

Contribution of Sanskrit in Mathematics and Astronomy

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यथा शिखा मयूराणां नागाणां मणयो यथा।

तथा वेदाङ्गशास्त्राणां ज्योतिषं मूर्धनिस्थितम् ॥

“Like the combs of the peacocks and the crest-jewel of the serpents, the science of, *Jyotiṣa* is situated at the head of all the sciences forming the auxiliaries of the vedās (*Vedāṅgas*)” says Lagadha in *Vedāṅga Jyotiṣa* .

Introduction

Sanskrit's contributions to the field of Mathematics and Astronomy are well known. Both Mathematics and Astronomy has played a vital role in all aspects of life in the Indian society. ‘*Jyotiṣam*’ an auxiliary of *Veda(s)*, is a branch of study that includes Mathematics, Astronomy and Astrology. Mathematics forms the base for the study of Celestial bodies. The study of mathematics was always systematic in the Indian tradition. This paper is an attempt to summarize briefly the notable contributions of Indian Mathematicians / Astronomers.

Mathematics and Astronomy in Vedic texts

India holds a very rich and long history in the field of mathematics. From the elementary to complex problems like solving indeterminate equations. . Origin of algebra can be traced back to *Śathapatha Brāhmaṇa*. The *Śulvasūtra-s* (800 bce) used the mathematical concepts in the construction of Vedic altars.

Vedic texts has many references to Sun, Moon, stars, planets meteors, comets, etc. There were mentions of 27 stars with which association of Moon was mentioned, six seasons and twelve months were also observed. It is also recognized that an average solar year nearly has 365 days and lunar month has nearly 29.5 days.

Vedāṅga jyotiṣam, is the earliest available text in India, exclusively devoted to astronomy. The text is attributed to Lagadha. Vedāṅga jyotiṣam is the first text in astronomy in India in the sūtra format, giving mathematical algorithms in astronomy. This sets the trend for later astronomical texts in India, which are all in the sūtra format.

Importance given to Proofs in Indian tradition

It was a prominent thought in western countries that Indian texts has just statements with no proofs, but Ancient Indian Scholars acknowledge the statements as valid only if they are with valid proof. In Siddhāntaśiromani, Bhāskarācārya gives the importance of proofs as:

यद्येवमुच्यते गणितस्कन्धे उपपत्तिमान् एवागमः प्रमाणम्

“For all that is discussed in Mathematics [section], only an authentic tradition which is supported by Upapatti will be Pramāṇa”.

Indian Mathematicians / Astronomers and their important works:

There is a long gap between Vedānga jyotiśa and Āryabhaṭīya which was composed in 499 CE. Āryabhaṭīya can be said to herald the full-fledged Siddhāntic tradition, and contains a systematic treatment of all the traditional astronomical problems. It is the first text in India to use trigonometry, including spherical trigonometry extensively. Indian Mathematicians / Astronomers have contributed much to the field of Astronomy. The contributions of Kerala Astronomers are notable. Here we intend to name a few notable Indian Mathematicians and their important works

Aryabhata (466 AD)	Āryabhaṭīya (499 AD)
Varāhamihira (505 AD)	Pañcasiddhāntika
Bhaskara I (600 AD)	Mahābhāskarīya , Laghubhāskarīya, Āryabhaṭīyabhāṣya
Brahmagupta (591 AD)	Brāhmasphuṭasiddhānta, Khandakhādyaka
Vateswara (880 AD)	Vaṭesvarasiddhānta
Bhāskarācārya II (1114 AD)	Siddhāntaśiromani with Vāsanābhāṣya
Mādhava (1340 – 1425 CE)	Venvāroha , Sphuṭacandrāpti
Nilakantha Somayāji (1444 – 1550 CE)	Tantrasaṅgraha , Āryabhaṭīyabhāṣya
Paramesvara (1360-1455 CE)	Dr̥ggaṇitam , Surya Siddhānta vivaranam
Jyestadeva (1500 – 1610 CE)	Yuktibhāṣā
Sankara varier 19 th Century CE	Ṣaḍratnamālā
Acyuta pisaradi 16 th Century CE	Karaṇapaddati

Few Contributions of Sanskrit to the field of Mathematics and Astronomy:

The Sine Table

Deriving the Sine values interested our Indian Mathematicians as it is much needed for computation of position of celestial bodies. Āryabhaṭa gives **the values for RSines** in a single verse. The verse was written by Āryabhaṭa, in his Āryabhaṭīya [Gaṇitapāda, verse 12], when decoded gives the sine values:

मखि भखि फखि धखि णखि जखि
डखि हस्झ किष्ण स्थकि किघ्व ।
घलकि किग्र हक्य धकि किच
सा झश ड्व क्ल प्त फ छ कलार्धज्याः ॥

The values given by the well-known Kerala Astronomer, Mādhava are more accurate and are similar to the modern values.

