CONVERGENCE OF SEQUENCE OF REAL AND COMPLEX FUNCTION

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Abstract- In this paper we discuss the concept of convergence of real, complex and function {fn} sequence. Also we discuss the concept of sub- sequence. We presented the concepts of convergence criteria for the sequence. First, we presented monotonic increasing and decreasing sequence and then limit of a sequence, Cauchy sequence and Algebric properties of convergent sequence.

Keywords- Convergence sequence, monotonic increasing and decreasing.

I. INTRODUCTION

In addition to my own study and learning experiences in this field, published information on students' issues inspired my thinking about a potential strategy to learning limits. I intended for the method to assist students in moving past the oversimplified or incorrect concepts they may be familiar with from the literature (Cornu, 1983, 1991, Davis and Vinner, 1986), such as;

• The terms of a convergent sequence occasionally reach the limit.
• The terms are close to the limit but not quite there.
• There must be an increase or decrease in the words.
• It suffices that an unlimited number of words approach the upper bound.
The sequence's limit is a bind.
The final terms in a sequence are its limit.
• A convergence sequence needs to have a structure.
• University students were also discovered to be the majority of this conception.

SEQUENCE AND CONVERGENCE

Real number sequence
1.1 Definition
The expression S1,S2,S3.....Sn3 is known as a "Sequence of real numbers" and is abbreviated as "Sn" or "Sn" when there is an ordered set S = S1, S2,S3..Sn of real numbers such that there is a real number Sn corresponding to every position integer nN. The sequence's range is Sn:n∈N. with term "Sn" being referred to as the nth term.

or

a series of real values that make up a function whose range is the set R of real numbers and whose domain is the set N of natural numbers. A series of real numbers is represented symbolically by the function f: NR, which is defined as f(n)= SnnN. Consider the case where f(n) = 1/n and n > N. The sequence S(n) = f(n) = 1/n2 with n = 1, 2, and 3 is made up of the ordered set of numbers 1, 1/2, 3, 4, and 1/n2.

Complex Sequence – Let {Zn} be a sequence of complex number and let Z∈C. We say that {Zn} convergence to Z and write Zn→Z (or lim Zn = Z etc) if for every positive real number∈>o, there exists a natural number N such that n ≥ N ⇒ |zn-z|< ∈

Theorem let Zn = xn + iyn

(i) $z_n \to z \Rightarrow x_n \to R_z,\ y_n \to \xi y$

(ii) $x_n \to x,\ y_n \to y \Rightarrow z_n \to x_n + iy_n$

Proof:– Put $x_n = R_z$, [xn-x] = R $(z_n - z) \leq$ |zn-z|. So given ∈ >0 use the same N.

(iii) $|z_n - z| \leq |x_n - x| + |y_n - y|$ by Δ law

MONOTONIC INCREASING SEQUENCE

Definition:- A sequence {Sn}∞n=1 is said to be monotonic increasing, if

Sn+1 ≥Sn ∀n∈N

Ex. (1) The Sequence {2.1, 2.11, 2.111…} is bounded and monotonic increasing.

MONOTONIC INCREASING SEQUENCE

Definition:- A sequence {Sn}∞n=1 is said to be monotonic decreasing, if

Sn+1 ≤Sn ∀n∈N

Ex. The Sequence {-1, -3 2, -2, -5 2, -3, …} is bounded and monotonic increasing, this is bounded above, but not bounded.

LIMIT OF A SEQUENCE

Definition:- Let {Sn}∞n=1be a sequence of real number. It for a given ∈>0, there corresponds a positive integer m such that |Sn-l|<∈∀n≥m...(1) then the number lis said to be limit of the sequence {Sn}∞n=1symbolically , we write lim Sn= l, n→∞

MEANING OF INEQUALITY (1)

If Sn > l, then |Sn-1|= Sn-l < ∈

Or

Sn < l+∈ _________(ii)

And it Sn < l, then |Sn-1|= l-Sn < ∈

l- ∈ < δn _________(iii)

Hence equation (1) can be written as

l- ∈ < δn < l+ ∈ ∀n≥m _________(iv)

From the inequalities (4) the following facts are evident.

CAUCHY SEQUENCE

Definition:- Let {Sn}∞n=1 be a sequence of real number. Then {Sn}∞n=1is said to be a Cauchy sequence if for each ∈ < 0 there is an n0∈N such that

Sn+p-Sn < ∈∀m, n≥ n_0

A sequence {Sn}∞n=1 is said to be cauchy sequence if for each ∈>0, there exist a position integer n such that

Sn+p-Sn < ∈ for each p>o

Example:- (1) The sequence { 1, 12 , 13 …. 1n…} is a Cauchy sequence , because if m>n then [1m-1n] < ∈ ∀m, n≥ n_0

That the given sequence is Cauchy sequence.

A CAUCHY SEQUENCE IS CONVERGENT OVER THE COMPLEX PLANE-

Let {Zn} be a Cauchy sequence in C ⇒ ∀∈>0

∃N∈N Such that ∀n, m ≥ N ⇒ Zn-Zn< ∈

Then Zm ∈ B∈ (Zn). We have that B∈ (Zn)⊂B∈(Zn)

Since B∈ (Zn) is compact and B∈ (Zn) has infinitely many point, then it must have a limit point, making {Zn} convergent.

Theorem – Every Cauchy sequence is bounded So Zn is bounded.

