

## **Construction of Optimal Portfolio using Sharpe's Single Index Model**

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### ***Abstract***

*Everyone has the perception that their investments in the stock market should provide the highest possible return, but they neglect to consider the risk factor. Risk and reward are therefore go hand in hand. The goal of an investment is to maximize return while reducing risk, however if investors choose equities, it will have high risk and high return, whereas debt has low risk and low return. In order to achieve the investment goal, a concept known as a portfolio is necessary. Portfolio will achieve the investment's goal of maximizing return while minimizing risk. Both individual investors and institutional investors find it challenging to build portfolios. So, this study was conducted to construct the optimal portfolio using Sharpe's single index model. Fifty NSE-listed firms have been chosen for this study. And out of 50 stocks listed in NSE, only three stocks were in the bucket of investment portfolio.*

**Key words:** - Optimal Portfolio, Investment, Nifty, Sharpe's single index model, Risk and Return, Stock market, Securities, Beta.

### **Introduction**

Investments refer to the use of cash on assets with the goal of generating a profit and capital growth. Every investment involves risk and potential reward. For making wise investing selections, an investor has to have a solid understanding of portfolio theory and security analysis. The investor must choose which assets to hold and how much money to put into each one. Basically, the two key aspects of a portfolio are risk and return. Taking into account the risk and return characteristics of all potential portfolios, the investor seeks to select the best portfolio. A rational investor aims to reduce risk while maximizing reward. A number of models contend that risk may be significantly decreased by include assets from several industries in a portfolio. When thinking about investing in securities, a potential investor must decide which securities to buy from a broad selection and how to distribute his money among them. Investors cannot make informed judgements about their investments if they lack enough understanding of investing and portfolio management.

All the steps taken in building and maintaining an investment portfolio are included in portfolio management. Security analysis, portfolio analysis, portfolio selection, portfolio modification, and portfolio evaluation are the main topics covered. The application of conceptual frameworks for the logical allocation of money and analytical procedures for analysis are used in portfolio management. The goal of portfolio management is to reduce risk and increase return for investing activity.

***Sharpe's Single Index Model (SIM)***

The Sharpe's Single Index Model's most significant characteristic is its simplicity. William Sharpe created the Single Index Model. According to Sharpe's single index model, a security's return depends on the market index's performance.

***Steps to calculate Sharpe's single index model***

1. The first step for making effective portfolio is to calculate Excess Return to Beta

Excess return to beta ratio is used to rank securities, with the highest score representing the securities suitability for include in a portfolio and the lowest being the least desirable. The significance of this ratio is that the excess return earned for every unit risk taken in the market.

$$\text{Excess return to } \beta = \frac{R_i - R_f}{\beta} \quad \text{Where, } R_i = \text{Return of Individual stocks}$$
$$R_f = \text{Risk free return}$$
$$\beta = \text{Beta}$$

Here  $R_f$  that is returns are computed using the three-year average return on closing price from 2020 to 2022. And the  $R_f$  is computed by taking 365 Treasury bill rate for 3 years.

And Beta is calculated by: -  $\beta = \frac{\sigma_x}{\sigma_m} \times r_{xm}$

$\sigma_x$  = Standard deviation of the individual stocks.

$\sigma_m$  = Standard deviation of the Market

And Standard deviation is computed by: -

$$\sigma = \sqrt{\sum \frac{(r - \bar{r})^2}{n-1}}$$

$r_{xm}$  = Correlation Between Market return and the Return of Individual stocks.

2. The stock must be arranged in order of highest to lowest rank after identifying the excess return to beta. After arranging rank wise we need to calculate cut off point.

Cut off points is calculated by:

$$Cut\ off\ point = \frac{\sigma_{m^2} \times \left[ \frac{\sum (R_i - R_f) \beta_i}{\sigma_{i^2}} \right]}{1 + \left[ \sigma_{m^2} \sum \left( \frac{\beta_i^2}{\sigma_{i^2}} \right) \right]}$$

In constructing the effective portfolio we consider the Total Risk, that is systematic risk and Unsystematic risk.

$$Total\ risk = Systematic\ Risk + Unsystematic\ Risk$$

$$Systematic\ Risk = \sigma_{m^2} \times \beta_{p^2}$$

$$Unsystematic\ Risk = \sigma_{i^2}$$

3. Having said that all stocks will not be considered for constructing an effective portfolio. The stocks which are above the C point are considered to be in portfolio. The C point is constructed by taking risk element of both systematic and unsystematic risk. The stocks which are above the C point will be selected for constructing a portfolio.

