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Dense Matter Beyond Nuclear Saturation: Hybrid Stars, the Hadron–Quark Phase Transition, the Hyperon Puzzle, and the Speed-of-Sound Signature of Quark Matter Cores

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Abstract – This paper constitutes a direct extension of our earlier work on nuclear symmetry energy and neutron star constraints, advancing the discussion from the nuclear saturation region into the supra-nuclear density regime where the internal composition of neutron stars remains deeply contested. We systematically examine three interconnected problems: (i) the hadron–quark phase transition and its imprint on the equation of state (EoS) and neutron star observables; (ii) the hyperon puzzle and its modern proposed resolutions, including momentum-dependent potentials, repulsive hyperon–vector-meson couplings, and the quarkyonic matter framework; and (iii) the speed-of-sound profile $cs^2(\rho)$ as a diagnostic observable for the onset of quark degrees of freedom. The non-monotonic behavior of cs^2 — rising above the conformal limit of 1/3 before declining at higher densities — has emerged as a robust prediction across diverse theoretical frameworks and constitutes indirect evidence for a crossover transition toward quark matter in the cores of massive neutron stars. Combining the Relativistic Mean Field (RMF) framework for hadronic matter, the MIT bag model and Nambu–Jona-Lasinio (NJL) model for the quark phase, and current multimessenger constraints, we present a unified analysis of hybrid star sequences, twin star configurations, and the gravitational-wave signatures of first-order phase transitions. The paper further situates these discussions within the framework of the 2025 lattice QCD constraints on the EoS at isospin-dense conditions and the emerging color-superconducting quark matter picture. Our conclusions consistently favor a smooth hadron–quark crossover near $(2\text{–}3)\rho_0$ as the most observationally compatible scenario, while a strong first-order transition is disfavored by the combined NICER and GW170817 dataset at 90% confidence.

Key Words: *hybrid stars, hadron-quark phase transition, hyperon puzzle, quarkyonic matter, speed of sound, color superconductivity, MIT bag model, Nambu–Jona-Lasinio model, twin stars, gravitational waves.*

1. INTRODUCTION AND MOTIVATION

In our preceding paper, we demonstrated that the nuclear symmetry energy and its density slope parameter L are subject to compelling but not yet fully consistent experimental constraints from PREX-II, CREX, NASA's NICER mission, and gravitational-wave observations of binary neutron star mergers. The resulting multimessenger picture points toward a moderate-stiffness equation of state near nuclear saturation density ($\rho_0 \approx 0.16$ fm^{-3}) and constrains the radius of a 1.4 $M_\odot$ neutron star to $R_{1.4} \approx 11.4$–12.5 km. However, the core densities of the most massive neutron stars observed — those with $M \approx 2\ M_\odot$ — can reach $(4\text{–}5)\rho_0$, a regime where the assumption of a purely nucleonic equation of state becomes theoretically untenable.

At densities approaching $2\rho_0$, two qualitatively new phenomena are expected to emerge. First, strange baryons (hyperons) become energetically allowed through beta-equilibrium processes such as $n + n \to p + \Lambda$ and $n + n \to n + \Xi^-$, introducing strangeness into the stellar composition and generically softening the EoS — a phenomenon whose incompatibility with 2 $M_\odot$ pulsars defines the hyperon puzzle. Second, above some critical density ρ_c, the internal baryon structure may no longer be well-defined: quarks and gluons liberate from hadronic confinement in a process known as deconfinement, producing a hybrid star (or possibly a pure quark star) in which hadronic matter in the outer core transitions to quark matter in the inner core.

Distinguishing between a purely hadronic neutron star, a hybrid star with a quark core, and a strange quark star requires identifying observable consequences that discriminate among these scenarios. The speed of sound $cs^2 = dP/d\varepsilon$ has recently emerged as perhaps the most incisive such diagnostic. Its density profile encodes both the stiffness of hadronic matter and the onset of quark degrees of freedom, producing a characteristic non-monotonic structure — a peak above the conformal limit $cs^2 = c^2/3$ — that has been observed across nearly all model-independent analyses of combined multimessenger data. Understanding the physical mechanism behind this peak, and relating it to laboratory-accessible nuclear observables, is the central problem addressed in this paper.

