CONTROLLABILITY OF IMPULSIVE DIFFERENTIAL SYSTEMS WITH NONLOCAL CONDITIONS

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Abstract

This paper is concerned with the controllability of impulsive functional differential equations with nonlocal conditions. Using the measure of non compactness and Munch fixed point theorem, we establish some sufficient conditions for controllability. Firstly, we require the equicontinuity of evolution system, and next we only suppose that the evolution system is strongly continuous. Since we do not assume that the evolution system generates a compact semi group, our theorems extend some analogues' results of (impulsive) control systems.

Key Words:

Controllability, Impulsive functional, differential systems, Nonlocal conditions Measure of non compactness Mild solutions.

1. INTRODUCTION

In this paper, we consider the following impulsive functional differential systems:

$$x'(t) = A(t)x(t) + f(t; x(t)) + (Bu)(t); \text{ a ,e: on } [0, b], \tag{1.1}$$

$$\Delta x(t_i) = x(t_i^+) - x(t_i^-) = I_i(x(t_i)) \quad i=1,2\ldots\ldots s \tag{1.2}$$

$$x(0) + M(x) = x_0 \; ; \tag{1.3}$$

Where A(t) is a family of linear operators which generates an evolution operator

$$U: \Delta = \{(t, s) \in [0, b] \in : \leq s \leq t \leq b\} \rightarrow L(x),$$

here, X is a Banach space, L(X) is the space of all bounded linear operators in X; f : [0, b] x X → X; 0 < t_1< …. < t_s < t_{s+1} = b; Ii : X → X; i = 1,2……. s are impulsive functions; M : PC([0, b];X) → X; B is a bounded linear operator from a banach space V to X and the control function u(.) is given in L^2 ([0, b], V).

Controllability for differential systems in Banach spaces has been studied by many authors [2; 4; 9] and the reference therein. Benchohra and Ntouyas [4], using the Martelli fixed-point theorem, studied the controllability of second-order differential inclusions in Banach spaces. Guo et al. [9] proved the controllability of impulsive evolution inclusions with nonlocal conditions.

The impulsive differential systems can be used to model processes which are subjected to abrupt changes. The study of dynamical systems with impulsive effects has been an object of intensive investigations [8; 14; 15]. The semi linear nonlocal initial problem was first discussed by Byszewski [5; 6] and the importance of the problem consists in the fact that it is more general and has better effect than the classical initial conditions. Therefore it has been studied extensively under various conditions on A(orA(t)) and f by several authors [1; 11; 13; 17].

Recently, Hernandez and O'Regan [10] point out that some papers on exact controllability of control systems contain a similar technical error when the compactness of semi group and the other hypotheses are satisfied, this is, in this case the application of controllability results are restricted to finite dimensional space. The goal of this paper is to find conditions guaranteeing the controllability of impulsive differential systems when the Banach space is non separable and evolution system U(t; s) is not compact, by means of Monch fixed-point theorem and the measure of non compact-ness. Since the method used in this paper is also available for evolution inclusions in Banach spaces, we can improve the corresponding results in [2; 4].

2. PRELIMINARIES:

Let (X, ‖.‖) be a real Banach space. We denote by C([0, b];X) the space of X-valued continuous functions on [0; b] with the norm $\|X\| = sup\|X(t)\|$,t∈ $[0, b]$ and by $L^1([0, b]; X)$ the space of X-valued Bochner integrable functions on [0,b] with the norm $\|f\|_{L^1} = \int_0^b \|f(t)\| dt$ for the shake of simplicity, we put J = [0,b]; J_0 = [0, t_1]; Ji = [t_i; t_{i+1}]; i = 1,……,s. In order to find the mild solution of the problem (1.1)-(1.3), we introduce the set PC([0, b];X) ={ u : [0, b] →X : u is continuous on Ji; i = 0, 1,……..,s and the right limit $u(t_i^+)$ exits ,i = 1,…….,s. It is easy to verify that PC([0, b];X) is a Banach space with the norm $\|X\|pc = \sup\{\|X(t)\|$,t ∈ $[0, b]\}$.

Let us recall the following definitions.

