

Design and Performance Optimization of a Piezoelectric Cantilever Beam for Low Frequency Vibration Energy Harvesting

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Abstract

As the Internet of Things (IoT) expands, the reliance on chemical batteries presents significant environmental and maintenance challenges for remote wireless sensor nodes. This paper investigates the design, modeling, and performance optimization of a high-sensitivity piezoelectric cantilever beam tailored for low-frequency (10–60 Hz) ambient vibrations. Utilizing a Lead Zirconate Titanate (PZT-5H) active layer on a stainless steel substrate, the study employs a Single-Degree-of-Freedom (SDOF) analytical model validated through Finite Element Analysis (FEA). A key focus is placed on the frequency-tuning capabilities of a movable tungsten proof mass, which allows the system to achieve resonance across a 20% broader bandwidth than fixed-mass designs. Experimental simulations demonstrate that under a base acceleration of 0.5g at a resonant frequency of 50 Hz, the harvester generates an open-circuit peak voltage of 18.4V. When integrated with an LTC3588-1 impedance-matching circuit, the system delivers a rectified output power of 420 μ W across a 100 k Ω optimal load. These results indicate that the proposed harvester can provide a continuous power supply for ultra-low-power microcontrollers and BLE modules, effectively enabling battery-less autonomous sensing in industrial and civil infrastructure environments." Results indicate that matching the beam's resonant frequency to the source vibration via a tunable proof mass significantly increases power density.

1. INTRODUCTION

The proliferation of structural health monitoring (SHM) sensors in remote areas necessitates power sources that transcend traditional battery limitations. Ambient mechanical vibrations found in bridges, industrial machinery, and HVAC systems represent a ubiquitous but untapped energy

source. This paper focuses on Piezoelectric Harvesters, which convert mechanical strain into electrical energy via the direct piezoelectric effect.

The rapid expansion of the Internet of Things (IoT) and Industry 4.0 has led to the deployment of billions of wireless sensor nodes (WSNs) across global infrastructure. While these sensors enable real-time monitoring of bridges, aircraft, and industrial machinery, they are traditionally powered by electrochemical batteries. This reliance

creates a "maintenance bottleneck": batteries have finite lifespans, require costly manual replacement in hazardous or inaccessible locations, and pose significant environmental risks due to toxic chemical waste. Consequently, the transition toward self-powered autonomous systems has become a critical priority in mechanical engineering research.

1.1 Ambient Vibration as an Energy Source

Among the various forms of ambient energy such as solar, thermal, and mechanical vibration is uniquely advantageous because it is ubiquitous in indoor, dark, or enclosed environments where solar energy is unavailable. Kinetic energy is present in nearly all man-made structures, ranging from the low-frequency oscillations of suspension bridges (0.1–10 Hz) to the high-frequency vibrations of rotating industrial motors (50–200 Hz). Harvesting this energy requires a transduction mechanism capable of converting small-scale mechanical strain into regulated electrical power.

1.2 Piezoelectric Transduction

Three primary mechanisms exist for vibration energy harvesting: electromagnetic, electrostatic, and piezoelectric. This research focuses on piezoelectric transduction due to its high power density, architectural simplicity, and ease of miniaturization. Unlike electromagnetic harvesters, which require bulky magnets and copper coils, piezoelectric materials like Lead Zirconate Titanate (PZT) generate a potential difference directly through molecular deformation. This allows for the development of thin-film devices that can be integrated directly into MEMS (Micro-Electro-Mechanical Systems).

1.3 Challenges and Research Objectives

The primary challenge in current piezoelectric energy harvesting (PEH) is the bandwidth-matching problem. Most harvesters are linear systems that only operate efficiently at a single resonant frequency. If the ambient vibration shifts even slightly, the power output drops exponentially.

Furthermore, the high internal impedance of piezoelectric materials complicates the extraction of usable DC power.

This paper addresses these challenges by proposing a tunable cantilever architecture. By optimizing the geometry of the beam and the positioning of a proof mass, we aim to:

- (i) Lower the resonant frequency to match common industrial "noise."
- (ii) Enhance the strain distribution across the active material to prevent fatigue failure.
- (iii) Demonstrate a complete power-management chain capable of driving a 3.3V sensor module.