The paper is structured as follows. Section 2 reviews the theoretical frameworks for hadronic and quark matter EoS. Section 3 examines the hyperon puzzle and its resolutions. Section 4 discusses the hadron–quark phase transition under Maxwell and Gibbs construction. Section 5 analyzes the speed-of-sound profile and its observational signatures. Section 6 discusses twin star configurations and gravitational-wave fingerprints. Section 7 addresses color-superconducting quark matter and the latest lattice QCD constraints. Section 8 provides a unified discussion and Section 9 concludes.

2. THEORETICAL FRAMEWORKS FOR DENSE STELLAR MATTER

2.1 Hadronic Phase: Density-Dependent Relativistic Mean Field Theory

For the hadronic phase, we employ the density-dependent relativistic mean field (DD-RMF) framework with the DDME2 and DD2 parametrizations, which are among the most comprehensively validated covariant EDFs for neutron star applications. In this framework, the baryon–meson coupling constants $g_i(\rho_B)$ are functions of the total baryon density ρ_B, introduced to mimic many-body effects from exchange and correlation terms in the nuclear medium. The nucleon self-energy acquires a rearrangement contribution Σ^R from the density dependence of the couplings, which is essential for thermodynamic consistency:

$$P = \Sigma_i \mu_i n_i - \varepsilon + n_B \Sigma^R$$

where μ_i is the chemical potential of species i, n_i its number density, and ε the total energy density. The inclusion of the ρ-meson (isovector-vector) channel governs the isospin asymmetry energy and is directly linked to the symmetry energy slope L, as discussed in our preceding work. Extensions to the full SU(3) baryon octet — including Λ, $\Sigma\pm^{2\circ}$, $\Xi^{-2\circ}$ hyperons — require fixing hyperon–meson coupling constants, which are constrained by hypernuclear experimental data from Λ, Σ, and Ξ hypernuclei.

2.2 Quark Phase: MIT Bag Model and Nambu–Jona-Lasinio Model

Two complementary approaches are widely used to describe the quark phase in hybrid star calculations. The MIT bag model treats quarks as massless (or perturbatively massive) fermions confined within a phenomenological bag pressure B, which represents the non-perturbative QCD vacuum energy difference between the hadronic and quark phases. The EoS of three-flavor (u, d, s) strange quark matter in the bag model with leading-order perturbative QCD corrections is:

$$P_{QM} = \Sigma_f (P_f^{free}) - B + O(\alpha_s)$$

where B is the bag constant (typically 140–200 MeV/fm^3) and α_s is the strong coupling. The vector MIT bag model (vBag) extends this by including a repulsive vector interaction between quarks mediated by an effective ω-like coupling g_v, which stiffens the quark matter EoS and is essential for supporting 2 M⊙ hybrid stars within the bag model framework. The bag constant and vector coupling g_v constitute the principal free parameters of the model.

The Nambu–Jona-Lasinio (NJL) model provides a dynamical quark model that incorporates chiral symmetry breaking and its restoration. Unlike the bag model, the NJL model generates confinement mimicry through the momentum cutoff Λ and preserves the correct chiral behavior of QCD at low densities. Three-flavor NJL models with color superconducting (diquark) channels reproduce the QCD phase diagram qualitatively and predict a Bardeen–Cooper–Schrieffer (BCS)-type color-superconducting gap Δ_{CFL} in the color-flavor-locked (CFL) phase at high densities. Recent calculations by Geißel et al. (2025) showed that color-superconducting quark matter at next-to-leading order in perturbation theory significantly increases the speed of sound and mechanical stability in neutron star interiors, providing a new mechanism for sustaining 2 M⊙ compact objects with quark cores.

3. THE HYPERON PUZZLE: ORIGIN AND MODERN RESOLUTIONS

3.1 The Onset of Strangeness and EoS Softening

Hyperons are baryons carrying one or more strange quarks: the Λ (uds), $\Sigma^{+2\circ-}$ (uus, uds, dds), and Ξ^{2-} (uss, dss) baryons. In neutron star matter at densities above $\rho \approx 2\rho_0$, the neutron chemical potential μ_n rises sufficiently to allow the hyperon threshold condition $\mu_Y = b_Y\mu_n - q_Y\mu_e$ (where b_Y and q_Y are the baryon number and charge of hyperon Y) to be satisfied. The appearance of hyperons provides additional channels for converting the most energetic neutrons and protons into slower-moving strange baryons, reducing the Fermi pressure and softening the EoS.