Definition 2.1: Let E^+ be the positive cone of an order Banach space (E, ≤). A function defined on the set of all bounded subsets of the Banach space X with values in E^+ is called a measure of noncompactness (MNC) on X if $\varphi(\overline{\omega}\,\Omega)$ = $\varphi(\Omega)$ for all bounded subsets $\Omega \subset$ X, where $\overline{\omega}\,\Omega$ stands for the closed convex hull of Ω.

The MNC φ is said:

(1) monotone if for all bounded subsets Ω_1, Ω_2 of X we have:

$$(\Omega_1 \subseteq \Omega_2) \Rightarrow (\phi(\Omega_1) \leq (\phi(\Omega_2))$$

(2) nonsingular if $\phi(\{a\} \cup \Omega) = \phi(\Omega)$ for every a ∈ X, $\Omega \subset$ X;

(3)regular if if $\phi(\Omega) = 0$ if and only if Ω is relatively compact in X.

one of the important examples of MNC is the non compactness measure of Hausdorff β define on each bounded subset of X by

$$\beta(\Omega) = \inf\{ \in > 0 ;\ \Omega \text{ has a finite } \in \text{- net in X}\}.$$

It is well known that MNC _ enjoys the above properties and the other proper-ties(see[3,12]): for all bounded subsets $\Omega, \Omega_1, \Omega_2$ of X.

(4) $\beta(\Omega_1 + \Omega_2) \le \beta(\Omega_1) + \beta(\Omega_2)$, where 1 + 2 = $\Omega_1 + \Omega_2 = \{x + y : x \in \Omega_1, y \in \Omega_2\}$;

(5) $\beta(\Omega_1 \cup \Omega_2) \le \max\{\beta(\Omega_1), \beta(\Omega_2)\}$;

(6) $\beta(\lambda\Omega) \le |\lambda| \beta(\Omega)$ for any $\lambda \in R$;

(7) If any map Q: D(Q) $\subseteq X \to Z$ is Lipschitz continuous with constant k, then

$\beta_z(Q\Omega\ )$ for any bounded subset $\Omega \subseteq$ D(Q), where Z is a Banach space.

Definition 2.2: A function x(.)$\in$ PC([0, b];X) is a mild solution of (1.1)-(1.3) if

$$x(t) = U(t,0)x(0) + \int_0^t U(t,s)(f + \beta u)(s)ds + \sum_{0<t_i<t} U(t,t_i)\, I_i\, x(t_i),$$

where x(0) +M(x) = x_0.

Definition 2.3:

The system (1.1)-(1.3) is said to be non locally controllable on J if, for every $x_{0,}x_1 \in X$ there exist a control $u \in L^2$(J,V) such that the mild solution x(.)of (1.1)-(1.3) satisfies

x(b) +M(x) = x_1.

A two parameter family of bounded linear operators U(t, s), $0 \le s \le t \le b$ on X is called an evolution system if the following two conditions are satisfied:

(i) U(s, s) = I, U(t, r)U(r, s) = U(t, s) for $0 \le s \le t \le b$;

(ii) (t, s) $\to$ U(t, s) is strongly continuous for $0 \le s \le t \le b$.

since the evolution system U(t, s) is strongly continuous on the compact set $\Delta = JxJ$, then there exist L_u >0 such that $\|U(t,s)\| \le L_u$ for any (t, s) $\in \Delta$.More details about evolution system can be found in [18].

Definition 2.4: A countable set $\{f_n\}_{n=1}^{+\infty} \subset L^1([0,b];X)$ is said to be semi compact if:

- the sequence $\{f_n\}_{n=1}^{+\infty}$ is relatively compact in X for a.a. tϵ [0, b];
- there is a function $\mu \epsilon L^1([0,b];\ R^+)$ satisfing $sup_{n\ge1}\|f_n(t)\| \le \mu(t)$ for a.e t$\in$[0, b].

The following interchange results about estimation are shown in [12] Theorems 4.2.2 and 5.1.1, respectively.