Concept and Operation with Zero:

The history of concept of zero can be traced back to Ṛg Veda. The following hymn gives a numeral with the numeral 'zero'

त्रीणि शता त्रीसहस्ताण्यग्निं रिंशश्च देवा नव चापसर्पयन् । औक्षन् घृतैरसृणन् बहिरस्मा ॥ ऋ वे

This corresponds to - 33 + 303 + 3003

The following Ṛg veda hymn gives the names for the powers of 10 upto the power of 17.

शताय स्वाहा सहस्राय स्वाहा युताय स्वाहा नियुताय स्वाहा प्रयुताय स्वाहार्बुदाय स्वाहा न्यर्बेदाय स्वाहा
समुद्रायमध्याय स्वाहान्ताय स्वाहा परार्धाय स्वाहा

Āryabhaṭīya gives a verse the place values in Gaṇitapāda, that means – in the sequence of 1, 10, 100, 1000, 10000,.. each place [value] is ten times the previous place

एकं दश च शतं च सहस्रं त्वयुत नियुते तथा प्रयुतम् ।
कोट्यर्बुदं च वृन्दं स्थानात् स्थानं दशगुणं स्यात् ॥

Brahmagupta was the first person to give the **operations of zero** explicitly in his work

Brāhmasphuṭasiddhānta, though the conceptual usage of 'zero' is seen in vedic texts and Āryabhaṭīya.

Brāhmasphuṭasiddhānta, an extensive astronomical treatise, is the first text to discuss mathematics of zero.

Chapter 12 – Gaṇitādhyāya , chapter 18 – Kuṭṭakādhyaya of Brāhmasphuṭasiddhānta are dedicated to

Mathematics. Pṛthūdakasvāmin wrote a commentary named Vāsanābhāṣya on Brāhmasphuṭasiddhānta .

Brahmagupta gives the rules for operations with zero in verses 30-34, chapter 18 –

Addition:

ऋणधनशून्ययोः शून्ययोश्शून्यं ।
ṛṇadhanaśūnyayoḥ śūnyayośśūnyam

Subtraction:

शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् ।
śūnyavihīnamṛṇamṛṇam dhanam dhanam bhavati śūnyamākāśam

Multiplication:

शून्यर्णयोः खधनयोर्खशून्ययोर्वा वधशून्यम् ।
śūnyarṇayoḥ khadhanayorkhaśūnyayorvā vadhaśūnyam

Division:

स्वोद्धृतमृणन्धनं वा तच्छेदमृणन्धनं विभक्तं खम् ।
svoddhṛtamṛṇandhanam vā tacchedamṛṇandhanam vibhaktam kham

Pythagoras theorem:

भुज कोटि कर्णन्यायः popularly known as **Pythagoras theorem** lays the foundation for the study of

Trigonometry. भुज कोटि कर्णन्यायः can be traced back to Baudhayana Sulba sūtra (belongs to Taittiriya Samhita of Krishna Yajur Veda) dated 8th – 6th BCE.

Pythagoras theorem statement by Baudhayana –

दीर्घचतुरश्रस्याक्षण्या रज्जुः पार्श्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरूतस्तदुभयं करोति ॥

Infinite series

The **infinite series** expansion for the sine and cosine functions are well known in the Aryabhata school. These were first discussed in the Kerala works on astronomy and mathematics. The full proofs of these are to be found in *Gaṇita-yukti-bhāṣā* of Jyeṣṭhadeva (composed around 1530 CE).

Ancient Indian mathematicians included many examples to keep the study of the subject interesting. Study of Mathematics was always an integrated study with Astronomy in Ancient India. Indian mathematicians

Shape of Earth:

Indian tradition always holds the Shape of the Earth as Spherical. The word ‘Bhūgola’ can be found in the Bhāgavata Purana. We also find a reference in Mārkaṇḍeya purana where the shape of the Earth is said to be spherical with bit elevated at the poles [which we agree now]. Āryabhaṭa describes Earth as sphere, in Golapāda, verse 7:

यद्वत् कदम्बपुष्पग्रन्थिः प्रचितः समततः कुसुमैः ।
तद्वद्वि सर्वसत्त्वैर्जलजः स्थलजश्च भूगोलः ॥

Just as the bulb of a Kadamba flower is covered all around by blossoms, just so is the globe of the Earth surrounded by all creatures, terrestrial as well as aquatic.