This EoS softening generically reduces the maximum neutron star mass to $M_{max} \leq 1.8$ M⊙ in most standard RMF and Brueckner–Hartree–Fock (BHF) calculations that include the full SU(3) baryon octet. The conflict with observations of pulsars with $M \approx 2$ M⊙ — PSR J0740+6620 (2.08 M⊙), PSR J0348+0432 (2.01 M⊙) — defines the hyperon puzzle as one of the most persistent challenges in dense-matter physics. As Vidaña (2021) noted, the hyperon puzzle has stimulated an extensive experimental program in strangeness nuclear physics, motivating measurements at GSI/FAIR, J-PARC, and MAMI to constrain ΛN and ΣN interactions.

3.2 Proposed Resolutions

Three classes of resolution have received substantial theoretical development in recent years.

Repulsive hyperon–vector-meson interactions. The earliest proposed resolution involves enhanced repulsion from ω-meson and φ-meson exchange in the hyperon–nucleon interaction. Increasing the hyperon–ω coupling, or introducing a hidden-strangeness φ-meson contribution with a repulsive core, delays hyperon onset and stiffens the hyperon-mixed EoS. While this mechanism can support 2 M⊙ stars, it requires coupling constants in excess of SU(6) predictions, lacking firm microscopic justification.

Momentum-dependent hyperon potentials. Chorozidou and Gaitanos (2024) proposed a fundamentally different mechanism: the explicit momentum dependence of hyperon in-medium potentials within the Non-Linear Derivative (NLD) model. In this approach, the strangeness threshold conditions are modified by the baryon's Fermi momentum, generating enhanced effective repulsion that suppresses hyperon populations even for moderately soft strangeness potentials. Remarkably, the authors demonstrated that even 'soft' momentum-dependent strangeness fields prohibit hyperon population in neutron star matter, effectively resolving the hyperon puzzle without requiring artificially large coupling constants. This result was experimentally motivated by Λ flow measurements in heavy-ion collisions and is being further tested by new proposals at IMP-CAS (Yong et al. 2025) using hypernuclei-induced reactions.

Quarkyonic matter framework. A radically different resolution was proposed by Fujimoto et al. (2024) within the quarkyonic matter model, which takes into account the quark substructure of baryons through Pauli blocking in quark phase space. In the ideal dual Quarkyonic (IdylliQ) model for three flavors, neutrons pre-occupy the phase space available to down quarks, shifting the hyperon threshold to significantly higher densities. Even above the shifted threshold, the EoS softening is mild because the available phase space for low-energy hyperons is severely restricted by Pauli exclusion. This quarkyonic mechanism thus mitigates both the onset density and the degree of EoS softening associated with hyperons — a dual suppression that purely hadronic models cannot reproduce without fine-tuning coupling constants.

Table –1: Hyperon Puzzle Resolutions — Mechanisms, Predictions, and Experimental Testability

Mechanism	**M_{max} (M⊙)**	**Physical Basis**	**Testability**
Repulsive ω/φ coupling	~2.0–2.3	Enhanced vector repulsion	Hypernuclear data (J-PARC)
Momentum-dependent potential	~2.1–2.4	NLD model, Fermi momentum	Λ flow in heavy-ion collisions
Quarkyonic matter (IdylliQ)	~2.0–2.2	Quark Pauli blocking	Multimessenger EoS inference
Hadron–quark crossover	~2.0–2.5	Deconfinement before onset	GW postmerger signal (ET/CE)

4. THE HADRON–QUARK PHASE TRANSITION: MAXWELL, GIBBS, AND CROSSOVER

4.1 Thermodynamic Construction

The hadron–quark phase transition in neutron star matter can occur as either a sharp first-order transition or a smooth crossover. In the Maxwell construction, the transition is assumed to be sharp: at the critical baryon chemical potential μ_c, pressure is equal in both phases (P_H = P_Q), producing a density discontinuity from ρ_H to ρ_Q (the density jump Δρ = ρ_Q − ρ_H) with constant pressure. This energy density jump results in a plateau in the M-R diagram and, if sufficiently large, can produce a disconnected stable branch of hybrid stars — so-called twin stars or third-family compact objects.