Lemma 2.2.1. [12]:

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Let $\{f\}_{n=1}^{\infty}$ be a sequence of f_n in $L^1([0,b];R^+)$. Assume that there exist $\mu,\eta \in L^1([0,b];R^+)$ satisfing $sup\|f_n(t)\| \leq \mu(t)$ and $\beta\{f\}_{n=1}^{\infty} \leq \eta(t)$ a. e t ∈ [0, b], we have $\beta(\{\int_0^t U(t,s)f_n(s)ds\colon n \geq 1\}) \leq 2\, L_u \int_0^t \eta(s)ds$.

Lemma 2.2.2[12]:

Let $(Gf)(t) = \int_0^t U(t,s)f(s)ds$. If $\{f\}_{n=1}^{\infty} \subset L^1([0,b];X)$ is semi-compact, then the set $\{Gf\}_{n=}^{+\infty}$ is relatively compact in $C([0,b];X)$ and moreover, if $f_n \to f_0$. Then for all t ∈[0, b] (Gf_0) as n→ +∞.

The following fixed-point theorem, a non linear alternative of Monch fixed-point theorem, plays a key role in our proof of controllability (see Theorem 2.2 in [16]).

Lemma 2.2.3:

Let D be a closed convex subset of a Banach space X and 0 ϵ D.
Assume that F : D →X is a continuous map which satisfies Monch condition, that is, M ⊆ D is countable, M ⊆ $\varpi(\{0\} \cup F(M)) \implies \overline{M}$ is compact.
Then, there exist x ϵ D with x = F(x).

3. MAIN RESULTS

We first give the following hypotheses:(H1) A(t) is the family of linear operator, A(t) : D(A) → X ; D(A) not depending on t and dense subset of X, generating an equicontinuous evolution system {U(t, s) : (t; s) ∈ Δ
i.e., (t; s) → {U(t; s)x : x ∈ B} is equicontinuous for t > 0 and for all bounded subset B.

(H2) The function f : [o, b] x X →X satisfies:
(i) for a.e. t ∈ [0, b], the function f(t, .) : X → X is continuous and for all x ∈ X,
the function f(. , x) : [o, b] → X is measurable;

(ii) there exist a function m ∈ $L^1([0,b];R^+)$ and a non decreasing continuous function Ω : $R^+ \to R^+$ suh that $\|f(t,x)\| \leq m(t)\,\Omega$ (‖x‖), X ∈ X, t ∈[0, b] and

$$lim_{n\to\infty} \inf \frac{\Omega\,(n)}{n} = 0$$

iii) there exist h ∈ $L^1([0,b];R^+)$ such that, for any bounded subset D ⊂ X ,
$\beta(f(t,D)) \leq h(t)\beta(D)$ for a.e t ∈ [0, b], where β is the Hausdorff MNC.

(H3) M : PC(J,X) → X is a continuous compact operator such that

$$\lim_{\|y\|pc\to\infty} \frac{\|M(y)\|}{\|y\|pc} = 0.$$

(H4) The linear operator W : $L^2(J,V) \to X$ is defined by $W_u = \int_0^b U(b,s)B(s)ds$
such that:

(i) W has an invertable operator W^{-1} which take values in L^2 (J,V)=ker W and there exist positive constants L_B and L_w

such that $\|B\| \leq L_B$ and $\|W^1\| \leq L_w$;

(ii) there is $K_w \in L^1(J, R^+)$ such that, for any bounded set Q ⊂ X,

$$\beta((W^{-1}Q)(t)) \leq K_w(t)\beta(Q)$$

(H5) Let Ii : X →X; i = 1,….,s be a continuous operator such that:

(i) there are non decreasing function li : $R^+ \to R^+$; i = 1,…., s such that

$\|I_i(x)\| \leq l_i(\|x\|)$ and $lim_{n\to\infty}\inf\frac{l_i(n)}{n} = 0$, i = 1,….,s;

(ii) there exist constants Ki ≥0, such that $\beta(I - i(D)) \leq K - i\beta(D)$, i = 1,…s;

for every bounded subset D ⊂X.