4. Construct the weight of investment of each stock.

$$Weight = (Excess\ return\ to\ beta - Cut\ off) \times \frac{\beta_i}{\sigma_{i^2}}$$

5. Now we have to give the proportion of each stock in the portfolio. The percentage to invest in each chosen security

$$Proportion = \frac{Weight\ of\ Individual\ Stock}{Total} \times 100$$

## Objectives

1. To determine the returns and standard deviation of all individual stocks
2. To determine the cut-off rate that acts as a benchmark for choosing stocks to be included in a portfolio and to empirically build an optimum portfolio using Sharpe's Single Index Model.
3. To know the proportion of each security in the portfolio.
4. To construct the portfolio using the Nifty 50 stocks.

## Methodology

Secondary resources were utilized for the research for this study. For the purpose of building the ideal portfolio, 50 firms from the Nifty 50 of the year 2020 were taken into account for the three-year period from 2020 to 2022. The tools used for the study were: -

1. Average returns
2. Standard Deviation
3. Excess return to beta
4. Cut-off points

The mean return of individual stock was found by averaging the daily returns.

## Analysis

Table 1 shows the list of Nifty 50 companies that we have consider with the average return calculated and the Beta value

*Table1: –*

S No.	Companies	Average Return	Beta
1	Adani ports	0.15061	1.21621
2	Airtel	0.09038	0.61443
3	Asian paints	0.06681	1.32058
4	Axis Bank	0.04519	0.65119
5	Bajaj Auto	0.09393	1.39898
6	Bajaj Finance	0.10611	1.29733
7	Bajaj FinServ	-0.01871	0.82743
8	Bharat petroleum	0.09731	0.78077
9	Britannia	0.06314	0.39258
10	Cipla	0.13370	0.35457
11	Coal India	0.04234	0.86566
12	Dr. Reddy lab	0.06823	0.42645
13	Eicher Motors	0.07810	0.97433
14	GAIL	0.06080	0.85632
15	Grasim	0.13950	1.00366
16	HCL technology	0.09759	0.82621
17	HDFC	0.05381	1.14287
18	Hero MotoCorp	0.04330	0.81914
19	Hindalco	0.16208	1.59540

20	Housing Development Financial Co.	0.03140	1.27336
21	HUL	0.04799	0.46177
22	ICICI Bank	0.09486	1.34046
23	Indian oil Corporation	0.01373	0.83932
24	Indus Tower	0.02185	0.79476
25	IndusInd Bank	0.04017	1.42276
26	Infosys	0.10592	0.93036
27	ITC	0.06443	0.67723
28	JSW steel	0.18359	1.30451
29	Kotak	0.03590	1.05792
30	L&T	0.08654	1.05423
31	Mahindra and Mahindra	0.14950	1.04673
32	Maruti	0.04867	0.83804
33	Nestle	0.05304	0.38708
34	NTPC	0.07135	0.74073
35	ONGC	0.06516	0.87998
36	Power Grid	0.07657	0.60151
37	Reliance	0.11207	1.02626
38	SBI	0.10700	1.33702
39	Shree cement	0.04469	0.91005
40	Sun Pharma	0.13169	0.67594
41	Tata Consultancy Services	0.15999	1.55841
42	TATA Motors	0.16474	1.38841
43	TATA steel	0.07060	0.73042
44	Tech Mahindra	0.05319	0.98878
45	Titan	0.12582	0.91554
46	Ultratech cement	0.09433	0.89513
47	UPL	0.07091	0.96585
48	Vedanta	0.16235	1.30893
49	Wipro	0.08042	0.89392
50	Zee	0.05521	1.15370

Table 2 shows the Excess return to beta with the ranking of the companies.

