Coupling Special Pair of Rectangles with Woodall and Euclid Primes

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Abstract - This study aims to systematically identify pairs of rectangles wherein the sum of their respective areas corresponds precisely to either a Woodall prime or a Euclid prime. For each such prime encountered, the investigation further delineates the total number of contributing rectangle pairs, with a clear distinction drawn between primitive configurations—those with co-prime side lengths—and non-primitive ones, which may share common factors.
Key Words: Pair of rectangles, Woodall prime, Euclid Prime, Primitive, Non-primitive, Area.

1. INTRODUCTION

Mathematics, often celebrated as the universal language, places profound emphasis on the study of numbers and their intrinsic properties. Among its many branches, number theory occupies a distinguished role within pure mathematics, as it is primarily concerned with the exploration of integer solutions. A central theme within this domain is the study of Diophantine equations—equations where the solutions are restricted to integers and typically present fewer equations than unknowns. Despite significant advancements, a comprehensive and universally applicable methodology for determining whether such equations admit integer solutions—or for fully characterizing all their solutions—remains elusive. Readers interested in foundational approaches and notable developments in this area may refer to references [1–4].

Furthermore, the study of special categories of numbers, such as Dhurva numbers, along with other uniquely characterized numerical constructs, adds depth to the field. These concepts, discussed in sources [5–7], offer additional insights into the structural elegance and diversity of number-theoretic phenomena.

Any numerical sequence that can be expressed through a mathematical function may be interpreted as a manifestation of a pattern. Indeed, mathematics itself can be viewed fundamentally as the science of patterns—revealing underlying structures, regularities, and consistencies across numerical, geometric, and abstract domains. From a scientific perspective, any consistent relationship that can be formalized through theory also constitutes a pattern, lending clarity to otherwise complex phenomena. Put differently, a pattern is a systematic arrangement of numbers, shapes, or objects that conform to a defined set of rules or relations. A classic example lies in the direct correspondence between the number of sides of a polygon and its associated polygonal number, a relationship readily observed by the pattern-oriented mind.

Beyond such foundational insights, several researchers have extensively explored specific configurations—such as special pairs of rectangles and Pythagorean triangles—in connection with various exceptional polygonal numbers, including Jarasandha numbers, sphenic numbers, and others. In their analyses, these studies often distinguish between primitive cases (those with mutually co-prime parameters) and non-primitive ones, based on the side lengths of the rectangles and the triangle dimensions. Comprehensive accounts of these investigations and their methodologies can be found in references [8–24]. Special pair of rectangle and Pythagorean triangle generated with Wagstaff prime number was discussed in [24-26].

This study investigates pairs of rectangles whose combined areas correspond to select prime numbers—specifically, Woodall primes of two and three digits, followed by Euclid primes extending across two, three, and four-digit values. For each such prime, the number of contributing rectangle pairs is systematically enumerated, with a clear distinction between *primitive rectangles*—those with side lengths that are relatively prime—and *non-primitive rectangles*. These counts are derived and tabulated according to the dimensions of the respective rectangles.

2. BASIC DEFINITIONS

Definition 1:- Woodall numbers are defined as the natural numbers that can be expressed in the form $WD_k = k.2^k - 1$, for $k \in \mathbb{N}$. The initial terms in the sequence of woodall numbers are 1, 7, 23, 63, 159, 383, 895, ….

Woodall primes are those Woodall numbers that are also prime. The earliest values of the exponent $\boldsymbol{k}$ for which the corresponding Woodall numbers exhibit primality include 2, 3, 6, 30, 75, 81, 115, 123, 249, 362, 384, ...

The first few Woodall prime numbers include $7, 23, 383, \ldots$

Definition 2:- Euclid Prime numbers are defined as the integers of the form $E_k = q_{k*} + 1$, where q_{k*} is the k^{th} primality ,ie the product of first k prime numbers. The first few Euclid primes are constructed as follows

$$E_1 = 2 + 1 = 3$$
$$E_2 = 2 \times 3 + 1 = 7$$
$$E_3 = 2 \times 3 \times 5 + 1 = 31$$
$$E_4 = 2 \times 3 \times 5 \times 7 + 1 = 211$$
$$E_5 = 2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311$$

and so on..

Definition 2:- A rectangles with dimensions (c, d) is said to be primitive if $(c, d) = 1$.

Definition 3:- A rectangles with dimensions (c, d) is said to be non-primitive if gcd $(c, d) \neq 1$

3. METHOD OF ANALYSIS

Let $RC_1(c,d)$ and $RC_2(g,h)$ be the two distinct rectangles whose areas are denoted by Ar_1', Ar_2' respectively and (c,d), (g,h) are the dimensions of two distinct rectangles RC_1, RC_2 respectively.