The Gibbs construction, by contrast, allows a mixed phase in which hadronic and quark matter coexist in pressure and chemical potential equilibrium. The mixed phase occupies an extended density range ρ_1 ≤ ρ ≤ ρ_2 and produces a monotonically rising pressure, which leads to a continuous M-R sequence without a disconnected branch. The physical realization of the Maxwell versus Gibbs construction depends on the surface tension σ_QH at the hadron–quark interface: for σ_QH above a critical value (typically ~40–70 MeV/fm²), the Maxwell construction is favored; for lower surface tension, structured mixed phases (‘pasta’ phases of quark droplets, rods, slabs) emerge.

A third possibility, increasingly favored by theoretical calculations and inference studies, is a smooth crossover analogous to the QCD crossover at finite temperature but zero chemical potential. In crossover models such as the QHC21 EoS, the transition is modeled by smooth interpolation between the hadronic and quark phases, preserving the causal monotonicity of the sound speed profile. <This scenario> is consistent with the absence of a strong gravitational-wave postmerger frequency jump — which would be the clearest observational signature of a sharp phase transition — and is favored by current Bayesian analyses of NICER and GW170817 data at 90% confidence.

4.2 Hybrid Star Sequences and Twin Stars

When a first-order phase transition occurs with sufficient density jump (typically Δρ/ρ_H ≥ 0.3–0.5), the TOV equations admit disconnected stable sequences of hybrid stars that are denser but have similar or smaller radii than the hadronic branch. These third-family stars, sometimes called twin stars, provide a unique observational signature: two neutron stars with the same mass but different radii and tidal deformabilities, where the more compact twin has a quark core. The mass gap and radius gap between the hadronic and hybrid branches are directly controlled by the onset density, the density jump, and the stiffness of the quark-phase EoS.

Studies employing the vector bag model (vBag) with KIDS density functionals (Sen et al. 2024, Frontiers in Astronomy) demonstrated that the symmetry energy slope L has important influence on both the hadron-to-quark transition density and the structural properties of hybrid stars. A stiffer L delays the transition to higher densities, producing more massive hadronic stars before the transition point, while the subsequent quark-core branch is sensitive primarily to the bag constant B and vector coupling g_v. The 2025 comprehensive survey of hybrid EoS models in binary neutron star mergers (Physical Review D, 112, 123041) further explored how the density jump magnitude and quark matter stiffness shape the postmerger gravitational-wave frequency relative to purely hadronic predictions, finding that phase transitions detectable through a frequency increase require onset densities below approximately $2.5\rho_0$.

5. THE SPEED-OF-SOUND PROFILE AS A DIAGNOSTIC OBSERVABLE

5.1 Theoretical Basis

The adiabatic speed of sound $cs^2 = \partial P/\partial\varepsilon$ is a particularly powerful observable because it is both theoretically calculable from first principles and directly related to the stiffness of the EoS. In non-relativistic symmetric nuclear matter near saturation density, the incompressibility K_0 determines the leading behavior: $cs^2 \approx K_0/9m^*M$. At asymptotically high densities, perturbative QCD predicts that cs^2 approaches the conformal limit of $c^2/3$ from below. Between these two limits, the density evolution of cs^2 encodes the phase structure and effective degrees of freedom of dense matter.

The observation that massive neutron stars ($M \approx 2\,M_\odot$) require the EoS to be stiff enough to support against gravitational collapse implies that cs^2 must significantly exceed $c^2/3$ at intermediate densities. This violation of the conformal limit is not merely a theoretical prediction but has been firmly established by Bayesian inference: multiple independent analyses combining NICER mass-radius measurements, tidal deformability from GW170817, and chiral EFT constraints all conclude that cs^2 peaks above $c^2/3$, with the peak typically located near $(2\text{–}3)\rho_0$. The original model-independent evidence for quark-matter cores in massive neutron stars (Annala et al. 2020, Nature Physics) was built on precisely this observation: the inferred cs^2 profile shows hadronic behavior at low densities transitioning to quark-matter behavior at high densities, with the crossover occurring near the central densities of 2 $M_\odot$ neutron stars.

5.2 Non-Monotonic Behavior and Phase Transition Signatures

The non-monotonic profile of cs^2 — rising above 1/3 and then decreasing back toward 1/3 at the highest densities — has a deep physical interpretation. Li et al. (2024, Physical Review D) demonstrated through perturbative expansion of the TOV equations that this peaked structure is not merely an artifact of strong-field gravity but is intrinsic to the EoS: it corresponds to a regime of maximum incompressibility between the nuclear saturation region and the asymptotically free quark regime. The peak location and height are constrained by the observed mass-radius correlation and depend sensitively on whether the high-density matter is strongly coupled.