Table 2: –

S No.	Companies	Excess Return to Beta	Ranking
1	Adani ports	0.1626	7
2	Airtel	-2.6296	48
3	Asian paints	-0.0485	25
4	Axis Bank	0.4569	4
5	Bajaj Auto	-0.0739	28
6	Bajaj Finance	-0.0482	24
7	Bajaj FinServ	-0.1471	37
8	Bharat petroleum	0.0239	16
9	Britannia	-0.0026	20
10	Cipla	-0.3175	45
11	Coal India	0.0595	12
12	Dr. Reddy lab	-0.1894	41
13	Eicher Motors	-0.1388	34
14	GAIL	2.2790	3
15	Grasim	0.0674	11
16	HCL technology	-0.0697	27
17	HDFC	-0.1033	30
18	Hero MotoCorp	-0.1622	38
19	Hindalco	0.0282	15
20	Housing Development Financial Co.	-0.0633	26
21	HUL	-0.6459	47
22	ICICI Bank	-0.0442	23
23	Indian oil Corporation	-0.1398	35
24	Indus Tower	2.8752	1
25	IndusInd Bank	-0.1859	40
26	Infosys	-0.0872	29
27	ITC	-0.1733	39
28	JSW steel	0.1652	6
29	Kotak	-0.1381	33
30	L&T	0.0295	14
31	Mahindra and Mahindra	0.0182	17
32	Maruti	-0.2608	44
33	Nestle	-2.8536	49

34	NTPC	-0.4624	46
35	ONGC	-0.1453	36
36	Power Grid	2.6349	2
37	Reliance	0.0110	19
38	SBI	-0.0250	22
39	Shree cement	-0.1283	32
40	Sun Pharma	0.1546	8
41	Tata Consultancy Services	-0.1935	42
42	TATA Motors	0.1146	9
43	TATA steel	-0.1283	31
44	Tech Mahindra	0.0704	10
45	Titan	-0.0178	21
46	Ultratech cement	0.0327	13
47	UPL	-3.3218	50
48	Vedanta	0.0180	18
49	Wipro	0.2156	5
50	Zee	-0.1966	43

Table 3

S no.	Companies	B <sup>2</sup>	SD <sup>2</sup> unsystematic risk	(R <sub>i</sub> -R <sub>f</sub> )B/Sd		Bi <sup>2</sup> /Sdi <sup>2</sup>		SDm <sup>2</sup>	SD <sup>2</sup> *R <sub>i</sub> -R <sub>f</sub>	SDm <sup>2</sup> *Bi/Sdi	Cut off
				Base	Cumulative	Base	Cumulative				
1	Indus tower	0.632	0.194	9.358	9.358	3.255	3.255	0.040	0.374	1.130	0.331
2	Power grid	0.362	0.097	9.802	19.160	3.720	6.975	0.040	0.766	1.279	0.599
3	GAIL	0.733	0.147	11.387	30.547	4.996	11.971	0.040	1.221	1.479	<b>0.826</b>
4	Bajaj auto	0.424	0.076	2.546	33.093	5.573	17.544	0.040	1.323	1.701	0.778
5	Wipro	0.799	0.096	1.799	34.893	8.348	25.892	0.040	1.395	2.035	0.685
6	JSW steel	1.702	0.167	1.684	36.576	10.191	36.083	0.040	1.462	2.442	0.599
7	Adani ports	1.479	0.148	1.625	38.202	9.995	46.078	0.040	1.527	2.842	0.537
8	Sun Pharma	0.457	0.088	0.805	39.006	5.207	51.285	0.040	1.559	3.050	0.511
9	TATA steel	1.928	0.190	1.161	40.168	10.131	61.415	0.040	1.606	3.455	0.465
10	Tech Mahindra	0.978	0.123	0.559	40.727	7.942	69.357	0.040	1.628	3.773	0.432
11	Grasim	1.007	0.115	0.590	41.317	8.759	78.116	0.040	1.652	4.123	0.401
12	Coal India	0.749	0.134	0.331	41.649	5.574	83.691	0.040	1.665	4.346	0.383
13	UltraTech cement	0.801	0.092	0.285	41.934	8.722	92.413	0.040	1.676	4.694	0.357
14	L & T	1.111	0.096	0.340	42.274	11.530	103.943	0.040	1.690	5.155	0.328
15	Hindalco	2.545	0.238	0.302	42.575	10.708	114.651	0.040	1.702	5.583	0.305
16	Bharti Airtel	0.610	0.106	0.138	42.713	5.776	120.427	0.040	1.708	5.814	0.294
17	Mahindra and Mahindra	1.096	0.134	0.149	42.862	8.189	128.616	0.040	1.713	6.142	0.279
18	Vedanta	1.713	0.238	0.129	42.992	7.188	135.804	0.040	1.719	6.429	0.267
19	Reliance	1.053	0.103	0.111	43.103	10.176	145.980	0.040	1.723	6.836	0.252
20	Britannia	0.154	0.060	-0.007	43.097	2.556	148.536	0.040	1.723	6.938	0.248
21	Titan	0.838	0.106	-0.140	42.956	7.890	156.426	0.040	1.717	7.253	0.237
22	SBI bank	1.788	0.137	-0.328	42.629	13.088	169.514	0.040	1.704	7.777	0.219
23	ICICI bank	1.797	0.132	-0.602	42.027	13.627	183.141	0.040	1.680	8.321	0.202
24	Bajaj FinServ	1.683	0.157	-0.515	41.512	10.698	193.839	0.040	1.659	8.749	0.190