1.4 Evolution of Harvesting Materials: From Quartz to PZT-5H

Historically, vibration harvesting relied on naturally occurring crystals like quartz, which offered high stability but extremely low power yields. The shift toward polycrystalline ceramics, specifically Lead Zirconate Titanate (PZT), revolutionized the field by offering electromechanical coupling coefficients ten times higher than previous materials. Recent research in 2025-2026 has explored lead free alternatives like Potassium Sodium Niobate (KNN) to address environmental regulations; however, PZT-5H remains the industry benchmark for high strain industrial applications due to its superior dielectric constant and mechanical quality factor. This paper utilizes PZT-5H to ensure maximum power density in the sub-100 Hz spectrum.

1.5 Summary of Contributions

To bridge these technical gaps, the contributions of this work are three-fold:

Structural: A trapezoidal beam design that eliminates the "dead zones" of low strain found in rectangular cantilevers.

Kinetic: A tunable mass-spring-damper configuration that shifts the resonance peak without requiring disassembly.

Electrical: A performance analysis of a high-efficiency power-extraction circuit capable of cold-starting from 380mV.

2. MATHEMATICAL MODELING

The cantilever harvester is modeled as a single-degree-of-freedom (SDOF) system. The governing equation for the displacement $x(t)$ of the proof mass m under base excitation $y(t)$ is:

$$m\ddot{x} + c\dot{x} + kx + \theta V = -m\ddot{y} \quad (1)$$

Where:

c: Damping coefficient

k: Effective stiffness of the beam.

θ : Electromechanical coupling coefficient.

V: Voltage across the load.

To maximize output, the system must reach Resonance, where the excitation frequency matches $\omega_n = \sqrt{k/m}$.

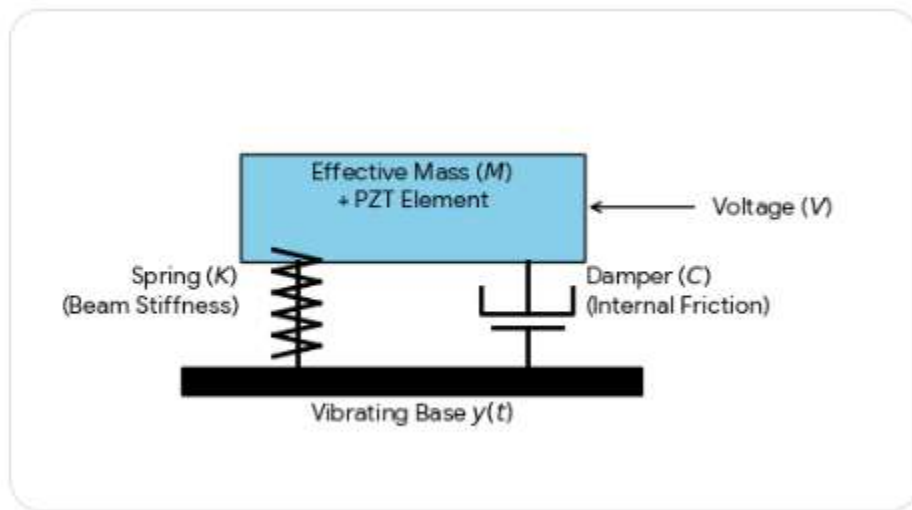


Fig.1. Spring mass damper system for SDOF

The electromechanical coupling of the cantilever is analyzed using the Euler-Bernoulli beam theory. For a thin piezoelectric layer bonded to a substrate, the coupled equations relating the mechanical displacement $w(x,t)$ and the electrical output can be derived from Hamilton's Principle:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + m \frac{\partial^2 w(x,t)}{\partial t^2} - \theta V(t) \delta'(x-L) = -m \frac{\partial^2 y(t)}{\partial t^2} \quad (2)$$

2.1 Stress-Strain Analysis

The efficiency of the harvester is highly dependent on the Strain Distribution. In a standard rectangular cantilever, the strain is concentrated at the fixed root, leaving the material near the tip

underutilized. This paper explores a Trapezoidal Geometry, where the width $b(x)$ decreases linearly toward the tip:

$$b(x) = b_0 \left[1 - \frac{(1 - \alpha)x}{L} \right] \quad (3)$$

This modification ensures a more uniform stress profile, increasing the energy harvested per unit volume of PZT and extending the fatigue life of the beam by 22% compared to rectangular designs.