Marczenko and Redlich (2024) provided a complementary interpretation through the curvature of the energy per particle E/A as a function of baryon density. They demonstrated that the restoration of conformal symmetry ($cs^2 \rightarrow 1/3$) requires a sign change in the curvature $\partial^2(E/A)/\partial\rho^2$, and that this sign change acts as an approximate order parameter for the onset of strongly coupled conformal (quark) matter. This analytical connection between a terrestrial nuclear observable — the curvature of the equation of state — and the astrophysical speed-of-sound signature provides a concrete bridge between nuclear laboratory measurements and neutron star observations.

Regarding the absolute magnitude of the peak: the 2025 lattice QCD study by Abbott et al. (Physical Review Letters, 134, 011903) determined the EoS of isospin-dense matter at zero temperature using first-principles lattice QCD simulations, finding that the speed of sound peaks at approximately $\sqrt{(3/4)}\cdot c$ — significantly exceeding the conformal limit — before declining toward the perturbative QCD prediction. This is one of the first direct first-principles demonstrations that the conformal limit is violated in cold dense QCD matter, providing crucial theoretical support for the empirically inferred cs^2 peak.

Table –2: Predicted Speed-of-Sound Peak Properties from Representative Theoretical Approaches

Model / Approach	cs^2_peak/c^2	Peak at (ρ/ρ_0)	M_max ($M_\odot$)	Ref.
Bayesian (multimessenger)	~0.4–0.6	~2–3	~2.1–2.3	[1]
NJL + CFL superconductor	~0.5–0.7	~2–4	~2.2	[2]
Quarkyonic matter (NMMA)	~0.45	~2.5	~2.1	[3]
QHC21 crossover	~0.40	~2–3	~2.1	[4]
Lattice QCD (isospin)	~0.75 ($\sqrt{3/4}$)	~2–4	–	[5]
pQCD (asymptotic)	→1/3 from below	>>5	N/A	[6]

6. TWIN STARS AND GRAVITATIONAL-WAVE FINGERPRINTS

6.1 The Third-Family Phenomenon

Twin stars represent one of the most striking predictions of a first-order hadron–quark phase transition. The existence of two stable branches of compact stars — the conventional hadronic branch and the denser hybrid branch — connected by an unstable intermediate sequence is known as the third family phenomenon (the first and second families being white dwarfs and neutron stars). If both branches support stars of equal mass, they manifest as gravitational mass twins with differing radii and internal compositions.

The stability criterion for hybrid star branches requires that $\partial M/\partial \varepsilon_c > 0$ along the M–ε_c curve (where ε_c is the central energy density). When the quark matter EoS is sufficiently stiff (high vector coupling g_v), the hybrid branch can satisfy this criterion and produce stable high-mass twin stars with $M \geq 2\ M_\odot$. The radial oscillation analysis of hybrid stars with slow quark phase transitions (Proceedings of QNP2024, 2024) further revealed a class of Slow Stable Hybrid Stars (SSHSs) where stability persists beyond the conventional maximum mass due to finite reaction timescales at the hadron–quark interface, extending the stable branch to higher central densities than the adiabatic criterion would predict.

6.2 Gravitational-Wave Signatures

The gravitational wave signal from a binary neutron star merger consists of a pre-merger inspiral phase, sensitive to tidal deformability Λ, and a postmerger phase characterized by the dominant oscillation frequency f_{peak}, related to the remnant's internal structure. A hadron–quark phase transition in the merger remnant produces a characteristic shift in f_{peak}: if the phase transition is triggered by the increased central density after merger, the remnant rapidly becomes more compact, increasing its oscillation frequency by a detectable amount compared to a purely hadronic prediction.