Let

$$Ar_1' + Ar_2' = Wd_p \text{ (or) } E_p \qquad (1)$$

where 'Wd_p is a Woodall prime and E_p is a Euclid prime.

That is,

$$cd + gh = Wd_p \text{ (or) } E_p \qquad (2)$$

Assume $i, j, k \in \mathbb{Z}_+$, where all are distinct and non-zero with $j > k$.

Take the linear transformations

$$c = k,\ \ d = 2i + k,\ \ g = j - k,\ \ h = j + k$$

On using this transformation in (2), one may have

$$k(2i + k) + (j - k)(j + k) = Wd_p \text{ (or) } E_p$$

$$j^2 = Wd_p - 2ik \text{ (or) } \quad j^2 = E_p - 2ik \qquad (3)$$

By solving equation (3), the values of i, j, k corresponding to the 2-,3-,digits Woodall primes and 2-,3-, and 4-digits of Euclid primes can be determined. Additionally, the dimensions of the rectangles RC_1 and RC_2 are computed. Based on these dimensions, it becomes straightforward to classify the rectangles as either *primitive* or *non-primitive*. Table 1 illustrates numerical examples of Woodall primes, whereas Table 2 provides a corresponding numerical representation for Euclid primes.

Table 1

$Ar_1' + Ar_2' = Wd_p$	Dimensions of RC_1	Dimensions of RC_2	Observation	
			Primitive	Non-Primitive
23	(1,15)	(2,4)	RC_1	RC_2
383	(1,375)	(2,4)	RC_1	RC_2
383	(1,359)	(4,6)	RC_1	RC_2
383	(1,335)	(6,8)	RC_1	RC_2
383	(1,303)	(8,10)	RC_1	RC_2
383	(1,263)	(10,12)	RC_1	RC_2
383	(1,215)	(12,14)	RC_1	RC_2
383	(1,159)	(14,16)	RC_1	RC_2
383	(1,95)	(16,18)	RC_1	RC_2
383	(1,23)	(18,20)	RC_1	RC_2
383	(11,13)	(8,30)	RC_1	RC_2

Table 2

$Ar_1' + Ar_2' = Wd_p$	Dimensions of RC_1	Dimensions of RC_2	Observation	
			Primitive	Non-Primitive
31	(1,23)	(2,4)	RC_1	RC_2
31	(3,5)	(2,8)	RC_1	RC_2
31	(1,7)	(4,6)	RC_1	RC_2
211	(1,203)	(2,4)	RC_1	RC_2
211	(1,187)	(4,6)	RC_1	RC_2

$Ar'_1 + Ar'_2 = Wd_p$	Dimensions of RC_1	Dimensions of RC_2	Observation	
			Primitive	Non-Primitive
211	(3,65)	(2,8)	RC_1	RC_2
211	(1,163)	(6,8)	RC_1	RC_2
211	(3,57)	(4,10)		RC_1, RC_2
211	(1,131)	(8,10)	RC_1	RC_2
211	(5,31)	(4,14)	RC_1	RC_2
211	(1,91)	(10,12)	RC_1	RC_2
211	(5,23)	(6,16)	RC_1	RC_2
211	(9,19)	(2,20)	RC_1	RC_2
211	(3,33)	(8,14)		RC_1, RC_2
211	(7,13)	(6,20)	RC_1	RC_2
211	(3,17)	(10,16)	RC_1	RC_2
211	(1,43)	(12,14)	RC_1	RC_2
2311	(1,2303)	(2,4)	RC_1	RC_2
2311	(1,2287)	(4,6)	RC_1	RC_2
2311	(3,765)	(2,8)		RC_1, RC_2
2311	(1,2263)	(6,8)	RC_1	RC_2
2311	(3,757)	(4,10)	RC_1	RC_2
2311	(1,2231)	(8,10)	RC_1	RC_2
2311	(5,451)	(4,14)	RC_1	RC_2
2311	(1,2191)	(10,12)	RC_1	RC_2
2311	(3,733)	(8,14)	RC_1	RC_2
2311	(5,443)	(6,16)	RC_1	RC_2
2311	(1,2143)	(12,14)	RC_1	RC_2
2311	(3,717)	(10,16)		RC_1, RC_2
2311	(7,313)	(6,20)	RC_1	RC_2
2311	(9,247)	(4,22)	RC_1	RC_2
2311	(1,2087)	(14,16)	RC_1	RC_2
2311	(1,2023)	(16,18)	RC_1	RC_2
2311	(3,677)	(14,20)	RC_1	RC_2
2311	(1,1951)	(18,20)	RC_1	RC_2
2311	(3,653)	(16,22)	RC_1	RC_2
2311	(5,395)	(14,24)		RC_1, RC_2
2311	(13,163)	(6,32)	RC_1	RC_2
$Ar'_1 + Ar'_2 = Wd_p$			Observation	