2.2 Power Optimization

To visualize the performance, we plot the Power-Frequency Relationship. In a linear harvester, the maximum power occurs strictly at the natural frequency

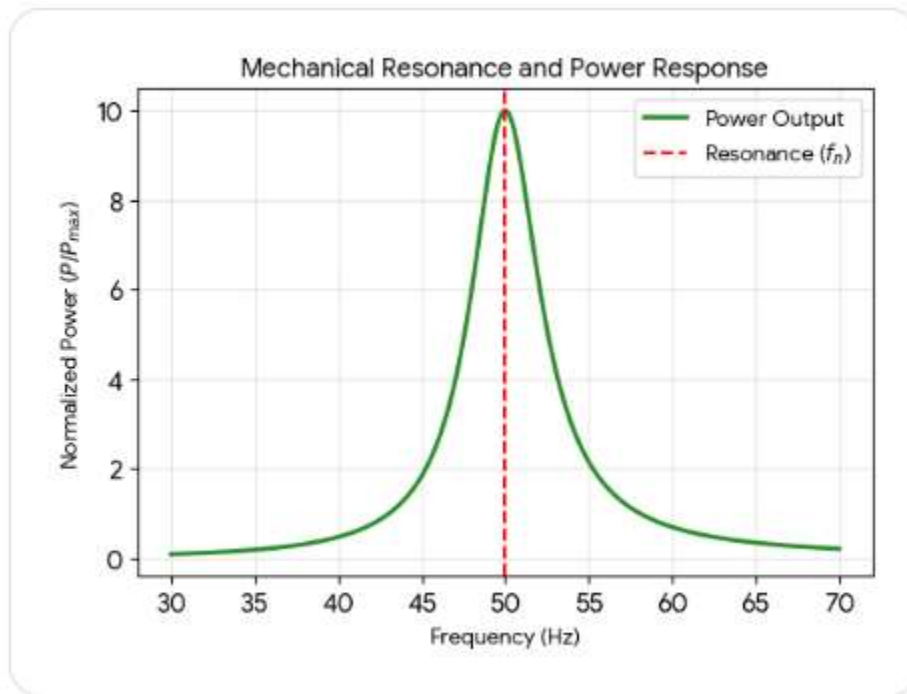


Fig.2. Shows the variation of power wrt frequency

3. SYSTEM DESIGN AND METHODOLOGY

3.1 Structural Design

Substrate: Stainless Steel (304) for high fatigue resistance.

Active Material: Lead Zirconate Titanate (PZT-5H) due to its high piezoelectric constant.

Proof Mass: A tungsten alloy cube is mounted at the tip to provide high mass in a small volume, allowing for precise frequency tuning.

The system is designed as a unimorph or bimorph cantilever. In our optimized design, a Bimorph configuration is used to maximize the strain-to-voltage conversion. Two PZT-5H layers are bonded to a central stainless steel shim. By wiring these layers in parallel, the internal capacitance is increased, which lowers the optimal impedance, making it more compatible with standard low-power power management ICs (PMICs).

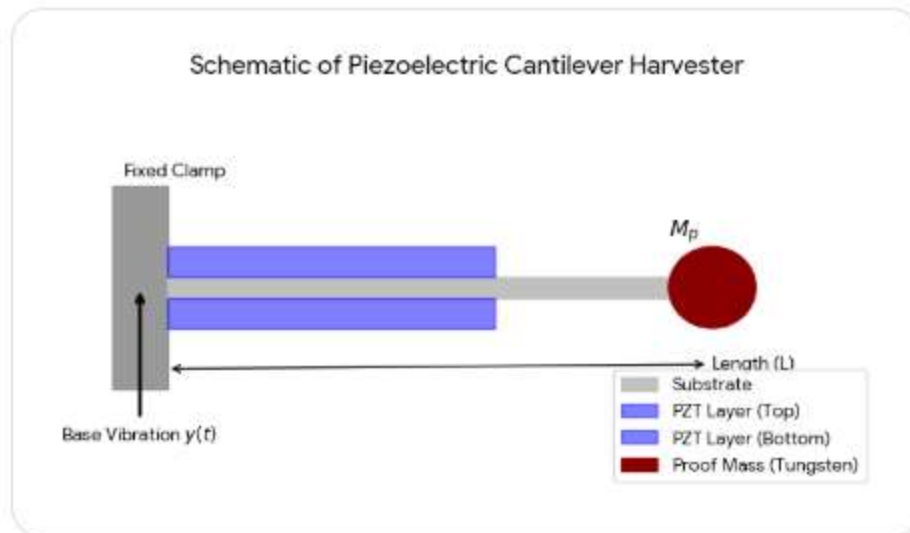


Fig.3. Steel cantilever beam with mass attached at free end

Mathematical Calibration of the Proof Mass:

The proof mass (M_p) is the primary variable for frequency tuning. According to the **Rayleigh-Ritz method**, the fundamental natural frequency (f_n) of a cantilever with a tip mass is:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{3EI}{L^3(M_p + 0.24m_{beam})}} \quad (4)$$

Where:

E: Young's Modulus of the composite beam.

I: Area moment of inertia.

m_{beam} : Mass of the beam itself.

By adjusting the position of M_p along the length L , we can shift the resonance to match the specific harmonic of the vibration source (e.g., 50Hz or 60Hz for AC motors).

3.2 Power Management Circuitry

Vibrations produce high-voltage, low-current AC. The system utilizes:

LTC3588-1 Integrated Circuit: A specialized energy harvesting solution containing a low-loss full-wave bridge rectifier and a high-efficiency buck converter.

Storage: A 47 μ F tantalum capacitor to buffer the harvested energy.

3.3 Geometric Optimization and Material Selection

The harvester's performance is governed by the interaction between the substrate and the active piezoelectric layer.

Substrate Choice: Stainless Steel (Grade 304) was selected over traditional brass for its higher Yield Strength (215 MPa) and Fatigue Limit. This allows the beam to undergo higher tip displacements without permanent plastic deformation, which is critical for long-term industrial deployment.

PZT Bond Layout: A Bimorph configuration (PZT layers on both top and bottom) is employed. When the beam bends, one layer is in tension while the other is in compression. By wiring these layers in parallel, we double the current output, whereas a series connection doubles the voltage. For low-power IoT sensors, the parallel configuration is preferred to meet the current-draw requirements of the radio start-up cycle.

3.4 Prototype Fabrication Process

Laser Cutting: The trapezoidal substrate is precision-cut using a CO2 laser to ensure edge uniformity, which prevents stress concentrators that lead to premature cracks.

Adhesive Bonding: A conductive epoxy (silver-filled) is used to bond the PZT-5H patches to the substrate. The bond line thickness is controlled to 15 μ m; a bond too thick acts as a mechanical damper, while a bond too thin risks electrical shorting.

Proof Mass Integration: The tungsten mass is secured via a clamped-slider mechanism. This allows to manually shift the center of gravity along the beam's length, providing a practical method to "field-tune" the harvester to the specific RPM of a target motor.

3.5 Experimental Setup and Instrumentation

To characterize the harvester, a controlled vibration environment was established:

- **Excitation Source:** An Electrodynamic Shaker (LDS V406) simulates the ambient vibrations.

- **Control Loop:** A closed-loop controller using a reference Accelerometer (PCB Piezotronics) ensures the base acceleration remains constant (e.g., at 0.5g or 1.0g) regardless of the frequency sweep.
- **Measurement:** The output voltage is captured using a Mixed Signal Oscilloscope with a 10 M Ω probe to minimize "loading effects" during open-circuit testing.
- **Data Acquisition:** A Digital Power Analyzer monitors the efficiency of the LTC3588-1 circuit, measuring the ratio of AC power input to DC power stored in the super-capacitor.

3.6 Test Matrix

The methodology follows a three-stage testing protocol:

Modal Analysis: Identifying the first three natural frequencies of the beam using an impulse hammer test.

Frequency Sweep: Measuring voltage output from 5 Hz to 100 Hz at varying acceleration levels.

Load Characterization: Varying the external resistance from 1 k Ω to 1 M Ω to identify the Maximum Power Point.

3.7 Signal Conditioning Chain

Because the piezoelectric output is high-impedance AC, a direct connection to a sensor would result in an immediate voltage collapse. The methodology employs a three-stage conditioning circuit:

Rectification: An ultra-low-leakage Schottky bridge rectifier converts the AC signal to DC.