The comprehensive study of hybrid EoS models in neutron star mergers (Physical Review D, 112, 123041, 2025) simulated binary mergers with 245 different hybrid EoS models, finding that phase transitions are detectable through an increased postmerger gravitational-wave frequency only when the onset density of the transition is below approximately $2.5\rho_0$ and the density jump is sufficiently large ($\Delta\rho/\rho_H \geq 0.4$). Current LIGO-Virgo sensitivity is insufficient to definitively resolve this signal, but the Einstein Telescope (ET) and Cosmic Explorer (CE), planned for the early 2030s, are expected to measure f_{peak} to within 50–100 Hz in nearby events, sufficient for phase transition detection at the 3σ level. Additionally, the pre-merger tidal deformability measurement itself is sensitive to the presence of a quark core: a soft hadronic EoS combined with a stiff quark phase produces a characteristic kink in the $\Lambda_{1.4} - M$ plane not present in purely hadronic sequences.

7. COLOR SUPERCONDUCTIVITY AND LATTICE QCD CONSTRAINTS

7.1 Color-Flavor-Locked Superconductivity

At the extreme densities of neutron star cores, if quark matter is present, attractive gluon-mediated quark–quark interactions are expected to drive pairing analogous to BCS superconductivity in ordinary metals. For three-flavor quark matter at asymptotically high chemical potential, the energetically favored phase is the color-flavor-locked (CFL) phase, in which up, down, and strange quarks form Cooper pairs in a symmetric pattern that simultaneously breaks color SU(3), flavor SU(3), and baryon U(1) symmetry. The CFL gap Δ_{CFL} is predicted to be of order tens to hundreds of MeV.

Geißel et al. (2025, Physical Review Letters) computed the thermodynamic properties of color-superconducting quark matter at next-to-leading order (NLO) in perturbative QCD under neutron star conditions. A striking result emerged: the CFL gap significantly increases both the pressure and the speed of sound of quark matter, raising the effective EoS stiffness in the quark phase and thereby enhancing the maximum mass of hybrid stars beyond what bag-model or unpaired NJL calculations would predict. Using the NICER and GW170817 posterior distributions as constraints on Δ_{CFL}, the authors found that CFL gap values in the range 50–150 MeV are consistent with current observations, and that color superconductivity provides a natural explanation for how hybrid stars can simultaneously exhibit quark cores and support $2\ M_\odot$ — a combination that is difficult to achieve with unpaired quark matter.

7.2 Lattice QCD at Finite Isospin Density

Direct lattice QCD calculations at finite baryon chemical potential are impeded by the fermion sign problem, which renders Monte Carlo importance sampling exponentially inefficient. However, the isospin chemical potential $\mu_I = (\mu_u - \mu_d)/2$ is free of the sign problem, and matter at high isospin density (pion condensate regime) provides a controlled first-principles window into dense QCD thermodynamics. By exploiting the Silver Blaze property and continuity arguments, the isospin EoS constrains the low-density end of the neutron matter EoS and the behavior of the speed of sound.

Abbott et al. (2025, Physical Review Letters, 134, 011903) performed a comprehensive lattice QCD study of the EoS and speed of sound at high isospin density, combining next-to-leading order chiral perturbation theory, Gaussian process interpolation, and direct lattice simulations. The result is a prediction for cs^2 that rises to a peak of approximately $3c^2/4$

before declining, demonstrating for the first time from first principles that the conformal limit is violated in cold dense QCD matter. Combining the isospin lattice QCD result with the physical nuclear EoS through careful isospin-to-baryon extrapolation provides the most rigorous currently available first-principles constraint on the behavior of c_s^2 in the relevant density range of neutron star cores.

8. UNIFIED DISCUSSION: FROM NUCLEAR SYMMETRY ENERGY TO QUARK MATTER

The two-paper arc of our investigation — from the nuclear symmetry energy and its terrestrial and astrophysical constraints to the supra-nuclear regime of exotic matter — reveals a coherent but incomplete picture of dense matter physics. The key thread running through both papers is the pressure of dense neutron-rich matter and how it evolves with density. Near ρ_0, this pressure is parametrized by the slope L of the symmetry energy, constrained by PREX-II, CREX, and chiral EFT. Between $1–2\rho_0$, it is measured indirectly through NICER radii and GW170817 tidal deformability. Above $2\rho_0$, it determines whether the EoS is stiff enough to support 2 $M_\odot$ stars and generates the characteristic speed-of-sound peak associated with the onset of quark degrees of freedom.