25	Axis bank	1.744	0.158	-0.535	40.977	11.035	204.874	0.040	1.638	9.190	0.178
26	Housing Development Financial Co.	1.621	0.123	-0.834	40.142	13.182	218.056	0.040	1.605	9.717	0.165
27	HCL technology	0.683	0.097	-0.490	39.653	7.025	225.080	0.040	1.585	9.998	0.159
28	Bajaj finance	1.957	0.170	-0.852	38.801	11.521	236.601	0.040	1.551	10.458	0.148
29	Infosys	0.866	0.091	-0.829	37.971	9.516	246.117	0.040	1.518	10.839	0.140
30	HDFC bank	1.306	0.093	-1.447	36.524	14.012	260.129	0.040	1.460	11.399	0.128
31	TCS	0.534	0.071	-0.971	35.553	7.566	267.695	0.040	1.421	11.701	0.121
32	Shree cement	0.828	0.113	-0.942	34.612	7.341	275.036	0.040	1.384	11.995	0.115
33	Kotak	1.119	0.113	-1.369	33.243	9.914	284.950	0.040	1.329	12.391	0.107
34	Eicher Motors	0.949	0.124	-1.062	32.180	7.652	292.602	0.040	1.286	12.697	0.101
35	Indian oil corp.	0.704	0.092	-1.065	31.115	7.617	300.219	0.040	1.244	13.002	0.096
36	ONGC	0.774	0.174	-0.645	30.470	4.441	304.660	0.040	1.218	13.179	0.092
37	Bharat petroleum	0.685	0.121	-0.832	29.638	5.655	310.315	0.040	1.185	13.405	0.088
38	Hero MotoCorp	0.671	0.106	-1.024	28.614	6.315	316.630	0.040	1.144	13.658	0.084
39	ITC	0.459	0.081	-0.983	27.630	5.674	322.304	0.040	1.105	13.884	0.080
40	IndusInd bank	2.024	0.243	-1.551	26.079	8.344	330.648	0.040	1.043	14.218	0.073
41	Dr. Reddy's Lab	0.182	0.076	-0.454	25.625	2.397	333.045	0.040	1.024	14.314	0.072
42	TATA motors	2.429	0.228	-2.064	23.561	10.668	343.712	0.040	0.942	14.740	0.064
43	Zee	1.331	0.287	-0.912	22.649	4.641	348.353	0.040	0.905	14.926	0.061
44	Maruti	0.702	0.106	-1.733	20.915	6.645	354.998	0.040	0.836	15.191	0.055
45	Cipla	0.126	0.087	-0.457	20.459	1.438	356.437	0.040	0.818	15.249	0.054
46	NTPC	0.549	0.104	-2.433	18.025	5.262	361.699	0.040	0.721	15.459	0.047
47	HUL	0.213	0.068	-2.023	16.003	3.132	364.831	0.040	0.640	15.585	0.041
48	Asian paints	0.378	0.086	-11.498	4.504	4.373	369.203	0.040	0.180	15.759	0.011
49	Nestle	0.150	0.060	-7.082	-2.578	2.482	371.685	0.040	-0.103	15.859	-0.006
50	UPL	0.933	0.152	-20.383	-22.962	6.136	377.821	0.040	-0.918	16.104	-0.057

Table 4 shows the cut off points of each company that will decide which company need to take into the portfolio