Diurnal motion of celestial objects:

Āryabhaṭa explains rotation of Earth, observation of the night sky, the stars going around the earth is due to the rotation of Earth. He describes the relative motion in a verse in Golapāda, verse 9, thus:

अनुलोमगतिर्नैस्थः पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥

“Just as a person in a boat moving in the forward direction observes the stationary objects to be moving in the opposite direction, so also the stationary stars seem to move directly westward for an observer at Lanka”

Chapter of False notions:

Indian astronomers were keen on breaking popular myths and give proper explanations. Lalla in his text has given a complete chapter on ‘false notion’. One such verse is given here where the proper understanding of the eclipses is given and the popular myth of the moon being swallowed is refuted.

अथ शापवशात् परिक्षयः स्याद्
विबुधैर्वा शशिनो निपीत मूर्तिः ।
गणितेन चरक्षयौ कथं स्तो
यदि पीतश्च समीक्ष्यते स कृष्णः ॥

If the moon decreases because of some curse or because it is being sucked by the Gods, how can the increase and decrease be determined by calculations.

Instruments:

Almost all the Indian astronomical texts hold a chapter on instruments. The description of the instruments that are used for practical methods used for observations are given in the chapter. The construction of the these instruments, the choice of the materials, the precision in which the instruments are to be constructed, the way to use the instruments, the alternatives, the error correction that is to be carried out over a period of time are also given in detail. Circle-type instruments, hemispherical bowl, celestial globe, clepsydra, gnomon are few of the instruments that are described in the astronomical works.

The Gnomon, known as *Śanku* and also as *nara-yantra*, is a widely used astronomical instrument. It is generally a vertical rod, but Vṛddha-vaśiṣṭha siddhānta mentions about a horizontal gnomon. Astronomers determined the direction, the latitude, celestial co-ordinates, time from the shadow of this vertical rod cast on a horizontal plane. The usage of gnomon can be traced back to Kātyāyana Śulbasūtra. In chapter 20 of Kautilya Arthaśāstra, we find the formula for finding the time elapsed after sunrise, for the given length of the gnomon. The chapter also gives the rule for the uniform variation of the noon-shadow from the solstices.

The [Celestial globe] gola-yantra is also popularly used. It is made of wood, perfectly spherical, uniformly dense all round but light in weight, should be made to rotate keeping pace with time with the help of mercury, oil and water by the application of one's own intellect.

काष्ठमयं समवृत्तं समन्ततः समगुरुं लघुं गोलम् ।
पारदतैलजलैस्तं भ्रमयेत् स्वधिया च कालसमम् ॥

Planetary Model

Around 1500 CE, Nīlakaṅṭha Somayājī contributed to the revision of the calculation of the planetary positions and gave his planetary model in his works. According to his model, the planets move in eccentric orbits around the Sun, which itself moves around the Earth. This was even before the Copernicus model (in 1542 CE). This is the same model as of Tycho Brahe's model, proposed around (1580 CE).

The Pancanga [calendrical computations], Eclipses compilation are notable contribution of Sanskrit to Mathematics and day-to-day life also.

References:

- *Āryabhaṭīya of Āryabhaṭa with commentary of Bhāskara I and Someśvara, K.SShukla, Indian National Science Academy, New Delhi, 1976, p.49
- *Brāhmasphuṭasiddhānta (with Hindi translation) 4 vols., Ed. By R.S.Sharma, Indian Institute of Astronomical and Sanskrit Research, New Delhi, 1966.
- *Gaṇita-yuktibhāṣā, Ed. with English Translation by K.V.Sarma, with Explanatory notes by K.Ramasubramanian, M.D.Srinivas and M.S.Sriram, Vol.I
- *The Kautilya Arthaśāstra, Part I (translation), 1972
- *The Gnomon in early Indian Astronomy, George Abraham, Indian Journal of History of Science, 16(2); 215-218, November 1981.
- *Astronomy in Gaṇita-Yukti-Bhāṣa, M.S.Sriram, 2010. *Algorithms in Indian Mathematics, M.S.Sriram, 2005.
- *M.D.Srinivas, 'Proofs in Indian Mathematics' NPTEL lecture, 2014
- *Mathematics and Astronomy in India before 300 BCE, Amartya Kumar Dutta and M.S.Sriram, 2016
- *NPTEL course on "Mathematics in India from Vedic Period to Modern Times", www.nptel.ac.in, 2014.