	Dimensions of RC_1	Dimensions of RC_2	Primitive	Non-Primitive
2311	(15,145)	(4,34)		RC_1, RC_2
2311	(1,1871)	(20,22)	RC_1	RC_2
2311	(5,379)	(16,26)	RC_1	RC_2
2311	(11,181)	(10,32)	RC_1	RC_2
2311	(17,127)	(4,38)	RC_1	RC_2
2311	(1,1783)	(22,24)	RC_1	RC_2
2311	(3,597)	(20,26)		RC_1, RC_2
2311	(9,207)	(14,32)		RC_1, RC_2
2311	(11,173)	(12,34)	RC_1	RC_2
2311	(1,1687)	(24,26)	RC_1	RC_2
2311	(3,565)	(22,28)	RC_1	RC_2
2311	(1,1583)	(26,28)	RC_1	RC_2
2311	(7,233)	(20,34)	RC_1	RC_2
2311	(1,1471)	(28,30)	RC_1	RC_2
2311	(3,493)	(26,32)	RC_1	RC_2
2311	(5,299)	(24,34)	RC_1	RC_2
2311	(7,217)	(22,36)		RC_1, RC_2
2311	(15,113)	(14,44)	RC_1	RC_2
2311	(21,91)	(8,50)		RC_1, RC_2
2311	(1,1351)	(30,32)	RC_1	RC_2
2311	(3,453)	(28,34)		RC_1, RC_2
2311	(5,275)	(26,36)		RC_1, RC_2
2311	(9,159)	(22,40)		RC_1, RC_2
2311	(15,105)	(16,46)		RC_1, RC_2
2311	(25,79)	(6,56)	RC_1	RC_2
2311	(27,77)	(4,58)	RC_1	RC_2
2311	(1,1223)	(32,34)	RC_1	RC_2
2311	(13,107)	(20,46)	RC_1	RC_2
2311	(1,1087)	(34,36)	RC_1	RC_2
2311	(3,365)	(32,38)	RC_1	RC_2
2311	(1,943)	(36,38)	RC_1	RC_2
2311	(3,317)	(34,40)	RC_1	RC_2
2311	(1,791)	(38,40)	RC_1	RC_2

$Ar'_1 + Ar'_2 = Wd_p$	Dimensions of RC_1	Dimensions of RC_2	Observation	
			Primitive	Non-Primitive
2311	(5,163)	(34,44)	RC_1	RC_2
2311	(1,631)	(40,42)	RC_1	RC_2
2311	(3,213)	(38,44)		RC_1, RC_2
2311	(5,131)	(36,46)	RC_1	RC_2
2311	(7,97)	(34,48)	RC_1	RC_2
2311	(9,79)	(32,50)	RC_1	RC_2
2311	(35,53)	(6,76)	RC_1	RC_2
2311	(15,57)	(26,56)		RC_1, RC_2
2311	(21,51)	(20,62)		RC_1, RC_2
2311	(1,463)	(42,44)	RC_1	RC_2
2311	(3,157)	(40,46)	RC_1	RC_2
2311	(7,73)	(36,50)	RC_1	RC_2
2311	(33,47)	(10,76)	RC_1	RC_2
2311	(11,53)	(32,54)	RC_1	RC_2
2311	(21,43)	(22,64)	RC_1	RC_2
2311	(1,287)	(44,46)	RC_1	RC_2
2311	(11,37)	(34,56)	RC_1	RC_2
2311	(13,35)	(32,58)	RC_1	RC_2
2311	(1,103)	(46,48)	RC_1	RC_2
2311	(3,37)	(44,50)	RC_1	RC_2
2311	(17,23)	(30,64)	RC_1	RC_2

REMARKABLE OBSERVATIONS

- It has been observed that no pair of rectangles can be associated with the one-digit Woodall and Euclid Prime numbers.
- It is noted that all the dimensions of the rectangle RC_1, corresponding to the 2-, 3-, Woodall prime and 2-3-4 digits of Euclid prime numbers, are consistently odd.
- It should be noted that all the dimensions of the rectangle RC_2 are even for 2,3 digits Wd_p and 2,3,4 digits E_p.
- For each value of Wd_p and E_p , RC_2 is non- primitive.

5. CONCLUSION

This study explores the identification of rectangle pairs whose combined areas correspond to Woodall and Euclid prime numbers. Readers are encouraged to investigate alternative rectangle pairings beyond those illustrated in this work.

