a protractor is a topic that we would have learned in our lower secondary school. With the existing approaches, we can only draw certain angles without deviation such as 30, 60, and 90, 120. However, using this method, we can easily draw any angle between 0 and 360 degrees without error. We were using the general version of this procedure at the beginning. Later, we decided to simplify it, which led to the discovery of the simplified version. This approach will be helpful to construct geometric figures including but not limited to triangles, quadrilaterals. Civil engineers can use this method to design and create prototypes of structures as well.

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