*Table4: –*

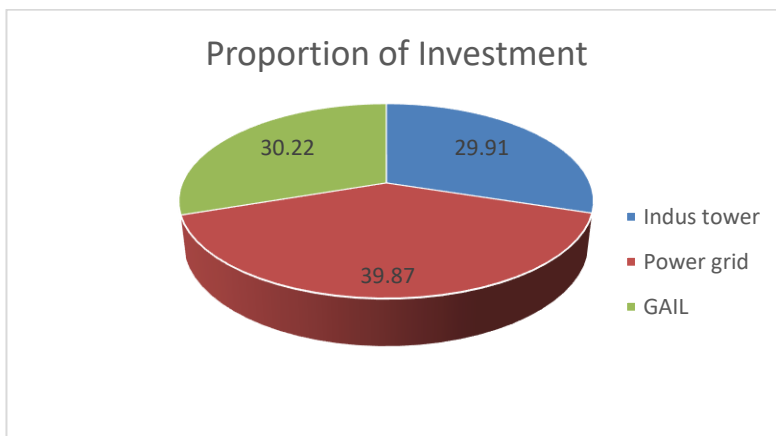
S no.	Companies	Cut off
1	Indus tower	0.3310
2	Power grid	0.5989
3	GAIL	0.8259
4	Bajaj auto	0.7776
5	Wipro	0.6854
6	JSW steel	0.5987
7	Adani ports	0.5373
8	Sun Pharma	0.5112
9	TATA steel	0.4647
10	Tech Mahindra	0.4316
11	Grasim	0.4006
12	Coal India	0.3831
13	UltraTech cement	0.3571
14	L & T	0.3278
15	Hindalco	0.3048
16	Bharti Airtel	0.2937
17	Mahindra and Mahindra	0.2790
18	Vedanta	0.2673
19	Reliance	0.2521
20	Britannia	0.2483
21	Titan	0.2368
22	SBI bank	0.2191
23	ICICI bank	0.2019
24	Bajaj FinServ	0.1897
25	Axis bank	0.1782
26	Housing Development Financial Co.	0.1651
27	HCL technology	0.1585
28	Bajaj finance	0.1483
29	Infosys	0.1400
30	HDFC bank	0.1281
31	TCS	0.1215
32	Shree cement	0.1154
33	Kotak	0.1072

34	Eicher Motors	0.1013
35	Indian oil corp.	0.0957
36	ONGC	0.0924
37	Bharat petroleum	0.0884
38	Hero MotoCorp	0.0838
39	ITC	0.0796
40	IndusInd bank	0.0733
41	Dr. Reddy	0.0716
42	TATA motors	0.0639
43	Zee	0.0607
44	Maruti	0.0550
45	Cipla	0.0536
46	NTPC	0.0466
47	HUL	0.0410
48	Asian paints	0.0114
49	Nestle	-0.0065
50	UPL	-0.0570

Table 5 indicates the weights on each security and the second shows the relative investment in each security.

*Table 5*

S no.	Companies	Weight	Proportion
1	Indus tower	8.392349887	29.9102343
2	Power grid	11.18758822	39.87243019
3	GAIL	8.478517747	30.21733551
		<b>28.05845586</b>	



## Findings

- Form the above study we can seen that from the Nifty 50 companies 11 companies are of the financial service provider, that gives the weightage of 38% in the nifty 50 index.
- From the top 50 nifty companies only top three securities based on ranking (excess return to beta ratio) namely
  - a) Indus Tower (1st Rank)
  - b) Power Grid (2nd Rank)
  - c) GAIL (3<sup>rd</sup> Rank) are becoming part of optimal portfolio.
- Weightage of each securities includes: -
  - a) Indus Tower – 29.91%
  - b) Power Grid – 39.87%
  - c) GAIL- 30.21%
- It has been found out that 20 companies out of 50 were having beta more than 1 which means these stocks are more volatile.

## Conclusion

Risk and reward are the two sides of each investment. Both are important when choosing an investment strategy. With the help of this study, we can draw the conclusion that having an ideal portfolio is essential for achieving the maximum return with the least amount of risk. Using the Sharpe's single index model will assist in identifying the ideal securities for our portfolio because it compares company returns to the market. This research intends to analyze the investment opportunities, both in terms of returns and risk, that are accessible to investors when investing in certain stocks of companies that are listed on the NSE Nifty 50. Only 3 equities from the 50 NSE stocks were included in the ideal portfolio.

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