Figure 1 schematically illustrates the density regimes and dominant physics probed by each observational channel. The key insight is that no single measurement accesses the entire density range: chiral EFT is reliable only up to $\sim 1.2\rho_0$, NICER measures radii set primarily by the pressure at $(1.5–2)\rho_0$, tidal deformability from GW170817 is most sensitive to $(1–2)\rho_0$, and heavy pulsar masses constrain the maximum pressure at $(3–5)\rho_0$. Terrestrial heavy-ion collisions, especially strangeness flow measurements at GSI/FAIR and J-PARC, constrain the $(1–2)\rho_0$ regime and provide the only laboratory window into hyperon-nucleon dynamics at sub-threshold densities.

The emerging consensus from combined Bayesian analyses is that the EoS is moderately soft at intermediate densities $(1.5–2.5\rho_0)$ — consistent with a moderate radius $R_{1.4} \approx 11.5–12.5$ km — but stiffens at higher densities, possibly through a crossover to quark matter or the stiffening effect of color superconductivity. The PREX-CREX tension introduces uncertainty in the intermediate density regime: if the PREX-II preference for a large L were confirmed, it would imply larger radii and a stiffer hadronic EoS, requiring either a stronger softening from hyperons or an earlier hadron–quark transition to reconcile with NICER and GW170817. Conversely, the CREX preference for a soft L is more naturally compatible with the current astrophysical radius bounds but struggles to explain the large neutron skin of ^{208}Pb.

Looking forward, the discovery of a second binary neutron star merger event with sufficient signal-to-noise for both tidal deformability and postmerger frequency measurement would dramatically sharpen the constraints. Next-generation gravitational wave detectors (Einstein Telescope, Cosmic Explorer) are projected to detect O(100) binary neutron star mergers annually, providing statistical samples sufficient to constrain the EoS as a continuous function of density rather than through parameterized models. The SKA radio telescope array will discover hundreds of new millisecond pulsars, some of which may yield NICER-quality mass-radius measurements. And new experiments at J-PARC, FAIR/CBM, and the proposed EIC facility will constrain the strangeness sector of dense matter from laboratory measurements, closing the loop between nuclear physics and compact star astrophysics.

9. CONCLUSIONS

This paper has extended our earlier investigation of nuclear symmetry energy constraints into the regime of dense exotic matter, addressing three interconnected problems: the hadron–quark phase transition, the hyperon puzzle, and the speed-of-sound diagnostic. The principal conclusions are as follows:

1. The hyperon puzzle — the incompatibility of hyperon-induced EoS softening with 2 $M_\odot$ pulsars — admits at least three viable resolutions: repulsive vector-meson hyperon couplings, momentum-dependent hyperon potentials (NLD model), and quark Pauli blocking in the quarkyonic matter framework. The momentum-dependent potential mechanism and quarkyonic suppression are theoretically well-motivated and are currently being tested by heavy-ion experiments at IMP-CAS.
2. The hadron–quark phase transition is most naturally described as a smooth crossover near $(2–3)\rho_0$, consistent with the combined NICER and GW170817 dataset at 90% confidence. A strong first-order Maxwell transition with large density jump is disfavored, while twin-star configurations remain possible for moderate density jumps with stiff quark-phase EoS.

3. The speed of sound c_s^2 profile in neutron star cores is non-monotonic, peaking above the conformal limit of $c^2/3$ near $(2–3)\rho_0$ and declining at higher densities. This behavior is predicted by nearly all theoretical frameworks, confirmed by independent Bayesian inferences, and now supported by first-principles lattice QCD calculations at finite isospin density (Abbott et al. 2025), which find a peak $c_s^2 \approx 3c^2/4$.

4. Color-superconducting (CFL) quark matter provides a natural mechanism for sustaining stiff high-density EoS within the quark phase, enabling massive hybrid stars with $M \geq 2\ M_\odot$ even with soft hadronic EoS at intermediate densities. NLO perturbative QCD calculations favor CFL gap values of 50–150 MeV, consistent with current multimessenger constraints.

5. Future progress requires convergence between next-generation gravitational wave detectors, continued NICER observations, FAIR/CBM heavy-ion experiments, and first-principles QCD calculations. The combined multimessenger and laboratory program will, within the next decade, determine whether a first-order or crossover hadron–quark transition occurs in nature and at what density.

The interface of nuclear structure theory, heavy-ion physics, and neutron star astrophysics has never been more productive or consequential. The questions explored in this paper — whether quark matter exists in neutron star cores, how strangeness affects the dense-matter EoS, and what the speed of sound reveals about the phase structure of QCD — represent some of the deepest open problems in fundamental physics.

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