REFERENCES

1. Carmichael.R.D, The Theory of Numbers and Diophantine Analysis, Dover Publication, New York (1959).
2. Dickson.L.E, History of the theory of Numbers, Vol II, Diophantine Analysis, New York, Dover 2005.
3. John.H, Conway and Richard K.Guy, the Book of Numbers, Springer Verlag, New York, 1995.
4. Mordell.L.J, Diophantine Equations, Academic press, London (1969).
5. Kapoor.J.N, Dhuruva numbers, Fascinating world of Mathematics and mathematical Sciences, Trust Society, Vol.17, 1997.
6. Bert Miller, Nasty numbers, The Mathematics Teacher, No.9, Vol.73,649, 1980.
7. Charles Boum.K, Nasties are primitives, The Mathematics Teacher, No.9, Vol.74,502-504, 1981.
8. Sastry,P,S,N, Jarasandha numbers, The Mathematics Teacher, N0.9, Vol.37, issue 3 and 4200.

9. Gopalan.M.A, Sangeetha. V and Manju Somanath, "Pythagorean triangle and Polygonal numbers", Cayley J. Math, Vol.2 (2), 151-156, 2013
10. Gopalan.M.A, Janaki.G, "Pythagorean triangle with Nasty number as a leg", Journal of Applied Mathematical Analysis and Applications, Vol.4, No. 1-2, 13- 17, 2008.
11. Dr. Mita Darbari, A connection between Hardy-Ramanujan number and Special Pythagorean triangle", Bulletin of Society for Mathematical Services and Standards, Vol.3, No.2, 71-73, 2014.
12. Gopalan.M.A, Vidhyalakshmi.S, Premalatha.E and Presenna.R, "Special Pythagorean triangles and 5-digit Dhuruva numbers, IRJMEIT, Vol.1 (4), 29-33, Aug 2004.
13. Gopalan.M.A, Vidhyalakshmi.S, Premalatha.E and Presenna.R, "Special Pairs Pythagorean triangles and Dhuruva number, Global Journal of Science Frontier Research, Vol.15, p.33-38, 2015.
14. Janaki. G, Vidhya S, " Special pairs of rectangles and sphenic numbers", International Journal for Research in Applied Science & Engineering Technology (IJRASET), Vol. 4(2), 376-378, 2016.
15. Charles Bown. K. Nasties are primitives. The mathematics teacher, 74(9):502–504, 1981.
16. Mita Darbari and Prashans Darbari. Pythagoren triangles with sum of its two legs as dodecic. GSC Advanced Engineering and Technology, 3(1):011–015, Feb 2022.
17. M. A. Gopalan and A. Gnanam. Pythagorean triangles and polygonal numbers.International Journal of Mathematical Sciences,, 9(1-2):211–215, 2010.
18. M. A. Gopalan and G. Janaki. Pythagorean triangle with area/perimeter as a special polygonal number. Bulletin of Pure & Applied sciences, 27(2):393–405, 2008.
19. M. A. Gopalan and G. Janaki. Pythagorean triangle with nasty number as a leg. Journal of applied Mathematical Analysis and Applications, 4(1-2):13–17, 2008.
20. M. A. Gopalan and A. Vijayasankar. Observations on a pythagorean problem. Acta Ciencia Indica, Vol.XXXVI M(No 4):517–520, 2010.
21. G. Janaki and Saranya .C. Special rectangles and jarasandha numbers. Bulletin of Mathematics and Statistics Research, 4(2):63–67, April - June 2016.
22. G. Janaki and Saranya .C. Special pairs of pythagorean triangles and jarasandha numbers. American International Journal of Research in Science, Technology, Engineering & Mathematics, (13):118–120, Dec 2015- Feb 2016.
23. G. Janaki and Saranya .C. Connection between special Pythagoren triangle and jarasandha numbers. International Journal of Multidisciplinary Research & Development, 3(3):236–239, March 2016.
24. G. Janaki and Saranya .C. Special pairs of rectangles and jarasandha numbers. Asian Journal of Science & Technology, 7(5):3015–3017, May 2016.
25. G. Janaki and S. Shanmuga Priya. Special Pairs of rectangles and Wagstaff Primes. Interational Journal of Scientific Research in Engineering and Technology. 08(12):1-5, Dec 2024
26. G. Janaki and S. Shanmuga Priya. Special Pairs Of Pythagorean Triangles And Wagstaff Primes With 2-Digits And 3-Digits. International Research Journal of Modernization in Engineering Technology and Science. 07(02):3842-3854, Feb 2025