(H6) The following estimation holds true:

L = $(L_U + 2L_U^2\, L_B\, \|K_w\|L^1)\, \sum_{i=1}^{s} K_i + \ (2L_U + 4L_U^2\, L_B\, \|K_W\|L^1)\|h\|L^1 < 1$,

where L_U= sup $\{\|U(t,s)\|, (t,s) \in \Delta\}$.

Theorem 3.1

Assume that the hypothesis (H1)-(H6) are satisfied, then the impulsive differential system 1.1, 1.2, 1.3 is non locally controllable on J

Proof:

Using hypothesis (H4) (i), for every x 2 PC(J,X), define the control

$$u_x(t) = W^{-1}\left[x_1 - M(x) - U(b,0)\big(x_0 - M(x)\big) - \int_0^b U(b,s)f\big(s,x(s)\big)ds - \sum_{i=1}^{s} U(t,t_i)I_i\big(x(t_i)\big)\right](t)$$

. We shall show that, when using this control, the operator, defined

$$(G_x)(t) = U(t,0)\big(x_0 - M(x)\big) + \gamma(f + Bu_x) + \textstyle\sum_{0<t_i<t} U(t,t_i)I_i(x(t)) \qquad (3.1)$$

Where $\gamma(f + Bu_x)$ (t) ∈ C(J, X) is defined by

$\gamma(f + Bu_x)$ (t) =$\int_0^b U(t,s)(f + Bu_x)(s)ds$ has a fixed point. This fixed point is then a solution of the system (1.1)-(1.3). Clearly

x1 - M(x) = G(x)(b); which implies that the system (1.1)-(1.3) is controllable. We define G = G1 + G2; where

$(G_{1x})(t) = U(t,0)\big(x_0 - M(x)\big) + \sum_{0<t_i<t} U(t,t_i)I_i(x(t_i))$,

$(G_{2x})(t) = \gamma(f + Bu_x)(t))$,

fo all t ∈ [0, b]. Subsequently, we will prove that G has a fixed point by using Lemma2.3.

Step 1: The operator G is continous on PC([0, b];X). For this purpose, we assume that xn → x in PC([0, b]; x). Then by hypothesis (H3) and (H5), we have that

$$\|G_1 x_n - G_1 x\|pc \le \|M(x_n) - M(x)\| + L_U \sum_{i=1}^{s} \big\|I_i\big(x_n(t_i)\big) - I_i(x(t_i))\big\| \quad (3.2)$$

Note that

$$\|G_2 x_n - G_2 x\|c \le L_U \int_0^b \big\|f\big(s, x_n(s)\big) - f(s, x(s))\big\|ds + L_U L_S \int_0^b \big\|u_{x_n}(s) - u_x(s)\big\|ds$$
$$\le L_U \int_0^b \big\|f\big(s, x_n(s)\big) - f(s, x(s))\big\|ds + L_U L_S \int_0^b \big\|u_{x_n}(s) - u_x(s)\big\|L^2$$

,(3.3)

$$\| Ux_n - Ux\|L^2 \le L_W \|M(x_n) - M(x)\| + L_U \|M(x_n) - M(x)\|$$
$$+L_U \int_0^b \big\|f\big(s, x_n(s)\big) - f(s, x(s))\big\|ds + L_U \sum_{i=1}^{s} \big\|I_i\big(x_n(t_i)\big) - I_i(x(t_i))\big\|$$

(3.4)

Observing (3.2)-(3.4), by hypotheses (H2), (H3), (H5) and domination convergence theorem, we have that

$\|Gx_n - Gx\|pc \le \|G_1 x_n - G_1 x\|pc + \|G_2 x_n - G_2 x\|c$ 0; as n →+∞;

i.e., G is continuous.

Step 2. There exists a positive integer $n_0 \ge 1$ such that $G(B_{n_0}) \subseteq B_{n_0}$, where $B_{n_0} = \{x \epsilon PC(J, X): \|x\| \le n_0\}$.

Suppose the contrary. Then we can find $x_n\, \epsilon PC(J, X), y_n = Gx_n\, \epsilon PC(J, X)$, such that $\|x\|pc \le n$ and $\|y_n\|pc > n$,

For every $n \ge 1$.