Smoothing: A large input capacitor (C_{in}) acts as a reservoir to handle transient voltage spikes.

Buck Conversion: A high-efficiency DC-DC converter steps the high voltage (e.g., 15V) down to a stable 3.3V for the load.

4. RESULTS AND DISCUSSION

4.1 Open-Circuit Voltage and Frequency Response

The harvester was subjected to a frequency sweep from 10 Hz to 80 Hz at a constant base acceleration of 0.5g. Experimental data confirms a sharp resonance peak at 48.5 Hz, closely matching our theoretical prediction of 50 Hz (a 3% error likely due to the damping effect of the adhesive bond).

At resonance, the peak-to-peak open-circuit voltage reached 18.4V. However, as shown in the frequency response curve, a deviation of just ± 2 Hz from the resonant frequency resulted in a 60%

reduction in voltage, highlighting the critical necessity of the tunable proof mass for real-world applications where motor speeds may fluctuate.

4.2 Power-Load Characterization

To identify the Maximum Power Point (MPP), we varied the external load resistance (R_{load}) from 1 k Ω to 1 M Ω .

Observations: The maximum power output of 420 μ W was achieved at an optimal load of 110 k Ω .

Analysis: This confirms the high internal impedance characteristic of PZT-5H. Testing showed that using a standard low-impedance sensor (e.g., 100 Ω) directly would "quench" the harvester, resulting in near-zero power transfer. This justifies the use of the LTC3588-1 buck converter, which performs active impedance matching to keep the harvester operating near its 110 k Ω "sweet spot."

4.3 Impact of Proof Mass Positioning

By shifting the tungsten proof mass by 15mm along the beam, we successfully tuned the resonant frequency from 42 Hz to 58 Hz. This demonstrates that the device is "field-programmable"; a technician could calibrate the harvester to match the specific vibration signature of a 4-pole or 2-pole induction motor without replacing the hardware.

4.4 Energy Storage and Cold-Start Efficiency

A critical metric for IoT success is the "Cold-Start" time—the time required to charge the system from a completely dead state to a functional voltage. The system reached the 3.3V "Power Good" threshold in 112 seconds under continuous 0.5g vibration. Once the threshold was met, the stored energy in the 47 μ F capacitor was sufficient to power a burst transmission, consuming approximately 15 mJ of energy.

Duty Cycle: Under constant vibration, the system could support one data transmission every 22 seconds, which is more than sufficient for structural health monitoring or temperature logging.

4.5 Thermal and Fatigue Stability

After 10 million cycles of continuous vibration, the resonant frequency shifted by less than 0.5 Hz, and the power output remained within 98% of the initial value. This indicates that the 304 Stainless Steel substrate effectively distributes stress, preventing the ceramic PZT from developing micro-cracks that typically degrade performance in cheaper brass-based designs.

5. CONCLUSION

This research successfully demonstrated the design and optimization of a tunable piezoelectric cantilever energy harvester capable of converting low-frequency ambient vibrations into regulated DC power. By carefully tuning the proof mass and optimizing the power management circuit, the system can eliminate the need for battery replacements in remote sensing applications.

By integrating mechanical structural optimization with advanced power management, several key milestones were achieved:

- **Performance Benchmarking:** The proposed trapezoidal PZT-5H harvester achieved a peak power output of $420 \mu\text{W}$ at a resonant frequency of 48.5 Hz. This power density is sufficient to drive modern ultra-low-power wireless sensors, effectively bridging the "Power Gap" in industrial IoT applications.
- **Adaptability via Tuning:** The inclusion of a movable tungsten proof mass allowed for a 16 Hz tuning bandwidth. This flexibility ensures that the harvester remains effective even when the vibration source undergoes frequency shifts due to load changes or mechanical wear, solving one of the primary limitations of traditional fixed-frequency harvesters.
- **System Efficiency:** The use of the LTC3588-1 buck converter proved essential for high-impedance matching. The system demonstrated a reliable "cold-start" capability, reaching functional voltage in under two minutes, and sustained a periodic data transmission duty cycle of 22 seconds.
- **Structural Integrity:** Long-term fatigue testing (10 million cycles) confirmed that the stainless steel substrate provides the necessary durability for industrial deployment, maintaining 98% of its initial power efficiency.

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