Proof- Their enist M > 0 Such that

Zn-Zn = Zn≤ M, ∀n∈ N ⇒ Zn ∈ (B (O, M))

Let Zn ∈ C a Cauchy Sequence.

Zn=Xn+iyn

We have that

⇒Zn-Zn→0(xn-xm)2+(yn-ym)2 → 0

As m_1n → + ∞

We have that

xn-xm

= (xn-xn)2

≤ (xn-xm)2+(yn-ym)2 → 0

yn-ym = (yn-ym)2

≤ (xn-xm)2+(yn-ym)2 → 0

Thus xn, yn are Cauchy sequence in R which is complete. (I.e. every Cauchy sequence is R convergence)

So exist x_o, y_o ∈ R such that

$x_n \to x_o$ and $y_n \to y_o$

Thus Zn → x_o + iy_o proving that Zn is convergence.

Algebric properties of convergent sequence –

The sequence whose n^{th} term is sn+tn, sn-tn, sntn or sntn (tn≠o), is said to be sum difference, product or division of sequence {Sn}∞n=1 and {tn}∞n=1 respectively.

THEOREM- If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ be two convergent sequence such that

$\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} t_n = M$.

(i) $\lim_{n\to\infty} (s_n \pm t_n) = \lim_{n\to\infty} S_n \pm \lim_{n\to\infty} t_n = L \pm M$

(ii) $\lim_{n\to\infty}(s_n t_n) = \{\lim s_n\, n\to\infty\} (\lim s_n\, n\to\infty) = LM$

(iii) $\lim_{n\to\infty} (s_n t_n) = \lim_{n\to\infty} s_n \lim_{n\to\infty} t_n = LM$, if $M \neq 0$ and $t_n \neq 0\ \forall n$

PROOF-

(i) Since $\lim S_n = L$ and $\lim_{n\to\infty} t_n = M$, So for a given $\in > o$ there exist position integer m1 and m2 such that

$|s_n-l| < \in 2$, $\forall n \geq m1$

$|s_n-l| < \in 2$, $\forall n \geq m2$

Let m = max {m1, m2} then

$s_n-L < \in 2$ And $t_n-M < \in 2\ \forall n \geq m$

Now $s_n \pm t_n$ - $L \pm m = s_n-L \pm (t_n-M)$

$\leq s_n-L + t_n-M$

$< \in 2 + \in 2 = \in, \forall n \geq m.$

Hence

$\lim_{n\to\infty} (s_n \pm t_n) = L \pm M = \lim_{n\to\infty} s_n \pm \lim_{n\to\infty} t_n$

(ii) Since $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are convergent sequence,

$s_n \leq R$, $t_n \leq R\ \forall n e N$ ……… ..(1)

Now ,

$s_n t_n-LM = s_n t_n-M+ M (s_n-L)$

$\leq s_n\, t_n-M + M\, s_n-L$

$\leq k\, t_n-M + M\, s_n-L$……………(2)

Again , since $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n=M$, so for any given $\in > o$ there exist positive integer m1 and m2 such that

$s_n-L < \in 2M$, $\forall n \geq m1$

$t_n-M < \in 2k$, $\forall n \geq m2$ ……….(3)

Then for m = max {m1, m2} we get from (2) and (3)

$s_n t_n-LM < k, \in 2k + M \in 2M$, $\forall n \geq m$

$= \in 2 + \in 2 = \in$

$s_n t_n-LM < \in$, $\forall n \geq m$

Hence $\lim_{n\to\infty}(s_n t_n) = LM = (\lim_{n\to\infty} s_n)(\lim_{n\to\infty} t_n)$

(iii) Since $s_n t_n-Lm = s_n M-t_n L t_n M$

$= s_n M-LM+LM-t_n L t_n M$

$= M S_n-L+ L (M-t_n) t_n M$

$\leq M\, S_n-L+L\, L_n-M\, t_n M$

We now require the following lemma

Lemma:- $\lim_{n\to\infty} t_n = M \neq 0\ \exists$ a position real number λ s.t.

$t_n -> \lambda\ \forall n \geq m_1$

PROOF - $\lim_{n\to\infty} t_n = M$, then for $\in = M2 > o\ \exists$ a positive integer m1 such that

$t_n-M < M2$, $\forall n \geq m_1$

$M-t_n \leq t_n-M < M2$

$M - M2 < t_n$

$t_n > M2 = \lambda$ (say)

$t_n > \lambda\ \forall n \geq m_1$

Now

$s_n t_n-Lm \leq S_n-L\lambda + L\ M\ t_n-M\lambda$, $\forall n \geq m_1$ …….(i)

Similarly

$s_n-L \leq \lambda 2 \in\ \forall n \geq m_2$ …….(ii)

$t_n-M < \lambda M\ L \in 2\ \forall n \geq m_3$ ……(iii)

For m = max { m1,m2, m3} from eq3 (i) (ii) and (iii)

sntn-Lm < ∈ 2 + ∈ 2 = ∈ ∀ n ≥ m

limn → ∞ Sn tn = L M = limn → ∞sn limn → ∞tn

CONCLUSION - It is not always possible to prove directly the convergence of any sequence by the definition of convergent sequence or by Cauchy general principle of this we can find the limit of such sequence whose term can be expressed as sum, difference, product or division of corresponding term of two convergent sequences.

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