Now we have that

$(y_n)(t) = U(t,0)\big(x_0 - M(x_n)\big) + \gamma\big(f + Bu_{x_n}\big)(t) + \sum_{0<t_i<t} U(t,t_i)I_i(x_n(t_i))$

So we get that

$$n < \|y_n\|_{PC} \le L_U \|x_0\| + \|M(x_n)\| + \|\gamma(f_n)\|_C + L_U \sum_{i=1}^{s} \|I_i(x_n)\|_{PC} \quad (3.5)$$

Note that, by (H2)(ii), (H3)(i), (H5)(i)

$$\|\gamma(f_n)\|_C \le sup_{t\in J} \int_0^t \|U(t,s)\| \; \|f(s, x(s))\| \, ds$$

$$\leq L_U \int_0^b m(s)\,\Omega\,(\|x\|_{PC})ds = L_U\,\Omega\,(\|x\|_{PC})\|\mathrm{m}\|_{\mathrm{L}} \tag{3.6}$$

$$\left\|\gamma(BU_{x_n})\right\|_C \leq sup_{t\in J}\int_0^t \left\|U(t,s)BU_{x_n}(s)\right\|\ ds$$

$$\leq L_U L_B \int_0^b \left\|U_{x_n}(s)\right\|\ \ ds \leq\ L_U L_B b^{\frac{1}{2}}\left\|U_{x_n}\right\|_L^2 \tag{3.7}$$

$$\left\|U_{x_n}(s)\right\|_L^2 = \left\| \begin{array}{c} W^{-1}\,[x_1 - M(x_n) - \\ U(b,0)(x_0 - M(x_n))\int_0^b U(b,s)f\big(s,x_n(s)\big)ds - \sum_{i=1}^{s} U(t,t_i)I_i\big(x_n(t_i)\big) \end{array} \right\|$$

(3.8)

Which is implies that

$$1 < \frac{1}{n}[C_1 + C_2\|M(x_n)\| + C_3\,\Omega\,(\mathrm{n}) + \mathrm{C}_4\sum_{\mathrm{i}=1}^{\mathrm{s}}\mathrm{l}_{\mathrm{i}}(\mathrm{n})\,] \tag{3.9}$$

Where

$C_1 = (L_U +\ L_U^2 L_B b^{\frac{1}{2}} L_W)\|x_0\| + L_U^2 L_B b^{\frac{1}{2}} L_W)\|x_1\|$,

$C_2 = (L_U +\ L_U L_B b^{\frac{1}{2}} L_W + L_U^2 L_B b^{\frac{1}{2}} L_W$

$C_3 = L_U\|m\|_L^1 + L_U^2 L_B b^{\frac{1}{2}} L_W\|m\|_L^1$

$C_4 = L_U L_U^2 L_B b^{\frac{1}{2}} L_W$

Observing (H2)(ii), (H3)(i), (H5)(i) by passing to the limit as $n \to +\infty$ in (3.9) we get $1 \leq 0$, which is a contradiction. Thus we deduce that there is $n_0 \geq 1$ such that $G(B_{n_0}) \subseteq B_{n_0}$.

Remark 3.3.1:

In Theorem 3.1 we require f to satisfy a compactness condition (H2) (iii),but not require the compactness of evolution system U(t,s). Note that if is compact or Lipschitz continuous, then condition (H2)(iii) is satisfied. Therefore, our work extends some previous results, where the compactness of T(t) and f, or the Lipschitz continuity of f are needed.

In the following, by using another MNC, we will prove the result of Theorem 3.1 in the case there is no equicontinuity of the evolution system U(t,s) and hypothesis (H6).Then the result we get is more general than most previous controllability results and it is interesting. Instead of (H5), we give the hypothesis (H5'):(H5') Let Ii : X → X; i = 1,…., s be a continuous compact operator such that there are non decreasing functions li : $R^+ \to R^+$; i = 1,….., s; satisfying

$\|I_i(x)\| \leq I_i(\|x\|)\ lim_{n\to\infty}\mathrm{inf}\frac{l_i(n)}{n} = 0$

n = 0; i = 1,…………,s:

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